Frictions and adjustments in firm-to-firm trade

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Abstract

We study bilateral trade adjustments in a dynamic Ricardian model of trade with search frictions. The model generates an endogenous network of firm-to-firm trade relationships that displays price bargaining within and across firm-to-firm relationships. Following a foreign shock, firms sourcing inputs from the country have three options: Absorb the shock, renegotiate with their current supplier or switch to a supplier in another country. The relative importance of these adjustment margins depends on the interplay between the Ricardian comparative advantages, search frictions and firms' individual characteristics, including the history of the relationship. We exploit French firm-to-firm trade data to estimate the model structurally and quantify the relative importance of these adjustment margins in 26 sectors and 14 EU countries.

1 Introduction

How does bilateral trade adjust to relative price shocks, through which margins, and at what pace? We answer these questions in a dynamic Ricardian firm-to-firm trade model with search frictions. In our model, once a shock hit a foreign country, firms sourcing from this country have three options: fully absorb the shock, bargain with their current supplier, switch to a supplier in another country. The relative importance of these margins depends on the interplay between exporting Ricardian comparative advantages, search frictions in the destination market, and firms' individual characteristics. We exploit panel data on the universe of transactions involving French exporters and their European partners to estimate search frictions across markets and quantify the importance of these adjustment margins for the dynamics of individual and aggregate trade flows.

We study an environment in which firms face frictions when taking decisions on the sourcing of their inputs. More specifically, our model assumes random search in a dynamic Ricardian model of trade in intermediates. To produce, firms purchase an endogenous set of inputs at

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the lowest possible quality-adjusted price. Following Eaton and Kortum (2002), intermediate producers are heterogeneous in terms of their cost of serving a given destination, the interaction of technology and geography shaping comparative advantages. We add random search to the Ricardian structure. Search frictions capture the various contractual and informational frictions affecting the establishment of production linkages that the literature before us has discussed (see, e.g., Antras and Helpman, 2004, Grossman and Helpman, 2021).[1] Because input markets display bilateral search frictions, each producer chooses among a discrete number of quality-adjusted prices, that expands randomly when new input suppliers are met. Random search introduces a rich structure of competition in which input suppliers compete to retain the buyers they have met by adjusting their price dynamically. In equilibrium, more competitive suppliers tend to sell to a broader set of buyers. This property of the model helps reproducing the heterogeneity across firms in the buyer's margin that the literature on firm-to-firm trade has already documented (Bernard et al., 2014, Carballo et al., 2013, Lenoir et al., 2022).

In addition to the cross-sectional pattern, the model reproduces three empirical facts about the dynamics of firm-to-firm relationships.[2] First, exporters' portfolio of foreign customers increases over time, at a decreasing path. Upon entry into a foreign market, exporters keep on randomly matching with foreign buyers, which drives the growth of their portfolio. Because dowstream firms endogenously decide who to source their inputs from, each supplier converges towards a finite portfolio. Second, the hazard rate of a relationship is shown to be decreasing over the length of the relationship. In our model, such pattern arises from endogenous selection. In the early stages of a relationship, the probability that the buyer meets a lower quality-adjusted cost supplier is high. Over time, only the best relationships survive and the probability of a separation decreases. Third, within a firm-to-firm relationship, we find evidence of prices being renegotiated downwards, which the model explains by bargaining "on-the-match", when the downstream firm meets with competing input suppliers. Price adjustments following a switch are however equally likely to be positive or negative. To reproduce this pattern in the data, our model combines exogenous separations and (unobserved) quality differentiation.

When search frictions are high, an increase in the relative price of foreign suppliers is mostly passed onto their downstream partners, that can hardly find alternative, less expensive suppliers. As time goes by, buyers meet with new suppliers which allows them to renegotiate the price downward or to switch to a more competitive supplier located elsewhere. Whereas such competitive pressures help reduce the pass-through of the cost shock onto downstream firms, their strength varies depending on the magnitude of search frictions and foreign firms' comparative advantage. In markets in which foreign firms have a strong Ricardian comparative advantage, the pass-through of foreign shocks is high as a result of downstream firms' low bargaining power. Our model thus predicts rich adjustment patterns following foreign shocks,

[1] See Eaton et al. (2013) who also study the dynamics of trade in a search-and-matching model. Alternatively, most of the literature on trade dynamics explain adjustments to shocks using models that display sunk costs of entry and an option value of starting to export (see, e.g., Roberts and Tybout, 1997, Das et al., 2007, Ruhl and Willis, 2017, Alessandria et al., 2021b).

[2] These dynamics patterns are recovered from an exhaustive dataset of firm-to-firm transactions involving French exporters and their European partners, between 2002 and 2006. The dataset is described in details in Bergounhon et al. (2018). A cross-section of these data has been exploited recently in Lenoir et al. (2022) and Eaton et al. (2022).

depending on the strength of search frictions and its interaction with comparative advantages. Moreover, whereas the previous adjustment patterns hold true on average, the strength and margin of the adjustment varies across firms within a market, depending on the history of its matches. Young firms that have not yet accumulated a wide network of potential input suppliers are left badly equiped in terms of bargaining power to renegotiate with foreign partners. They are therefore more exposed to foreign cost shocks. The model can thus qualitatively explain the heterogeneity in trade adjustments across products, over time and across firms within a particular market, that the literature before us has documented.[3]

We investigate the quantitative performances of the model using a structural approach. To this aim, we first use transaction-level panel data to recover estimates of search frictions and Ricardian comparative advantages at the sector and country level. The estimation relies on unconditional inference and maximum likelihood. Identification is achieved using transaction and switch frequencies that we observe in the data for all European importers interacting with French exporters at a point in time. Intuitively, the rate at which importers switch from a French exporter to another, conditional on a transaction, is informative about the magnitude of the frictions. However, the mapping between the empirical moments and the structural parameters is complex due to unobserved quality-adjusted cost differences between French exporters and unobserved switches towards non-French exporters. Working with unconditional hazard rates in the model and the data solves the first issue. We further exploit the model's structure to take into account the endogenous censoring in the data and match it with observed trade shares. In the end, the simulated maximum likelihood estimator makes it possible to recover the relative size of search frictions faced by French exporters in each of their (European) export markets, together with estimates for the overall meeting rate and Ricardian advantage of French firms there.

We use the estimator together with transaction and bilateral trade data to estimate the structural parameters for 14 EU countries and 26 different sectors. Results reveal a substantial degree of heterogeneity in the magnitude of Ricardian comparative advantages and the level of search frictions, across countries and sectors. Over the 331 country-sector pairs that constitute our sample of estimated parameters, we find that search frictions explain as much as 42% of the observed variance in French firms' market shares. However, the importance of Ricardian comparative advantages increases when we focus on the heterogeneity across sectors within a destination, or across destinations within a sector. Point estimates suggest that relative search frictions faced by French firms in European markets are lower in neighboring countries and in sectors such as chemical products, motor vehicules, electrical products or beverages, i.e. in markets that constitute the core of France's international competitiveness. Interestingly, some of these strengths are in markets that are estimated to display high overall search frictions, such as beverages or the average Belgian market.

Armed with these structural parameters, we quantify the extent to which heterogeneous

[3]The literature on the heterogeneity of trade elasticities across sectors and products is vast, see Caliendo and Parro (2015) and Imbs and Mejean (2015), among many others. The dynamics of trade adjustments is also the topic of a large empirical literature, e.g. Boehm et al. (2020) for a recent contribution. Besides the sector-specific dynamics, the use of granular firm-level data reveals heterogeneity across firms in terms of their elasticity to a common shock (**?**Fitzgerald and Haller, 2014, Amiti et al., 2014, **?**, Garetto, 2016).

search frictions contribute to explaining the dynamics of firm- and product-level trade in the aftermath of relative price shocks. More specifically, we can simulate a relative price shock in the model and compare the implied dynamics to estimated trade elasticities. Preliminary results show that estimated search frictions explain 6% of the variance in elasticities estimated using a local projection method applied to the trade data aggregated at the product and quarterly level.[4] This number increases to 40% once we control for the heterogeneity in the initial level of French firms' market shares. The elasticity of bilateral trade to relative price shocks is found systematically larger in markets in which the relative meeting rate of French firms is estimated lower, conditional on French firms initial market share.

Related literature: Our paper contributes to the burgeoning literature on firm-to-firm trade in international markets, and the more established literature on the dynamics adjustments of trade flows. Like several recent contributions, we examine firm-to-firm trade in the context of a Ricardian model with search frictions (Eaton et al., 2022, Lenoir et al., 2022, Chor and Ma, 2020).[5] We incorporate dynamics in this type of setting and propose a richer view of firm pricing strategies. Indeed, firms in the model bargain over prices on the match and charge non-constant markups. In this respect, our work is related to Kikkawa et al. (2019) and Alviarez et al. (2021) who also study pricing in a (static) model of firm-to-firm relationships, both theoretically and empirically. Unlike these papers, endogenous markups in our model arise from frictions in a Ricardian setting. Furthermore, our focus is on export values rather than prices, and our framework allows us to examine the dynamic adjustment of exports, a dimension that is absent from these two papers.

In doing so, we also contribute to the theoretical and empirical literature on the dynamics of trade flows recently reviewed in Alessandria et al. (2021a).[6] Most of this literature explains the dynamics of trade at the intensive and extensive margins using models that embody firm-level heterogeneity, uncertainty about the determinants of future profits and market entry costs. These extensive adjustments include firms' net entry into export and/or into additional export destinations, as well as adjustments at the product or customer margins (Kehoe and Ruhl, 2013, Alessandria and Choi, 2014, Bricongne et al., 2012, Fitzgerald et al., 2016). A strand of the literature also emphasizes customer accumulation as a source of firms' dynamics (Arkolakis, 2010, Drozd and Nosal, 2012, Gourio and Rudanko, 2014, Fitzgerald et al., 2017, Piveteau, 2020). Here as well, increasing fixed costs associated with serving a larger customer base are used to explain the heterogeneity observed into the data. Our model does not display any sunk cost. We instead emphasize the role of search frictions, and we add the firm-to-firm dimension to this literature. Finally, our estimation strategy borrows from the literature in labor, most notably Bagger et al. (2014). We exploit the structure of the model to identify search frictions for each product and country, using the observed transitions of importers in

[4]Since the data are for the EU, we use real exchange rates as our price shifter.

[5]Firm-to-firm trade has also been analyzed in monopolistic competition settings without search or matching frictions (Bernard et al., 2018, Carballo et al., 2018). See Bernard and Moxnes (2018) for a review.

[6]See, among many others, Baldwin and Krugman (1989), Roberts and Tybout (1997), Das et al. (2007), Arkolakis (2010), Nguyen (2012), Eaton et al. (2013), Impullitti et al. (2013), Ruhl and Willis (2017), Lim (2018), Berman et al. (2019),Alessandria and Choi (2019), Alessandria et al. (2021b).

and out of relationships with French firms.

The rest of the paper is organized as follows. Section 2 presents the data and documents new facts on firm-to-firm trade dynamics. Section 3 sets up the model whereas Section 4 discusses the estimation. In Section 5 we discuss the extent to which the estimated model can explain the dynamics of trade following relative price shocks observed in the data. Finally, Section 6 concludes.

2 Data and stylized facts

2.1 Data

The dataset is provided to us by the French customs and covers each single export transaction involving a French firm and one of its partners in the European Union. Importantly, these data identify the French exporter by its siren number *and* the European importer, identified by an anonymized VAT number that includes the iso2 code for the buyer's country of origin i. French exporters will be used to identify the sellers s in the model developed in next section and the European buyers will be the empirical counterpart of the buyers b. The transaction is also characterized by a product category p at the 8-digit level of the combined nomenclature and the date t of the transaction identified by a particular month within a year. Whereas the dataset is exhaustive, the customs form is simplified below a threshold defined over annual exports in the European Union. Unfortunately, the product category which is heavily used in the estimation is missing for firms below the threshold and we thus chose the estimation period so as to minimize the associated attrition.[7] Finally, we observe the value of the transaction and the quantity exported, which we use to compute the unit value of each transaction, our proxy for prices.[8]

The analysis covers the 2002-2006 period which does not incur any substantial change in the combined nomenclature, nor in the declaration rules for exports. The sample is further restricted to the 14 members of the European Union. Product codes affected by yearly changes in the combined nomenclature are harmonized over time using the algorithm proposed by Behrens et al. (2019). As the raw data goes back to 1995, we can use the pre-sample period to control for censoring. In this case, the matching of firm-to-firm relationships in- and out-of-sample is based on hs4 products which definition is invariant over time.

In the rest of the analysis, the focus is on European importers, and their interactions with French sellers. We follow the history of transactions involving a particular buyer b for a specific product p and various French sellers s, over time. The model explains the decision to purchase goods from a particular seller in the context of frictional good markets whereby importers are

[7] Before 2011, the declaration threshold was set at 150,000 euros. Since 2011, it has more than doubled, at 460,000 euros.

[8] For the vast majority of products, the quantity is declared in kilograms. For some particular 8-digit products, the customs ask firms to declare the quantity of exports in some identified physical units (e.g. liters for wine, number of units for living animals, number of carats for diamonds, etc), sometimes complemented with the weight of the merchandise. We use the physical quantity whenever available and the weight of the merchandises elsewhere. Using different units across products is innocuous, as our analysis always controls for product fixed effects.

willing to purchase a particular input p, to French or other producers. In this model, input purchases cannot be intermediated through wholesalers. We thus exclude from the analysis all transactions that involve a wholesaler, whether on the export or on the import side. Appendix A explains how we identify these wholesalers and how much they contribute to aggregate trade.

Table A1 in Appendix shows statistics on the dimensionality of the estimation sample. After having dropped intermediaries, the data covers a total of 27 million transactions that involve almost fourty thousands French exporters and 744 thousands European importers. In the rest of the analysis, we define a "relationship" as the set of transactions involving a particular pair of firms interacting over a specific product. There are 5.6 million such relationships in the estimation sample, that we thus see interacting over five transactions, on average. Of course, the intensity of these relationships is strongly heterogeneous, with a number of relationships being short-lived whereas other induce a large number of subsequent transactions. Our analysis mostly exploits this heterogeneity to discuss the dynamics of firm-to-firm trade relationships.

2.2 Five stylized facts on firm-to-firm relationships

2.2.1 Cross-sectional connectedness

Previous papers using similar data on firm-to-firm relationships have documented a strong degree of heterogeneity in firms' in- and out-degrees, i.e. in the number of partners an exporter sells to as well as the number of exporters an importer is connected to (Carballo et al., 2013, Bernard et al., 2014, Lenoir et al., 2022). In our setting, the degree of connectedness varies depending on whether we focus on a single cross-section, i.e. a subset of transactions observed over a particular month, or if we cumulate relationships over time. Results are also different when conditioning on a particular product being traded or if we cumulate statistics across products within a firm. We show and discuss insights recovered from cumulated distributions of firms' in- and out-degrees in Appendix A.2. The conclusion of the analysis is that the vast majority of importers (more than 95%) interact with a single French firm over a particular month and product. The distribution of importers' indegrees is however shifted down when their partners are cumulated over time, which is indicative of importers switching across French exporters, over time. Moreover, around 25% of importers source multiple inputs from France, at a point in time. At the other side of the graph, a substantial share of French exporters serve more than one foreign buyer at a point in time. Within a particular destination, 28% of French exporters serve at least two partners with the same product over a particular month. When we sum across destinations, the proportion increases to almost 60% among which 10% interact with more than ten European importers over a particular month. These firms are large on average, thus cumulating almost 50% of French exports.[9]

This leads us to the first of our motivating stylized facts:

[9]The heterogeneity in exporters' ability to serve a large number of foreign partners is explained in the model by the interaction of exporters' productivity heterogeneity and the history of their matches with foreign firms. From that point-of-view, we follow the recent literature on matching in international good markets (Lenoir et al., 2022, Eaton et al., 2022). The same data pattern can also be rationalized using models of two-way heterogeneity as discussed in Carballo et al. (2013) and Bernard et al. (2014).

Fact 1 *In the cross-section of transactions observed over a particular product, the graph of firm-to-firm transactions displays many-to-one matching: Importers tend to interact with a single input supplier whereas intermediate producers often serve several buyers simultaneously. Importers however switch across suppliers over time. Moreover, importers can source several inputs simultaneously. Finally, large exporters serve more buyers, on average.*

2.2.2 Exporters' accumulation of buyers

The above cross-sectional structure of firm-to-firm networks has been discussed in the previous literature. Less studied is the dynamics observed in the panel dimension of these data. We now turn to this dimension, starting with sellers' accumulation of buyers, over time. These statistics echo the literature on export dynamics, which studies the evolution of a firm's exports, posterior to entry into a destination (Fitzgerald et al., 2016, for instance). In comparison with this literature, we are able to further dig into the structure of a firm's export portfolio, over time. As illustrated in Figure A2, exporters indeed display an heterogeneous number of foreign partners in their portfolio. Using the time-dimension of the data, we can investigate the extent to which the heterogeneity in part reflects firms' accumulation of partners, over time.

To this aim, we estimate the following equation:

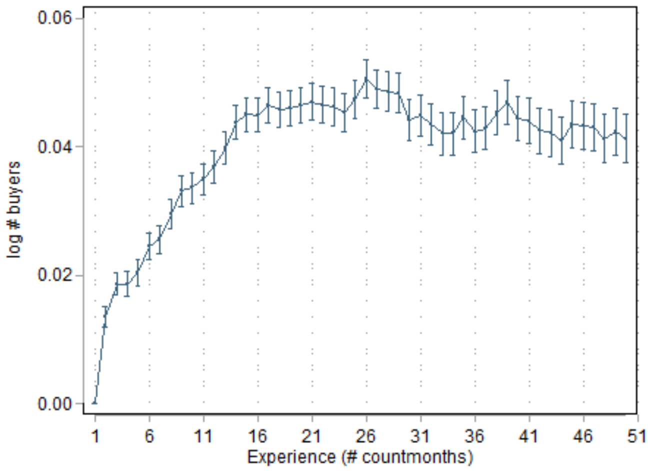
$$\ln n_{spit} = FE_{spi} + FE_t + \sum_{k=2}^{K} \alpha_k \mathbb{1}(Experience_{spit} = k) + \varepsilon_{bpst} \tag{1}$$

where n_{spit} is the number of buyers from country i served by seller s with product p at time t. FE_{spi} and FE_t are individual and time fixed effects, respectively. $\mathbb{1}(Experience_{spit} = k)$ is a dummy variable equal to one if the seller's experience in country i is equal to k. We tested with two alternative definitions of a seller's experience, either the number of periods or the number of export transactions since first entry into the destination.[10] This specification allows to vizualize the mean growth of sellers' stock of buyers, over time, controlling for unobserved heterogeneity across sellers and periods. The α_k coefficient can be interpreted as the (log of the) number of clients a seller with experience k has on average at time t in destination i, normalized by the number of its clients at entry.

Results are shown in Figure 1 and reveal a clear positive correlation between a firm's experience in a destination and the number of clients it serves there. The relationship is concave meaning that the accumulation of buyers is especially strong in the early stages of the firm's export experience. After 6 months in the market, the number of clients served has increased by about 2.3%. After two years, the number of clients served is on average 4.5% larger than at entry. Whereas these numbers may seem small in light of the average number of partners served by a firm, it needs to be noted that the regression is affected by composition effects. Not all firms remain active over two consecutive years and those that do tend to be the largest

[10] We control for censoring by using information prior to the estimation period to recover the full history of a firm's experience into a destination. As a consequence, exporters do not necessarily enter the estimation sample used to recover the coefficients of equation (1) with an experience of one. Figure 1 defines experience in terms of the number of periods since first entry into the destination and Figure A3 reproduces the same picture using the cumulated number of transactions.

Figure 1: Acquisition of buyers, over time



Note: The figure shows the evolution of a seller's stock of buyers, over time, recovered from equation (1). The figure reports the estimates and their 95% confidence intervals. Experience is measured by the number of periods since first entry.

ones. Since their number of clients at entry is already larger than the average, the 4.5% growth is significant for these firms. This leads us to the second stylized fact.

Fact 2 *Posterior to entry into a destination, a seller's portfolio of clients tends to grow over time, at a decreasing rate.*

2.2.3 Dynamics of firm-to-firm relationships

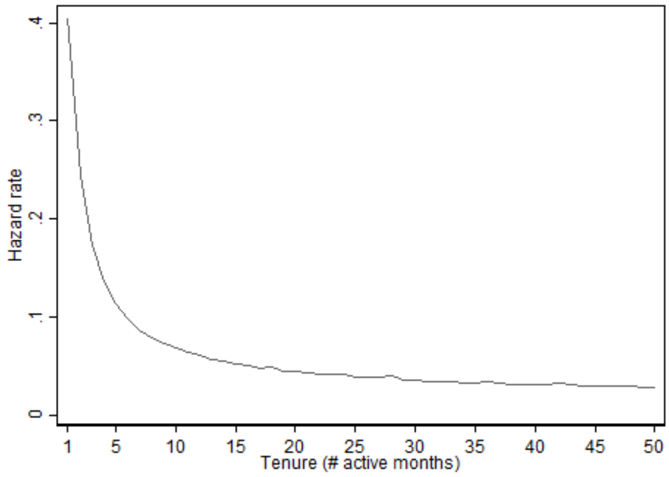
We next focus on the likelihood that a particular firm-to-firm relationship ends. Our data indeed displays significant heterogeneity in the duration of relationships, with 60% of the relationships that do not survive the first transaction whereas a significant number are long-lasting. We now use the dynamics within a firm-to-firm relationship to measure how the probability of a relationship ending evolves over the curse of the relation.

More specifically, Figure 2 illustrates the evolution of hazard rates, over the history of a relationship. The hazard rate is defined as the probability that a relationship ends after x months, conditional on the relationship having survived until there.[11] The declining pattern is consistent with the survival rate increasing over the curse of a relationship. The dynamics is especially strong during the first year of the relationship, whereas the probability of the relationship ending stabilizes after 1.5 to 2 years, around 4%. This leads us to the third of our stylized facts.

Fact 3 *The probability of the relationship ending declines over the length of a firm-to-firm relationship, before stabilizing after 18 months, on average.*

[11] Here as well, left-censoring is controlled for using data prior to 2002 to recover the full length of a relationship. In figure A4, we reproduce the graph using the number of cumulated transactions in the relationship to measure tenure.

Figure 2: Hazard rate, over time



Notes: The hazard rate is defined as the probability of the relationship ending, conditional on tenure into the relationship and is calculated as the ratio of the density to the survival rate at tenure k. The figure is recovered from the 2002-2006 sample using the cumulated number of periods in the relationship as measure of tenure.

2.2.4 Price dynamics within and across relationships

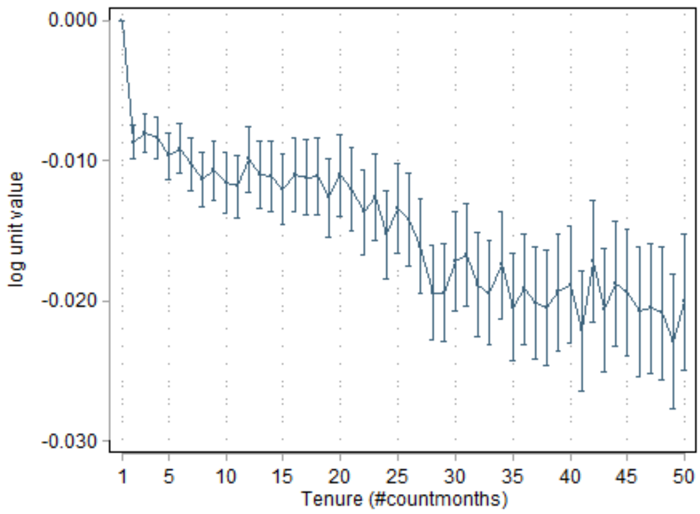
The last set of stylized facts concerns the dynamics of unit values, over time. Since we observe the unit value at the level of each firm-to-firm transaction, it is possible to measure the extent to which export prices change over the curse of a relationship, and in case of a switch. To this aim, we first estimate the following equation:

$$\ln p_{bpst} = FE_{bps} + FE_{ipt} + \sum_{k=2}^{K} \alpha_k \mathbb{1}(Tenure_{bpst} = k) + \varepsilon_{bpst} \tag{2}$$

where p_{bpst} is the unit value set on the transaction involving exporter s, product p, importer b that occurs at time t. The presence of relationship-specific fixed effects (FE_{bps}) implies that the identification of other coefficients is within a firm-to-firm relationship. The baseline regression also controls for country×product×period fixed effects (FE_{ipt}) to account for destination-specific inflation trends. The coefficients of interest are the α_k coefficients that measure the average price change after a tenure of k.

Results are reported in Figure 3. They show a negative trend in prices, at least over the first two years. The relationship becomes fuzzier over long tenures due to the small number of long relationships. But the price decline seems to persist. Note that the rate at which prices decline is moderate. After a year, prices are on average 1.2% lower than in the initial transaction. After 3 years, they are 2% lower. Finally, we show in Figure A6 that the same pattern is observed if we estimate the relationships separately on short and long tenures. On the other hand, the dynamics in the value of exports does not show any clear pattern over time. Whereas the value of the transaction seems to increase consistently between the first and the second transaction,

Figure 3: Price dynamics, within a firm-to-firm relationship



Note: This figure shows the evolution of prices within a firm-to-firm relationship. Coefficients are recovered from equation (2). The figure reports the estimates and their 95% confidence intervals. Tenure is measured by the number of periods since the beginning of the relationship.

the dynamics after the second transaction is either relatively stable or decreasing.[12] This leads us to our fourth stylized fact.

Fact 4 *Within a firm-to-firm relationship, prices tend to decrease over time.*

While the price decline within a firm-to-firm relationship is statistically significant, the behaviour of prices following a switch is far less clear. This is illustrated in Figure 4 which shows the kernel density of price changes, conditional on a switch. Namely, we compute the price growth between the last transaction within a firm-to-firm relationship and the next transaction involving the same buyer and product but a different seller.[13] The density is centered around zero with a mean at .006 and a median at 0. This means that a firm's switching to a new partner is equally likely to incure a drop than a raise in the unit value it pays for the same good. This leads us to the fifth and final stylized fact.

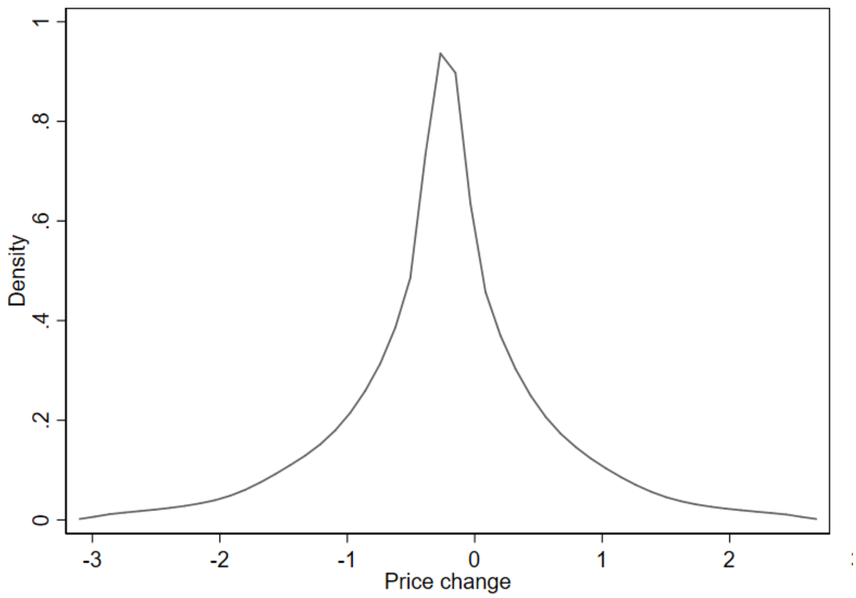
Fact 5 *After a switch, the unit value paid by the importer is equally likely to increase or decrease.*

In the next section, we develop a dynamic search model that reproduces the main stylized facts just described.

[12]The jump observed between the first and the second transaction may be indicative of a form of partial month effect or a learning phenomenon whereby importers first test the seller over relatively small quantities, before establishing a more stable relationship over larger quantities. Given the dynamics does not persist after the second transaction, we do not seek to model it afterwards.

[13]Here a switch denotes a situation in which we observe the same importer interacting with different French exporters. In the model, a switch will also designate a situation in which an importer terminates a relationship to start purchasing its input from a firm located in a different country.

Figure 4: Price changes, conditional on a switch



Note: This figure shows the kernel density of price changes, conditional on a switch. Price changes are computed in log differences, between the last transaction within a seller-buyer-product pair and the next transaction involving the same buyer×product but a different seller.

3 A Search Model of the International Markets

Our model pictures an environment where heterogeneous sellers are matched randomly on the international markets with buyers of intermediate inputs. The sellers compete to retain the buyers by changing prices over time. Moreover, intermediate inputs are of different qualities and competition hinges on the price-to-quality ratio rather than the input price only.

3.1 The demand for intermediate goods

In each country i, final good producers produce using inputs bought to intermediate good producers. Their production function is assumed CES and inputs are vertically differentiated. As it will be apparent in the next subsection, the quality chosen by the final good producer for each input and the number of inputs depend on the producer's network. We denote M_b the (endogenous) number of inputs used by producer b, q_j the quality of input j and p_j the price of that input. The producer chooses the quantity x_j of each intermediate input to minimize its costs, given the quantity x_b to be produced, which is taken as exogenous.[14]

[14] We discuss in Appendix C how this partial equilibrium model of trade in intermediates can be integrated into a general equilibrium structure whereby the demand expressed by final good producers is determined by their relative price, in a continuum of horizontally differentiated varieties.

$$\begin{cases} \min_{x_j} \sum_{j=1}^{M_b} p_j x_j \\ \text{s.t.} \\ x_b = \left(\sum_{j=1}^{M_b} (q_j x_j)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \end{cases}$$

The solution of this program gives the demand addressed to each input provider, as a function of its quality-adjusted price and the firm's network of input suppliers:

$$p_j x_j = x_b \left(\frac{p_j}{q_j} \right)^{1-\eta} \left(\sum_{j=1}^{M_b} \left(\frac{p_j}{q_j} \right)^{1-\eta} \right)^{\frac{\eta}{1-\eta}} \tag{3}$$

and the final good producer's marginal cost of production:

$$mc_b = \left(\sum_{j=1}^{M_b} \left(\frac{p_j}{q_j} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{4}$$

From this, it becomes clear that it is optimal for the final good producer to choose, for each intermediate input, the seller offering the lowest price adjusted for quality p_j/q_j, to minimize its marginal cost of production. Conditional on a chosen seller, the value of the transaction depends on the demand shifter which is specific to the buyer, on the price-quality ratio offered by the seller and on the buyer's marginal cost of production. x_b and mc_b are buyer-specific and vary over time. As explained below, $\frac{p_j}{q_j}$ is seller specific and varies over time, both within a seller-buyer match and when the buyer switches to a new input provider.

3.2 A model of heterogeneous input suppliers

To understand the dynamics of input purchases, we need to describe both the matching process between the final good producers and the intermediate good producers, and how the prices are set. Buyers can be located in different countries and so do the sellers. For the sake of clarity and following the limitation of our data, we focus here on sellers located in France (country F) and, even if it doesn't change the exposition of the model, it is useful to keep in mind that we only observe buyers in the EU. Finally, we assume that search occurs (simultaneously) on as many separate markets as there are input types. What follows describes the output of the search and matching process for a given type j. To alleviate notations, we no longer specify the type of inputs although all parameters in this subsection need to be understood as being input-specific. All coefficients that are indexed by F are further assumed to be heterogeneous across producing countries.

Intermediate good producers produce with a constant-return-to-scale technology and face iceberg transportation costs. They differ in terms of their productivity, noted e, the quality q of their input and the cost of the input bundle, which is assumed country-specific. The unit cost of serving market i for a French firm of productivity e reads:

$$\frac{\nu_F d_{iF}}{e}$$

where d_{iF} is the bilateral iceberg cost, ν_F a unit cost shifter that determines how France compares with respect to other countries in terms of producing costs. As it will be clear when we consider how buyers choose between competing sellers, quality-adjusted productivities and quality-adjusted serving costs are the key variables. We thus define the quality-adjusted serving cost $c_{iF}(e, q)$, as,

$$c_{iF}(e, q) = \frac{\nu_F d_{iF}}{e\ q} \equiv \frac{\nu_F d_{iF}}{z} = c_{iF}(z)$$

When a buyer and a seller meet, the quality-adjusted productivity, $z = e \times q$, is drawn in a sampling distribution $F(z)$, with support on $[\underline{z}, +\infty]$. We follow Eaton and Kortum (2002) by assuming that the measure of firms in France with efficiency above z reads

$$\mu_F^Z(z) = T_F z^{-\theta}$$

which implies that $F(z) = 1 - (z/\underline{z})^{-\theta}$ and that the total measure of sellers in France (noted S_F) is $T_F \underline{z}^{-\theta}$. Hence, whenever a French seller and a buyer from i are matched, the probability that the serving cost is below c reads

$$F_{iF}(c) = 1 - F\left(\nu_F d_{iF}/c\right) = \bar{F}\left(\nu_F d_{iF}/c\right) \tag{5}$$

Symmetrically, we define the probability that a buyer from i meets with a seller from another country ζ that offers a cost below c as $F_{i\zeta}(c) = \bar{F}\left(\nu_\zeta d_{i\zeta}/c\right)$.

3.3 Matching and pricing on the intermediate good market

Buyer-seller matching. In our framework, the buyers buy intermediate goods of different qualities to produce the final goods. Under the CES assumptions, any buyer-seller match is potentially profitable. There are B_i buyers in country i. A buyer exits the market at exogenous rate μ and is replaced by a new buyer. New buyers start unmatched but, over time, they enter in contact with multiple sellers and maintain links. A buyer meets with French sellers at rate γ_{iF} and with sellers from other countries at a rate of $\gamma_{i\bar{F}}$ which is the sum of the non-French meeting rates.

Borrowing from Postel-Vinay and Robin, 2002's model of the labor market, we assume that sellers Bertrand compete to supply goods to buyers and that there is no collusion between suppliers. We further assume that buyers have always the option to recall one of their previous sellers and that there is no commitment beyond the current transaction. Both are important assumptions that simplify the price setting.[15] Assuming a buyer with n potential sellers, we index these sellers by their quality-adjusted serving cost : $c_1 \leq c_2 \leq ... \leq c_n$.

[15]Namely, the wage equation in Postel-Vinay and Robin (2002) is affected by intertemporal considerations that are absent from our setting. The reason is that workers cannot recall previous employers whereas buyers can recall previously met input suppliers.

Price dynamics. Among all the possible sellers known by the buyer for a given input, the best supplier is the one able to serve with the lowest quality-adjusted price that minimizes the buyer's marginal cost (4). Consider the two sellers with the lowest quality-adjusted serving cost $c_1 \leq c_2$, which respective qualities are denoted q_1 and q_2. The best supplier is able to set the price p such that the buyer is indifferent between buying the good from her or from the other seller when that seller offers its best quality-adjusted price c_2:

$$p\left(q_1, c_2\right) = q_1 c_2$$

The price setting mechanism thus pushes the quality-adjusted price (p/q) to be equal to the quality-adjusted cost of the second-best supplier (c_2).

Prices can be renegociated over time as buyers meet with new, potentially more productive, sellers. Consider that the buyer matches with a new seller with quality-adjusted serving cost c'. We can distinguish three cases. First, if the new cost is *below* c_1, the next transaction (if there is no new match before) will be with this new seller at price

$$p\left(q', c_1\right) = q' c_1$$

Interestingly, that price can be above or below the previous price if the quality of the good offered by the new seller is high enough. This is consistent with evidence in Figure 4: Conditional price changes are equally likely to be positive or negative. However the quality-adjusted price p/q is always lower.[16] Second possibility, the new seller has a quality-adjusted cost above the second best, $c' \geq c_2$ and, in that case, nothing happens: the next transaction will be with the incumbent supplier at the same price.

Last case, the new seller has a quality-adjusted cost in between the best supplier and the last second best: $c_1 \leq c' < c_2$. In that case, the next transaction will be with the same supplier but at a lower price because the incumbent supplier has to match the utility level that the new supplier could provide:

$$p\left(q_1, c'\right) = q_1 c'$$

Hence, since $c' < c_2$, the new price offered by supplier 1 will be lower. Our model thus predicts that *within* a buyer-seller relationship, the price tends to decrease, as confirmed by evidence in Figure 3.

3.4 Distributions and shares

The distribution of suppliers. The overall meeting rate for a buyer, noted γ_i, is the addition of the rates at which it meets French and non-French suppliers, that is $\gamma_i = \gamma_{iF} + \gamma_{i\bar{F}}$.

[16] Although testing this prediction of the model is tricky as quality-adjusted prices are not observed, Figure A7 in the appendix presents suggestive evidence that are consistent with the prediction. Namely, we proxy a firm's position in the quality-adjusted price distribution by the individual fixed effect recovered from the estimation of equation (1). In theory, differences in the mean ability of input suppliers to accumulate a large number of buyers in a destination reflect their competitiveness there. In a second stage, we regress this proxy on buyer-specific fixed effect and a dummy for the rank of the seller in the sequence of the buyer's French partners. As expected, we observe buyers climbing the distribution of sellers' attributes, over time.

When a buyer is new to a given input market she meets her first supplier at that rate and buyers exit the market at an exogenous rate μ. To ensure steady state, we assume that when a buyer exits, she is replaced by an unmatched buyer. Hence, the share u_i of buyers that are unmatched satisfies $B_i u_i \gamma_i = B_i(1-u_i)\mu$ and thus

$$u_i = \frac{\mu}{\gamma_i + \mu}$$

After the first match, buyers keep on searching for new suppliers. The overall quality-adjusted serving cost distribution is a mixture of the country-specific ones, noted $F_i(c)$, with

$$F_i(c) = \frac{\gamma_{iF}}{\gamma_i} F_{iF}(c) + \frac{\gamma_{i\bar{F}}}{\gamma_i} F_{i\bar{F}}(c) \tag{6}$$

Before looking at how buyers are distributed among sellers, it is useful to remark a useful relationship between the distributions of French and non-French suppliers:

$$F_{iF}(c) \equiv \left(\frac{\nu_F d_{iF}}{c\underline{z}}\right)^{-\theta} = \tau_{iF}^{-\theta} F_{i\bar{F}}(c)$$

where $\tau_{iF} \equiv \left(\frac{\nu_F d_{iF}}{\nu_{\bar{F}} d_{i\bar{F}}}\right)$ measures the comparative advantage of foreign suppliers over French firms. The distribution of French costs is a translation of the distribution for the other countries with $\tau_{iF}^{-\theta}$ being the cost shifter.[17]

Armed with this model, it now becomes possible to derive the distribution of costs faced by final producers in country i. We denote $L_i(c)$ its cumulated distribution function and $\ell_i(c)$ its probability density function. As long as buyers always choose the lowest cost supplier that they have met, $\ell_i(c)$ satisfies

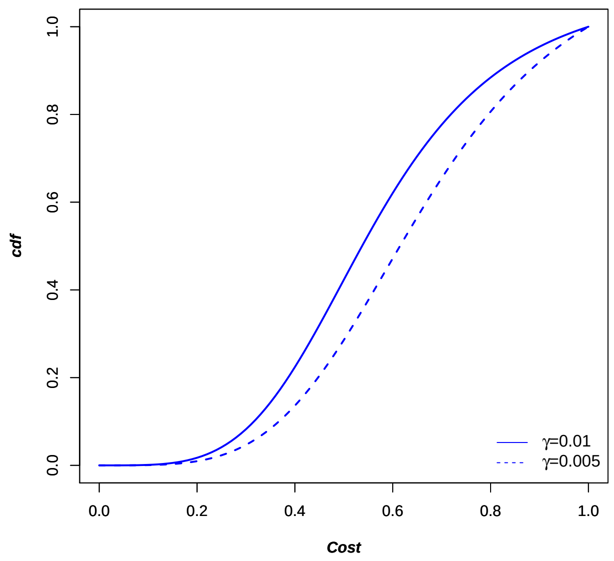
$$\underbrace{B_i(1-u_i)\ell_i(c)\left(\mu + \gamma_i F_i(c)\right)}_{\text{outflows}} = \underbrace{B_i(1-u_i)\gamma_i \bar{L}_i(c) f_i(c) + B u_i \gamma_i f_i(c)}_{\text{inflows}}$$

with $\bar{L}_i(c) \equiv 1 - L_i(c)$. The outflows are the sum of buyers exiting the market (μ) and buyers switching when they meet with a lower quality-adjusted cost supplier ($\gamma_i F_i(c)$). The inflows correspond to unmatched buyers meeting a cost-c supplier ($\gamma_i f_i(c)$) and buyers previously matched with sellers of serving cost higher than c ($\gamma_i \bar{L}_i(c) f_i(c)$). Simplifying and integrating by part, one gets

$$L_i(c) = \frac{\mu + \gamma_i}{\mu + \gamma_i F_i(c)} F_i(c) \tag{7}$$

[17] Remember that the maximum serving cost for France is $\nu_F d_{iF}/\underline{z}$ and $\nu_{\bar{F}} d_{i\bar{F}}/\underline{z}$ for the other countries.

Figure 5: Cumulated distribution of the costs paid by individual buyers as a function of meeting frictions



Notes: The figure illustrates the simulated cdf of input costs paid by buyers in a particular market, for two arbitrary values for the meeting probabilities.

The distribution of buyers among sellers hinges on the distribution of matches, $F_i(c)$, but also on the meeting frictions that slow down reallocations, hence the efficiency of the market. The relationship between search frictions and the efficiency of the matching process is illustrated in Figure 5 which shows the distribution of input costs, conditional on matches, for two values of meeting probabilities. Decreasing the meeting rate pushes the whole distribution of input costs to the right, i.e. lower meeting rates increase the mean cost of inputs for buyers.

The distribution of buyers among French suppliers. Consider the shares of buyers matched with a French seller, π_{iF}. Again, in equilibrium, flows in and out are balanced such that the density of buyers matched with a French-seller at cost c, noted $\ell_{iF}(c)$, satisfies

$$\underbrace{(1-u_i)\pi_{iF}\ell_{iF}(c)\,(\mu+\gamma_i F_i(c))}_{\text{outflows}} = \underbrace{u_i\gamma_{iF}f_{iF}(c) + (1-u_i)\bar{L}_i(c)\gamma_{iF}f_{iF}(c)}_{\text{inflows}} \tag{8}$$

Substituting $\bar{L}_i(c) = 1 - L_i(c)$ by its expression in equation (23) and using $u_i = \mu/(\mu+\gamma_i)$, one gets

$$\pi_{iF}\ell_{iF}(c) = \frac{\gamma_{iF}}{\gamma_i}\frac{\mu(\mu+\gamma_i)}{(\mu+\gamma_i F_i(c))^2} f_{iF}(c) \tag{9}$$

and similarly if we consider the density of buyers matched with non-French sellers

$$(1-\pi_{iF})\ell_{i\bar{F}}(c) = \frac{\gamma_{i\bar{F}}}{\gamma_i}\frac{\mu(\mu+\gamma_i)}{(\mu+\gamma_i F_i(c))^2} f_{i\bar{F}}(c) \tag{10}$$

Remember that $f_{iF}(c)/f_{i\bar{F}}(c) = \tau_{iF}^{-\theta}\ \forall c$. Equations (9) and (10) thus imply that

$$(1-\pi_{iF})\tau_{iF}^{-\theta}\gamma_{iF}\ell_{i\bar{F}}(c) = \pi_{iF}\gamma_{i\bar{F}}\ell_{iF}(c) \tag{11}$$

From that, we obtain by integration the share of buyers matched with a French seller, when μ is sufficiently close to zero:

$$\pi_{iF} = \frac{\gamma_{iF}}{\gamma_{iF} + \gamma_{i\bar{F}}\tau_{iF}^{\theta}} \tag{12}$$

Analytical details together with the formulas recovered in the general case when μ can take any value are provided in Appendix C.2. The interpretation of equation (12) discussed below goes through in the general case.

Two forces shape the market share of French suppliers, the strength of matching frictions and Ricardian comparative advantages. The ratio of γ_{iF} over $\gamma_{i\bar{F}}$ tells how easy it is for a buyer to meet a French supplier in comparison with non-French suppliers. τ_{iF} instead reflects French suppliers' competitiveness, conditional on a match. Both an increase in γ_{iF} over $\gamma_{i\bar{F}}$ and a decrease in τ_{iF} improve the likelihood that a French supplier serves market i. As already discussed in Lenoir et al. (2022), introducing search frictions in a Ricardian model of trade helps refine our understanding of the geography of trade. As in Eaton and Kortum (2002), the interaction of technology and geography reflected in τ_{iF} shapes the Ricardian advantage of French firms in market i. In comparison with the frictionless world in Eaton and Kortum (2002), heterogeneity in bilateral search frictions however distorts trade in favor of relatively low search / high meeting rates countries.

Using (12) together with (9), we finally derive the distribution of buyers among French sellers

$$\ell_{iF}(c) = \frac{\gamma_{iF} + \gamma_{i\bar{F}}\tau_{iF}^{\theta}}{\gamma_i}\frac{\mu(\mu+\gamma_i)}{\left(\mu+\gamma_i F_i(c)\right)^2} f_{iF}(c) \tag{13}$$

Using the expression for $\ell_{iF}(c)$ and the fact that $f_{iF}(c)/f_{i\bar{F}}(c) = \tau_{iF}^{-\theta}$, one can show that

$$\ell_{iF}(c) = \ell_i(c) \tag{14}$$

which means that buyers are identically distributed in terms of serving costs whatever the origin of their suppliers. As discussed in appendix C.2, the invariance of serving costs across origin countries, conditional on a match, also implies that the expression for π_{iF} in equation (12) defines the share of country i's absorption that is sourced from France. As in Eaton and Kortum (2002), the geography of bilateral trade flows is entirely summarized by the probability that a buyer in i ends up purchasing inputs from France.[18]

[18]Using this result, we can use equation (12) to define $\tau_{iF}^{-\theta}$ as a function of the meeting rates and the *observed* shares

$$\tau_{iF}^{-\theta} = \frac{\pi_{iF}}{(1-\pi_{iF})}\frac{\gamma_{i\bar{F}}}{\gamma_{iF}} \tag{15}$$

This will be used in the estimation.

3.5 Buyer acquisition on the seller's side.

The stylized facts in Section 2.2 characterize three things, how prices change as buyers move between sellers, how they change as the buyer-seller relationship last, and how sellers acquire buyers over time. We now focus on the acquisition of buyers.

Buyer acquisition. Over time, within a product category, French sellers meet with buyers from i at rate λ_{iF} and consistency implies: $\gamma_{iF}B_i = \lambda_{iF}S_F$. Consider now a French seller that can serve market i at quality-adjusted cost c. Its number of buyers, noted $n_i(c)$, evolves as new profitable links are formed and old links are dissolved when a buyer exit (rate μ) or meets a seller with a lower quality adjusted serving cost. $n_i(c)$ dynamics thus follows

$$\dot{n}_{it}(c) = -n_{it}(c)(\mu + \gamma_i F_i(c)) + \lambda_{iF}\left((1-u_i)\bar{L}_i(c) + u_i\right)$$

with $n_{i0} = 0$. Hence, given equation (7), the expected number of buyers in period t has a simple solution

$$n_{it}(c) = \frac{\lambda_{iF}\mu}{(\mu + \gamma_i F_i(c))^2}\left(1 - e^{-(\mu+\gamma_i F_i(c))t}\right)$$

The expected number of buyers is thus increasing over time as it converges towards a steady state[19]

$$n_i(c) = \frac{\lambda_{iF}\mu}{(\mu + \gamma_i F_i(c))^2} \tag{16}$$

This implies that the number of buyers served is higher for sellers with lower serving-costs. Moreover, before reaching steady state, they grow at higher pace since they retain current buyers and find new buyers to serve more easily (see equation (3.5)). The relationship between a seller's experience and the expected size of her portfolio of customers is illustrated in Figure 6 and can be compared with Figure 1 in the data. The heterogeneity in sellers' number of buyers at the steady state helps explain the cross-sectional heterogeneity in sellers' outdegrees discussed in Fact 1 and in the literature before us.

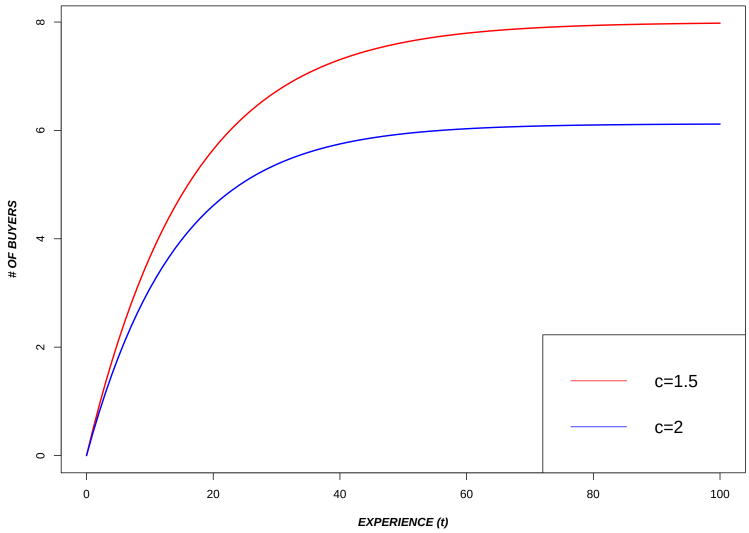
4 Estimation method

4.1 Estimating the model

We estimate our model using an unconditional inference method and maximum likelihood. This method only uses transaction and switch frequencies to recover the value of the structural parameters. In the following, we present that method and discuss identification. As identification does not use information contained in actual trade transactions (i.e. prices and quantities), we

[19] At the aggregate level, the number of buyers for each suppliers follows a stationary distribution with mean $n_i(c)$ and where the probability to have k buyers, noted $p(k, i, c)$, reads $p(k|i, c) = \frac{1}{k!}n_i(c)^k e^{-n_i(c)}$.

Figure 6: Expected number of buyers as a function of the seller's experience, for two levels of quality-adjusted costs



Notes: The figure shows the simulated expected number of buyers in a seller's portfolio as a function of its experience in the market. The blue (resp. red) line is for a high (resp. a low) quality-adjusted cost seller.

will later evaluate the performance of the model by considering untargeted moments computed from price and quantity data.

Before jumping into the estimation procedure, it is important to notice that our framework models the determinant of matches and transitions while, in the data, any observation implies a transaction. A new match or a switch is only observed at the time of a transaction. For that reason, we need to make assumptions about the transaction process. As for any event in the model, transactions are assumed to be exponentially distributed. We note t_{iF}, the rate at which a buyer from country i makes a transaction with a French seller. We allow for a discrete heterogeneity in the transaction rates : a buyer can be of type 1 with probability p (transaction rate noted t^1_{iF}), or type 2 with probability $1-p$ (transaction rate noted t^2_{iF}). Since the estimation is performed at the level of a specific product and destination, these probabilities are allowed to vary in these dimensions. While we allow for heterogeneity, a strong assumption is that the transaction rate is uncorrelated with the match quality.

The structural parameters of our model are the matching rates γ_{iF} and $\gamma_{i\bar{F}}$, the parameters shaping the distributions of serving costs, θ and τ_{iF}, the transaction rates t^1_{iF} and t^2_{iF}, and the rate at which a buyer exits the market, μ. Our dataset records transactions but also switches when buyers are observed making a transaction with a new supplier. Notice that a switch can be intermediated first by i) an unobserved switch to a foreign seller before the buyer being match again with a (more efficient) French supplier, or ii) a switch with a French seller that does not give rise to a transaction. Considering which moments of the data contribute to the identification of these parameters, the transaction rates are going to be identified by the frequencies of the transactions. Our assumption that they are independent of the match quality obviously eases identification. In the same way, the frequency at which we observe switches between French sellers helps to pin down γ_{iF}.[20] However, this is not as straightforward as for the transaction rates for two reasons. First, the switch rate between two French sellers depends on the cost at which the buyer is currently served, cost for which we have no direct information. Second, the switch can be indirect if the buyer first made an unobserved transition.

With respect to the first problem, our method uses the fact that *unconditional* hazard rates only depend on the structural parameters. As an example, consider the overall hazard rate for a buyer matched with a French seller of quality c. It adds the exit rate and the matching rate and reads

$$H(c) = \mu + \gamma_{iF} F_{iF}^N(c) + \gamma_{i\bar{F}} F_{i\bar{F}}^N(c) \tag{17}$$

As noted, we don't observe c in the data but our model gives us the distribution of buyers among French sellers $\ell_{iF}(c)$. Using (12) and (9), we notice that

$$\int_{c_{inf}}^{c^{sup}} H(c) dL_{iF}(c) = \frac{\gamma_{iF}\tau_{iF}^{-\theta} + \gamma_{i\bar{F}}}{\gamma_{iF} + \gamma_{i\bar{F}}} \int_{c_{inf}}^{c^{sup}} \frac{\mu(\mu + \gamma_{iF} + \gamma_{i\bar{F}}) dF_{i\bar{F}}(c)}{\mu + \gamma_{iF}\tau_{iF}^{-\theta} F_{i\bar{F}}(c) + \gamma_{i\bar{F}} F_{i\bar{F}}(c)}$$

$$= \frac{\gamma_{iF}\tau_{iF}^{-\theta} + \gamma_{i\bar{F}}}{\gamma_{iF} + \gamma_{i\bar{F}}} \int_0^1 \frac{\mu(\mu + \gamma_{iF} + \gamma_{i\bar{F}})}{\mu + \gamma_{iF}\tau_{iF}^{-\theta} x + \gamma_{i\bar{F}} x} dx \tag{18}$$

What equation (18) is showing is that *unconditional* hazard rates, that is expectations of rates over the observed buyer population, solely depend on the matching rate parameters γ_{iF} and $\gamma_{i\bar{F}}$, the exit rate μ, and τ_{iF}, the relative cost advantage of France vs the rest of the world. This reasoning is true for the densities/hazard rates of any type of events described by our model since they are all a combination of the matches/transaction rates and serving-cost distribution, integrated by $\ell_{iF}(c)$ (or $\ell_{i\bar{F}}(c)$).

Given that we already have to estimate γ_{iF} and $\gamma_{i\bar{F}}$, it is unclear how τ_{iF}, θ and μ, would be identified separately. For that reason, we calibrate μ the exit rate to 0.01. Exits are never directly observed and would have to be essentially inferred using censoring rates. However we already need that information to pin down $\gamma_{i\bar{F}}$: we don't observe switches towards non-French sellers, but if μ is given and the transaction rates identified, it is identified by the share of observations that disappear after one transaction while γ_{iF} is identified on the switching rate. τ_{iF} or even $\tau_{iF}^{-\theta}$ doesn't need to be estimated. Indeed, $\tau_{iF}^{-\theta}$ can be replaced by a function of the matching rates and the observed French share, using equation (12).

The remaining difficulty is that observed switches can be intermediated by a number of unobserved switches and especially by switches towards foreign sellers. Because of that, it is impossible to derive the related densities that would be necessary for a regular maximum likelihood procedure. The solution is to rely on a *simulated* estimation method: for given values

[20] In principle, our model could be estimated as a duration model or using frequencies/number of switches and transactions. We chose the later for the sake of practicality but durations and frequencies are the two faces of the same coin: since the events are exponentially distributed, the underlying Poisson process also describes the distribution of the number of events within a certain time frame.

of the structural parameters we simulate our model and compute the needed frequencies. We then choose the estimated parameters that best reproduce the empirical frequencies. Section B in Appendix details how the simulated likelihood procedure is implemented in practice.

4.2 Estimation results

The model is estimated for 331 sector×country pairs for which the number of observations and switches is sufficient for identification. Because the estimation strategy is demanding, we indeed chose to pool observations observed at the level of a (product-specific) transaction across products within broader CPA2/NACE sectors and constrain the model's parameters to equality across products within a sector. Whereas this implies loosing granularity on the estimated parameters, we gain a lot in terms of the precision of estimates. Figures A8 and A9 in Appendix display how we match the distributions of transactions and the distributions of switches : we compare the distribution in our data with the distribution we get in simulated data where we simulate each market at the estimated values. We have an almost perfect match of the distribution of transactions. This is not surprising : transaction frequencies are high in the data thus offering a useful source of identification and, even if the transaction rates are not formally separately identified, they only depend on the other parameters through censoring. If we consider the distribution of the number of switches, we match the data reasonably well, although the distribution on simulated data is more skewed than the data counterpart.

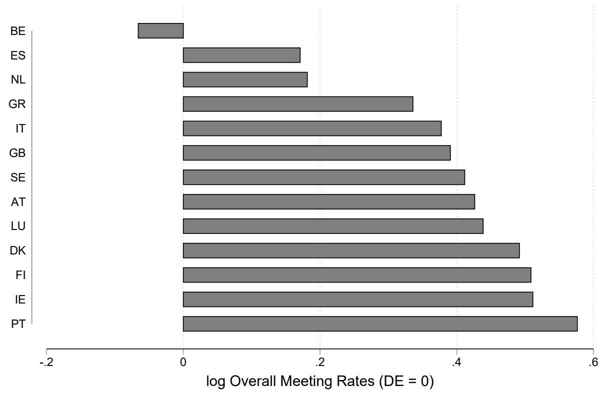
Figure A10 in Appendix displays the distributions of the estimated parameters, by country. The figures show the large dispersion in estimates: the distributions for the meeting rates γ_{iF} and $\gamma_{i\bar{F}}$, for the transaction rates $p\, t^1_{iF} + (1-p)t^2_{iF}$ and for French comparative advantages $\tau^{-\theta}_{iF}$ are skewed. French firms tend to be at a disadvantage in foreign markets, from the point of view of their Ricardian comparative advantage and from a meeting rate point of view, i.e. $\tau^{-\theta}_{iF}$ and $\gamma_{iF}/\gamma_{i\bar{F}}$ tend to cluster below one. This is not surprising given the parameters systematically compare French exporters to all possible competitors located in any other country, including the destination country itself.

To recover more interesting insights about how estimated coefficients vary across countries and sectors, Figure 7 and 8 show the mean value of estimated parameters, by country and sector, respectively. In both cases, the variability in the dimension that is ignored is controlled for using a fixed effect in a two-way fixed effect decomposition of estimated parameters. The dispersions are shown for estimated overall meeting rates (top panels) and the relative meeting rate of French firms (bottom panels), with the former being indicative of the overall magnitude of frictions whereas the later says something about the relative position of French firms in terms of meeting rates. Figure A11 shows similar histograms for estimated Ricardian advantages.

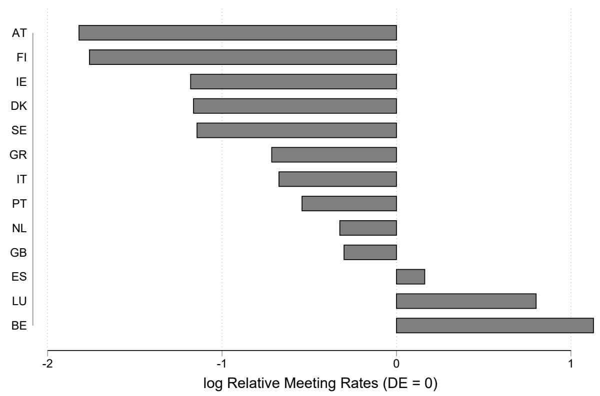
Consider first the dispersion across countries, shown in figure 7. The decomposition reveals sizable heterogeneity in the overall degree of frictions, with the overall meeting rate being 60% larger in Portugal than in Belgium, on average. However, the relative advantage of French firms in terms of meeting rates is hardly correlated with the overall level of frictions. Here, geographical and cultural proximity seems to help since the destinations in which French firms are relatively high in terms of meeting rates are all neighboring countries, namely Belgium, followed by Luxembourg, Spain, Germany and the UK. This result is consistent with the literature in trade that attributes part of the gravity structure of trade to the impact of information frictions and their correlation with distance (Rauch, 1999, 2001).

Figure 7: Dispersion in estimated search parameters, across countries

Overall Meeting rates ($\gamma_{iF} + \gamma_{i\bar{F}}$)



(b) Relative Meeting rates ($\gamma_{iF}/\gamma_{i\bar{F}}$)



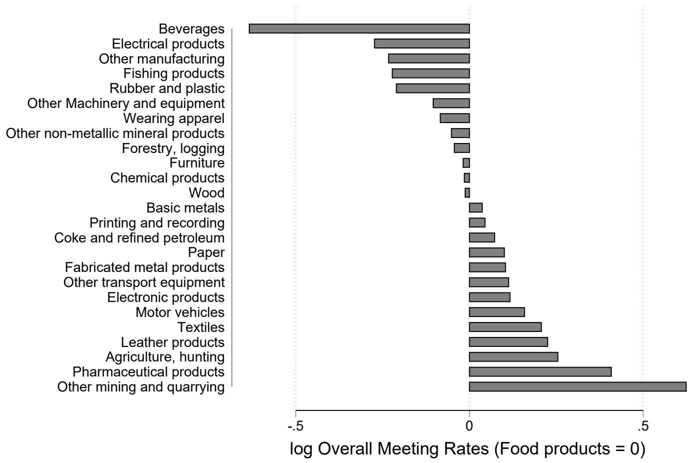
Note: The figure shows the mean value of estimated parameters, by country. All values are normalized by the mean estimate for Germany.

The dispersion in average meeting rates across sectors is even stronger with a 120% gap between the least frictional sector ("Other Mining and quarrying") and the most frictional one ("Beverages"). In relative terms, French firms are sometimes at their advantage in more frictional markets such as beverages or electrical products. But the two sectors in which relative meeting rates are the highest are chemical products and motor vehicules, two sectors that are at the core of France's international competitiveness. Finally, although these average statistics are useful, it is important to point out that our estimates display significant heterogeneity across products within a country and across countries within a product. These dimensions of heterogeneity explain 86 and 54% of the variance of overall and relative meeting rates, respectively.

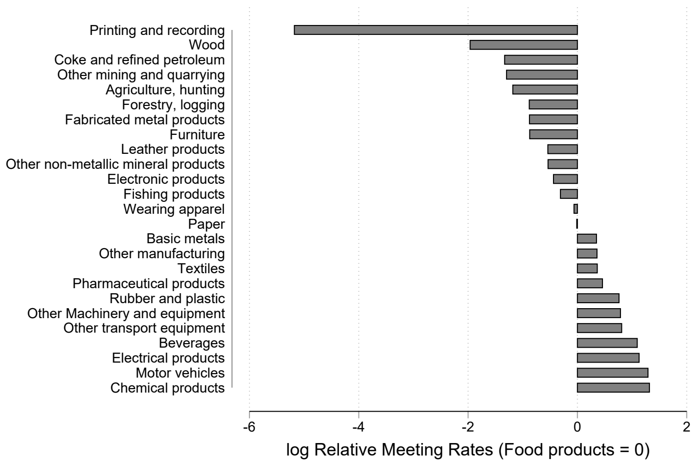
What do these estimates say in terms of the impact of search frictions on trade? As explained in Section 3, a contribution of our model is to introduce search frictions into a Ricardian framework. As we estimate all parameters of the model, we can quantify the extent to which

Figure 8: Dispersion in estimated search parameters, across sectors

Overall Meeting rates ($\gamma_{iF} + \gamma_{i\bar{F}}$)



(b) Relative Meeting rates ($\gamma_{iF}/\gamma_{i\bar{F}}$)



Note: The figure shows the mean value of estimated parameters, by sector. All values are normalized by the mean estimate of the Food products industry.

search frictions help explain the geography of trade. This is what we do in Table 1. Consider first columns (1) and (2) that give the unconditional variance decomposition of trade shares in terms of search frictions and comparative advantages. Overall, relative meeting rates explain 42% of the variance in French firms' foreign market shares, the rest being attributable to Ricardian comparative advantages. Whereas the contribution of search frictions is reduced when we focus on the variance of trade shares across products within a country or across countries within a product, as in columns (3) and (4), the explanatory power of search frictions remain sizable.

5 Implication for trade dynamics (In progress)

Armed with our estimates, we can now dig into the model's properties in terms of trade adjustments to relative price shocks. As explained in the introduction, the dynamic structure of firm-to-firm relationships translates into rich adjustment patterns that we can then compare

Table 1: Ricardian versus frictional determinants of trade

	Dep. Var			
	$\ln\frac{\gamma_{iF}}{\gamma_{i\bar{F}}}$	$\ln\tau_{iF\bar{F}}^{-\theta}$	$\ln\frac{\gamma_{iF}}{\gamma_{i\bar{F}}}$	$\ln\frac{\gamma_{iF}}{\gamma_{i\bar{F}}}$
	(1)	(2)	(3)	(4)
$\ln\frac{\pi_{iF}}{1-\pi_{iF}}$	$.418^a$	$.582^a$	$.298^a$	$.254^a$
	(.075)	(.075)	(.075)	(.082)
# Obs	331	331	331	330
Adjusted R^2	.083	.151	.209	.217
Country FE	No	No	Yes	No
Product FE	No	No	No	Yes

Note: The RHS is based on predicted trade shares using:

$$\frac{\pi_{iF}}{1-\pi_{iF}} = \frac{\gamma_{iF}}{\gamma_{i\bar{F}}}\tau_{iF\bar{F}}^{-\theta}$$

The variance decomposition in columns (1) and (2) is thus exact.

with existing empirical evidence. In this section, we will more specifically discuss what the model implies in terms of the long-run elasticity of trade to relative price shocks, the pass-through of shocks onto consumer prices and the dynamics of trade adjustments.

5.1 Trade elasticities in frictional markets

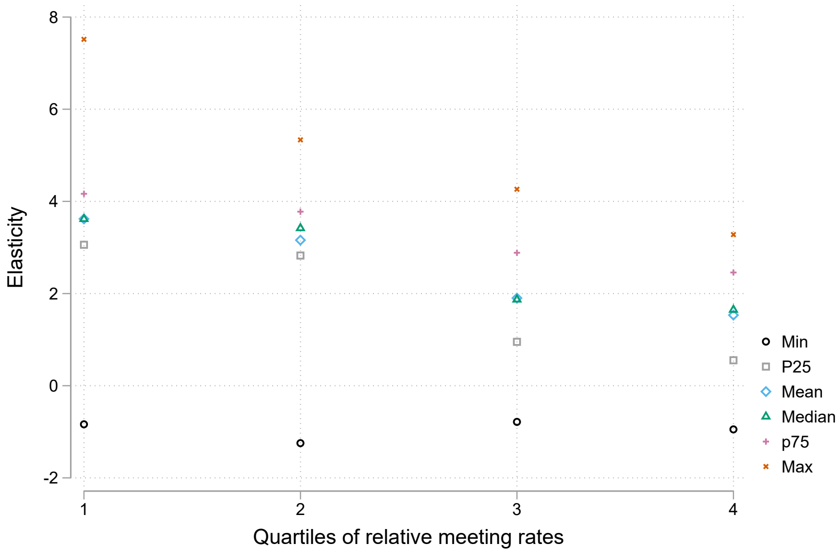
In the steady state, the geography of trade is shaped by the interaction of comparative advantages and relative meeting rates. Simple comparative statics thus convey insightful intuitions on how the *elasticity* of trade varies with search frictions. Using the approximation of trade shares in equation (12), we have:

$$\varepsilon_{iF} = \frac{\theta\pi_{iF}}{\tau_{iF}^{-\theta}\gamma_{iF}/\gamma_{i\bar{F}}} = \frac{\theta}{1+\frac{\gamma_{iF}}{\gamma_{i\bar{F}}}\tau_{iF}^{-\theta}}$$

where $\varepsilon_{iF} \equiv -\frac{d\ln\pi_{iF}}{d\ln\tau_{iF}}$ denotes the elasticity of market shares with respect to a shock on France's relative costs. For low-enough French market shares, it is also the elasticity of bilateral trade to the shock. As in Eaton and Kortum (2002), the elasticity of trade with respect to relative price shocks is muted in markets in which French firms have a high comparative advantage, where the relative price shock needs to be large to induce buyers to switch and trade to adjust at the extensive margin. Conditional on Ricardian comparative advantages, the elasticity of trade is also predicted lower in markets in which the relative meeting rate of French firms is larger. A high meeting rate for France indeed implies that the direct competitors of French firms in foreign markets are more likely to be French. As a consequence, the shock does not deteriorate French firms' competitiveness as much as it would in a market in which their competitors are mostly non-French.

This interaction is illustrated in Figure 9 that displays statistics on the distribution of elasticities in each quartile of the estimated distribution of relative meeting rates. As expected,

Figure 9: Distribution of elasticities, along the distribution of relative meeting rates



Notes: The figure shows statistics on the distribution of trade elasticities, in each quartile of the distribution of estimated relative meeting rates.

elasticities tend to be muted in higher quartiles of the distribution of relative meeting rates. This is true along the distribution of elasticities, as well as on average. The mean elasticity of trade thus decreases from 3.6 to 1.5 when moving from the first to the fourth quartile of the distribution of meeting rates. However, the figure also illustrates the dispersion in elasticities, conditional on the level of search frictions. The reason is that Ricardian comparative advantages also matter for the size of the elasticity, the decreasing relationship between ε_{iF} and $\frac{\gamma_{iF}}{\gamma_{i\bar{F}}}$ being muted in markets where French firms have a high comparative advantage. The interaction of search frictions and comparative advantages is thus a key component to understand how trade react to relative price shocks, in the long-run.

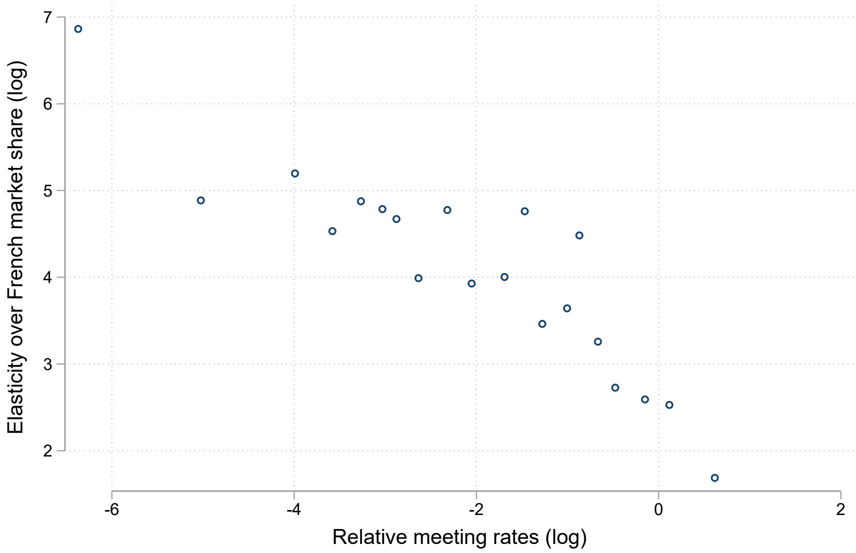
We confront this prediction of the model with actual data in Figure 10. We first estimate the long-run elasticity of trade to real exchange rate shocks for each sector and country using an error correction model inspired from Alessandria et al. (2021b).[21] In a second stage, the sector- and country-specific elasticities are correlated with estimated relative meeting rates. As expected, the correlation is negative and significant, i.e. the estimated sensitivity of trade to RER shocks is magnified in country×sector that are estimated more frictional from the point

[21] Namely, we first aggregate the trade data at the monthly and product level. The panel is merged with country-specific real exchange rate series recovered using Eurostat data on nominal exchange rate and PPI for France and each of its European partner. Finally, we estimate the following model for each sector×country pair:

$$d \ln X_{pct} = \beta^{SR} d \ln RER_{ct} + \beta^2 \ln RER_{ct-1} + \beta^3 \ln X_{pct-1} + FE_{pt} + \varepsilon_{pct}$$

where X_{pct} is the value of exports of product p to country c at time t, RER_{ct} is the real exchange rate between France and country c, defined such that an increase in RER denotes a real appreciation for French firms, and FE_{pt} is the product×period fixed effect. In this equation, β^{SR} estimates the short-run elasticity of trade to RER shocks while the long-run elasticity is defined as $\beta^{LR} = -\beta^2/\beta^3$.

Figure 10: Estimated trade elasticities and relative meeting rates



Notes: The figure shows a bin scatter plot of normalized trade elasticities against estimated relative meeting rates. The normalized trade elasticities are defined as the ratio of the estimated long-run elasticity of trade over French firms' market share. The ratio is taken in logs, which mechanically exclude sector×country pairs for which the estimated elasticity is negative.

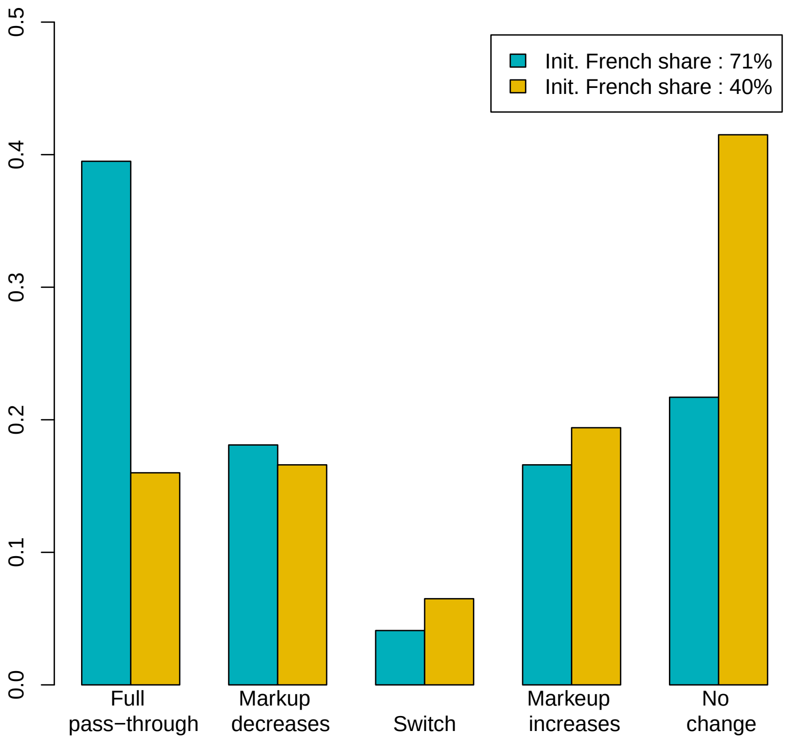
of view of French firms. In the dataset, the correlation is sizable, at -61%.

5.2 The short and long-run pass-through of relative price shocks

A novel feature of our model lies in its rich predictions regarding the incidence of relative price shocks. Depending on the impact of the shock on the strength of competition in each firm's network of suppliers, a shock can go from having zero consequences on the buyer's input prices to being fully passed through. The incidence on buyers' prices is complete when the buyer is matched with a French firm and the second best price remain French after the shock. In this case, both the supplier and its direct competitor see their cost competitiveness deteriorates so that the buyer does not gain market power and the pass-through is complete. At the other side of the spectrum, the buyer is left unaffected by the shock if both its current supplier and the supplier's direct competitor are non-French. In between, the incidence on the buyer is all the lower since the shock increases the strength of competition within its network, which forces its French supplier to reduce its markup or even leads to a switch. Finally, the relative price shock can deteriorate the bargaining power of the buyer vis-à-vis its non-French partners, in case the second lowest cost supplier is French. Under such circumstances, the buyer can see its input prices increase, even though it does not directly source its inputs from France.

The model can thus encompass a rich structure of pass-through rates following a relative cost shock that deteriorates French firms' competitiveness. Such richness is useful in as much as it can help understand the strong degree of heterogeneity in empirical pass-through rates. The heterogeneity in pass-through rates is indeed the topic of an old literature, surveyed in

Figure 11: Incidence of relative price shocks on impact



Notes: The figure shows the result of simulations of a 10% relative price shock. The simulations assume: θ=4, γ_F=0.005, $\gamma_{\bar{F}}$=0.01.

Burstein and Gopinath (2014). The heterogeneity in pass-through rates across firms has long been pointed out, see e.g. Berman et al. (2012). Cavallo et al. (2021) notably discuss the role of margin adjustments at various points of the distribution chain. Finally, Amiti et al. (2019) provide evidence of strategic complementarities using micro-data for the Belgian manufacturing sector.

In our model, the heterogeneity is again shaped by the interaction of Ricardian comparative advantages and search frictions. The interaction is illustrated in Figure 11 which shows the prevalence of the different types of price adjustments, in two cases in which the initial share of French firms is lower and higher. In markets in which French firms cumulate a high market share, thanks to the combination of Ricardian and search comparative advantages, the degree of pass-through is higher, because a cost shock does not transmit into a sizable deterioration of French firms bargaining power. Instead, in markets in which French firms account for a smaller share of the destination's absorption, most buyers are not exposed to the shock and, when they are, the incidence is small and mostly attributable to non-French suppliers gaining in terms of bargaining power.

5.3 To be completed

6 Conclusion

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A Data appendix

A.1 Construction of the estimated sample

Wholesalers: Whereas the raw data cover each single transaction involving French exporters and their partners in the European Union, the model tackles the choice of input suppliers in frictional markets in a context with no trade intermediation. Ideally, we would thus be willing to exclude from the sample all intermediated transactions. For French firms, we can use information from INSEE about the firm's sector of activity and remove all firms that are either wholesalers or retailers. As noted by Bernard et al. (2015), intermediaries are important traders in international markets. In our data, French intermediaries represent 40% of the population of exporters and 15% of the total value of exports. Unfortunately, we do not know the activity of the importing firm. As a proxy for wholesaling activities, we thus measure the maximum number of French sellers a particular foreign firm interacts with for a given product, over a particular month. Our argument is that firms purchasing the same product to many different French exporters are more likely to intermediate trade than firms which purchase a particular good to a single French exporter. In our data, only 5% of importers ever purchase the same product from two different French exporters in a particular month but some importers simultaneously interact with more than 50 producers of specific accessories of motor vehicules or Bordeaux wine. Despite their small number, the combined share of overall trade intermediated by these multi-seller importers is high, at 23%, which again is consistent with evidence in Bernard et al. (2015). We thus exclude from the estimation sample the one percent of importers that display the maximum number of simultaneous supliers within the same month. This excludes all firms that ever interacted with more than three French exporters in the same month. The remaining sample covers 75% of the total value of trade in the raw data and 4.7 million importer$\times$product pairs.

A.2 Statistics on the connectivity of the graph

A now standard way of measuring the connectivity in such seller-buyer networks consists in measuring the in- and out-degrees at each node, i.e. the number of partners firms at each side of the newtork are connected to. Figures A1 and A2 illustrate the heterogeneity in this measure of connectivity, among European importers and French exporters, respectively.

Focusing first on importers, Figure A1 illustrates the strong sparsity of this side of the network as the vast majority of European importers are connected with a single French exporter. As shown in the upper-left panel, more than 95% of the European importers that ever interact with a French exporter over the 2002-2006 period never interact with more than one firm within a particular month and over a particular product.[22] This number decreases somewhat, to 75%, when we do not condition over a particular product (upper-right panel), which means that a non-negligible number of European importers interact with several French exporters

[22] As explained in section A.1, this number is somewhat inflated artificially since we dropped firms purchasing the same product to many different exporters on the ground of the argument that these are more likely to be wholesalers. Remember however that the selection is based on the top 1% of firms with the highest indegree and does not change this figure much as a consequence.

Table A1: Dimensionality of the estimation sample

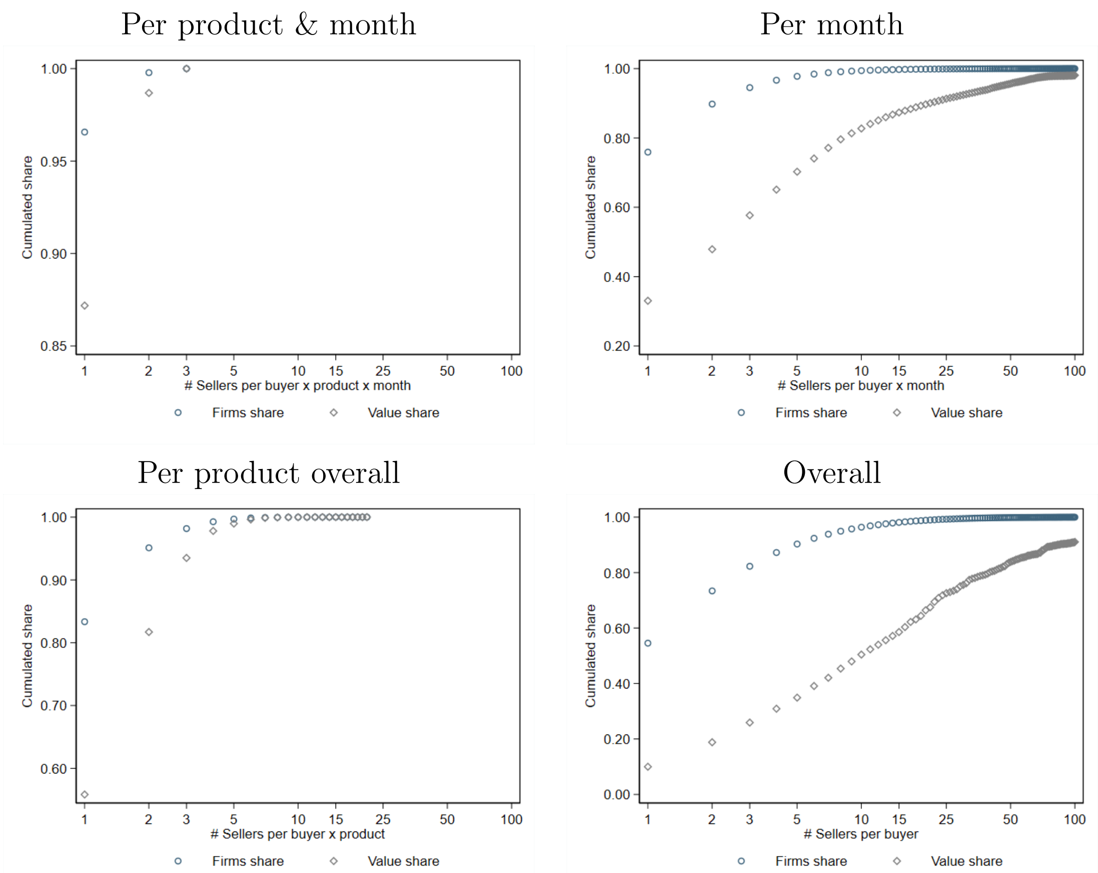
	Transactions (1)	Exporters s (2)	Importers $b(i)$ (3)	$sb(i)j$ Triplets (4)
All	27,442,785	39,751	744,118	5,646,587
Austria	787,990	9,669	20,765	157,550
Belgium	4,501,923	27,786	86,174	927,695
Denmark	577,165	9,478	14,326	116,695
Finland	357,670	6,261	7,718	69,181
Germany	5,731,010	24,683	181,630	1,122,918
Greece	634,143	8,415	14,950	136,556
Ireland	426,605	7,221	9,207	104,659
Italy	3,613,227	20,395	129,124	812,073
Luxembourg	479,248	10,922	8,047	97,417
Netherlands	1,869,157	17,344	46,071	375,632
Portugal	1,165,765	12,625	26,545	259,340
Spain	3,639,465	21,362	104,745	732,013
Sweden	637,453	8,975	15,298	121,086
United Kingdom	3,021,964	19,885	79,518	613,772

Notes: This table shows statistics on the dimensionality of the estimation dataset. The dataset covers the period from 2002 to 2006 and the EU15 countries. Trade intermediaries are neglected on the sellers' and buyers' sides, as explained in Section A.1.

simultaneously, to purchase different products. Whereas firms connected with multiple partners are relatively rare in the cross-section of the data, their number naturally increases when we cumulate their partners over time as in the bottom panels of Figure A1. Then, the share of firms that we never see interacting with two different exporters over the same product is reduced to 83%. This result is particularly important for the purpose of our exercise as the estimation exploits moments on firms that switch from one supplier to the other, after accumulating contacts over time. The shift of the distributions between the upper and the bottom panels of Figure A1 indicates that such switches are not uncommon.

Whereas importers interacting simultaneously with several exporters are rare, the reciprocal is not true, as illustrated in Figure A2. The upper left panel thus shows the cumulated distribution of sellers that interact with a given number of importers from a particular destination over a given month and for a particular product, as well as their contribution to aggregate trade. 70% of the sample is composed of French exporters that interact with a single firm in their typical destination at a point in time. When we cumulate across destinations as in the middle left panel, there are still 40% of exporters that serve a single importer in a single destination. These firms are however small, on average, and cumulate only 17% of the overall value of trade. At the other extreme of the distribution about 10% of exporters interact with more than ten European importers but they cumulate almost 50% of French exports. The heterogeneity in exporters' ability to serve a large number of foreign partners is explained in the model by the interaction of exporters' productivity heterogeneity and the history of their matches with foreign firms. The deterministic dimension can explain why the distribution of these outdegrees is not fundamentally different when we cumulate French exporters' partners over time as in the bottom left panel. Whereas cumulating over time significantly shifted the distribution down when we were focusing on importers in Figure A1, the same is not true when

Figure A1: Cumulated distribution of European importers' indegrees



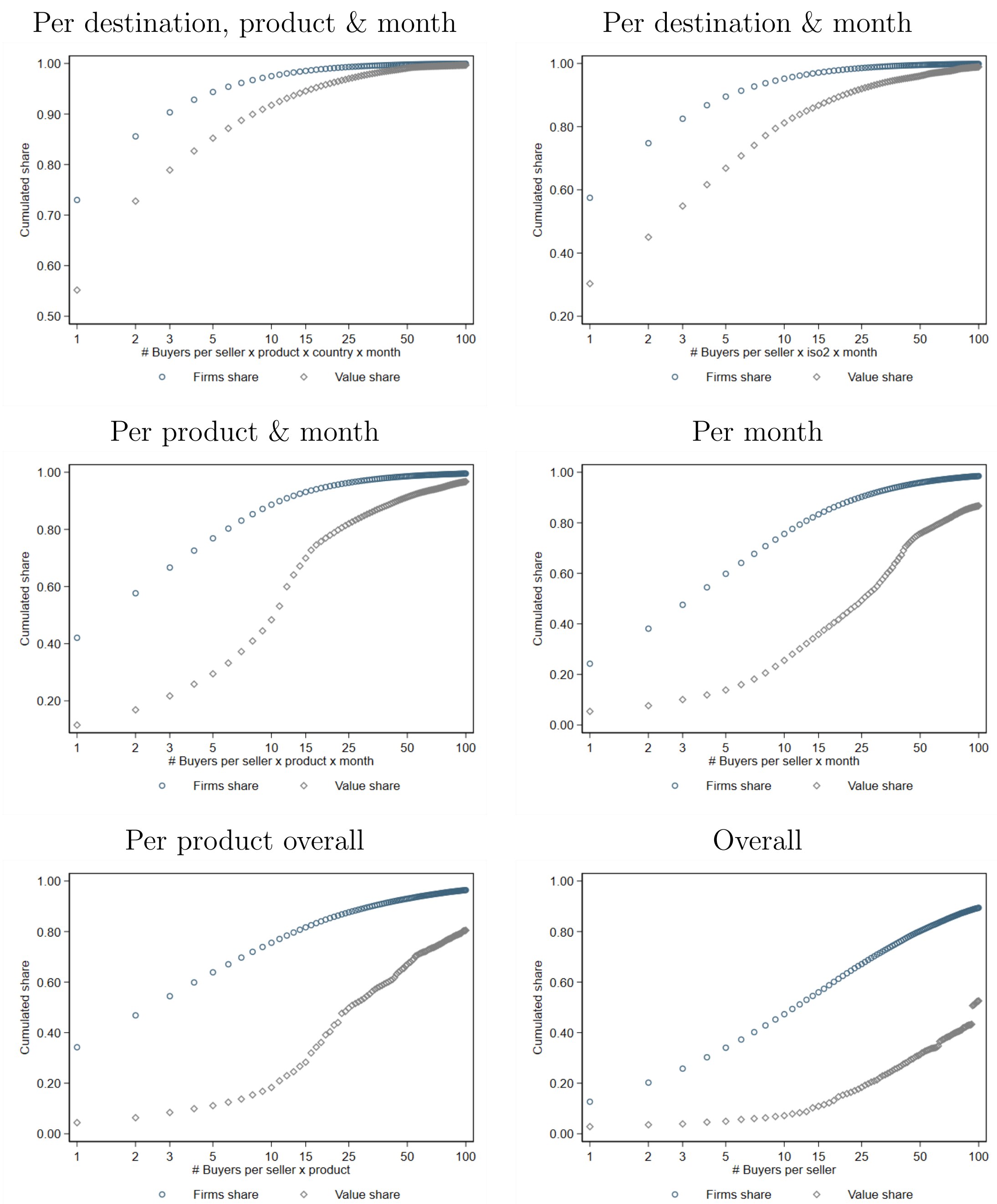
The figure illustrates the heterogeneity across European importers' in their "indegrees", i.e. their number of partners in France. The x-axis corresponds to a number of partners and the y-axis is the cumulated share of firms (blue circles) with x sellers or less, and their cumulated contribution to aggregate trade (grey squares). The two upper panels measure a firm's indegree in the cross-section, i.e. within a particular month. The bottom panels instead cumulate partners over the whole period of activity of the firm. The left panels treat multi-product importers as independent units whereas the right panels cumulate partners over the firm's portfolio of imported products.

we take the point of view of exporters. Here as well, cumulating partners over the exporter's portfolio of products as we do in the right panels of Figure A2 shifts the distributions down. The reason is that the vast majority of exporters do not serve the same importers with their different products.

A.3 Trade elasticity estimates

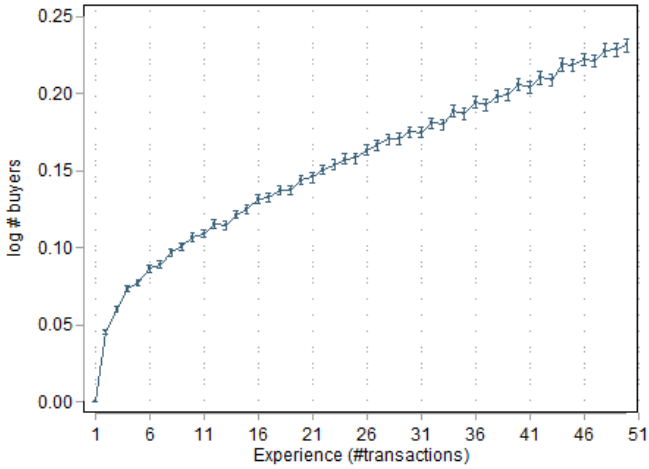
Trade data The monthly series of bilateral trade at product-level are directly recovered from our main dataset. Namely, we aggregate all transactions at the level of a patricular month, destination country, and for a particular month, over a period from January 1999 to December 2019. We obtain an unbalanced panel of 27 countries and 252 months, which we then merge with control variables recovered from external sources.

Figure A2: Cumulated distribution of French exporters' outdegrees



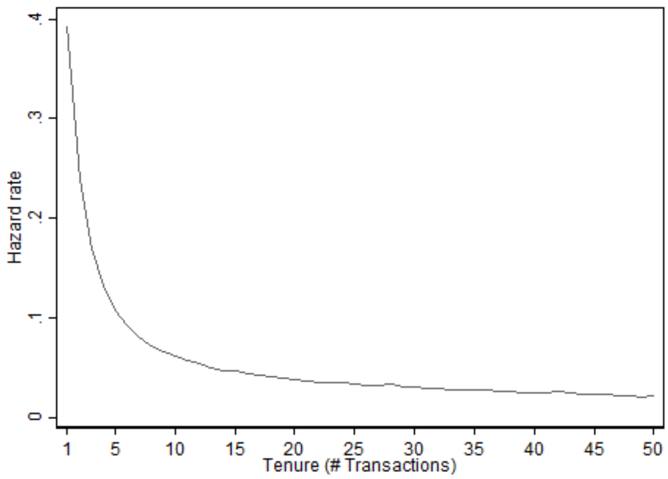
The figure illustrates the heterogeneity across French exporters' in their "outdegrees", i.e. their number of partners within the European Union. The x-axis corresponds to a number of partners and the y-axis is the cumulated share of firms (blue circles) with x buyers or less, and their cumulated contribution to aggregate trade (grey squares). The two upper panels measure a firm's outdegree in the cross-section, i.e. within a particular month. The bottom panels instead cumulate partners over the whole period of activity of the firm. The left panels treat multi-product firms as independent units whereas the right panels cumulate partners over the firm's portfolio of products.

Figure A3: Acquisition of buyers over time, Alternative definition of experience



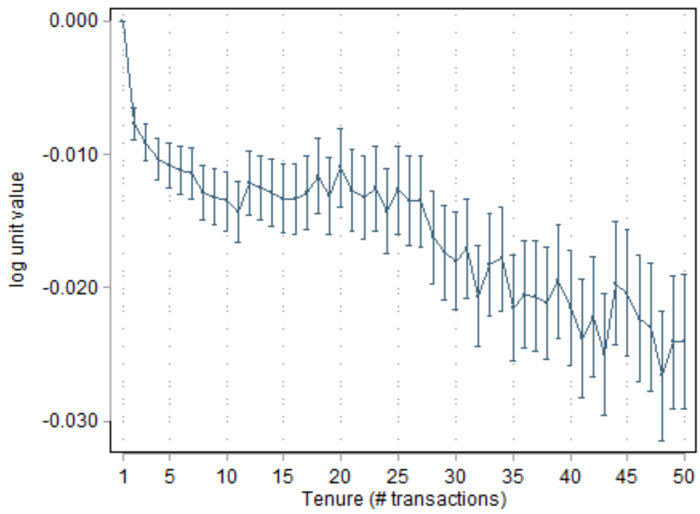
Note: The figure shows the evolution of a seller's stock of buyers, over time, recovered from equation (1).The figure reports the estimates and their 95% confidence intervals. Experience is measured by the cumulated number of transactions since first entry.

Figure A4: Hazard rate over time, Alternative tenure definition



Notes: The hazard rate is defined as the probability of the relationship ending, conditional on tenure into the relationship and is calculated as the ratio of the density to the survival rate at tenure k. The figure is recovered from the 2002-2006 sample using the cumulated number of transactions since the start of the relationship as measure of tenure.

Figure A5: Price dynamics within a firm-to-firm relationship, Alternative tenure definition



Note: This figure shows the evolution of prices within a firm-to-firm relationship. Coefficients are recovered from equation (2). The figure reports the estimates and their 95% confidence intervals. Tenure is measured by the cumulated number of transactions since the beginning of the relationship.

Nominal Exchange Rate The bilateral nominal exchange rates are sourced from Eurostat (series *ert_bil_eur_m* and *ert_bil_conv_m* for pre-Euro exchange rates among euro area countries) and correspond to the average monthly value.

Producer Price Index We obtain producer price indices (PPI) from Eurostat (series *sts_inpp_m*). We use the index that covers the largest set of sectors, namely industries B to E in the NACE classification.[23] Indices are seasonally adjusted, and normalized at 100 in January 2015.

Real Exchange Rate The real exchange rate is computed using the previous variables, as:

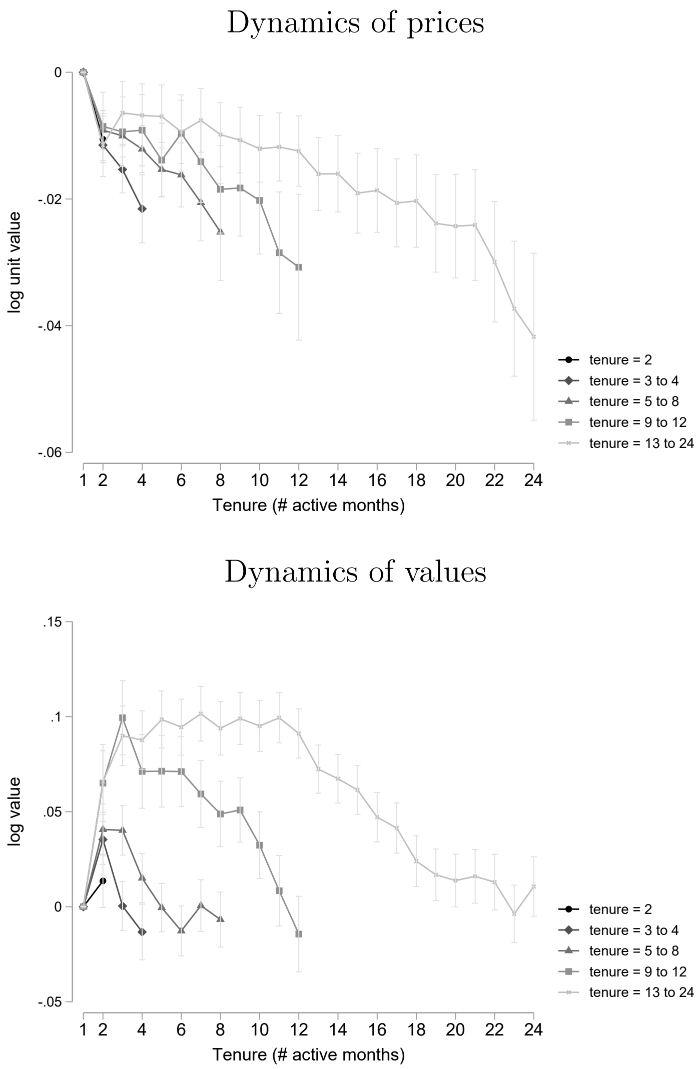
$$RER_{jt} = \frac{NER_{jt} \cdot PPI_{Ft}}{PPI_{jt}}$$

where $j = F$ stands for France.

Variations in the real exchange rate for euro area countries hence solely comes from variations in relative PPIs. For the non-EMU countries, nominal exchange rate movements add a lot more volatility. Summary statistics about the volatility of the real exchange rate in each country can be found in table A2, which shows the mean and median of the rolling 12-month standard deviation of the variable.

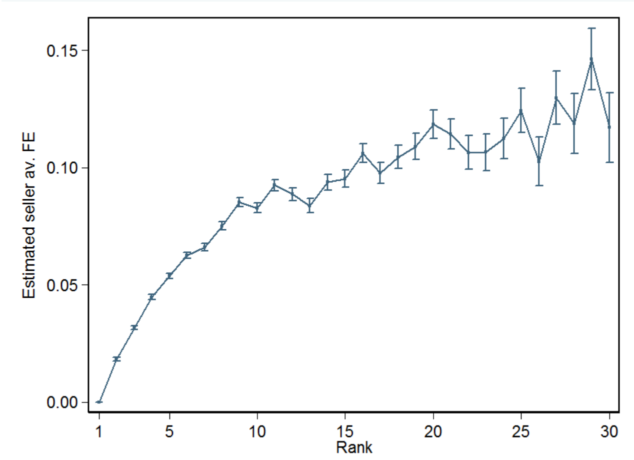
[23] B-Mining and quarrying, C-Manufacturing, D-Electricity, gas, steam and air conditioning supply, E-Water supply; sewerage, waste management and remediation activities

Figure A6: Price and quantity dynamics within a firm-to-firm relationship, by tenure



Note: This figure shows the evolution of prices and exported values within a firm-to-firm relationship. Coefficients are recovered from equation (2) which is estimated separately by subset of tenure lengths. The figure reports the estimates and their 95% confidence periods. Tenure is measured by the number of months since the beginning of the relationship.

Figure A7: Evolution of sellers' attributes, within a buyer's sequence of French partners

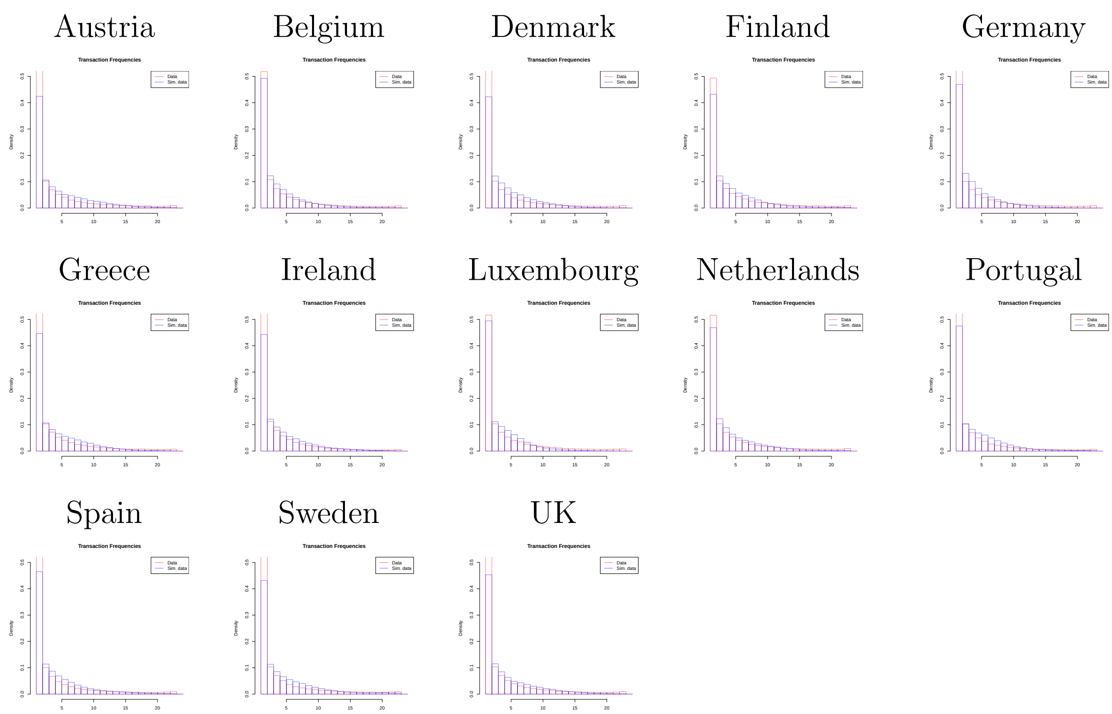


Note: This figure shows how buyers climb along the distribution of sellers' attributes, over time. It is recovered from the estimation of:

$$\hat{FE}_{spi} = FE_{bp} + \sum_{l=2}^{K} \alpha_l \mathbb{1}(Partner_{bps} = l) + \varepsilon_{bps}$$

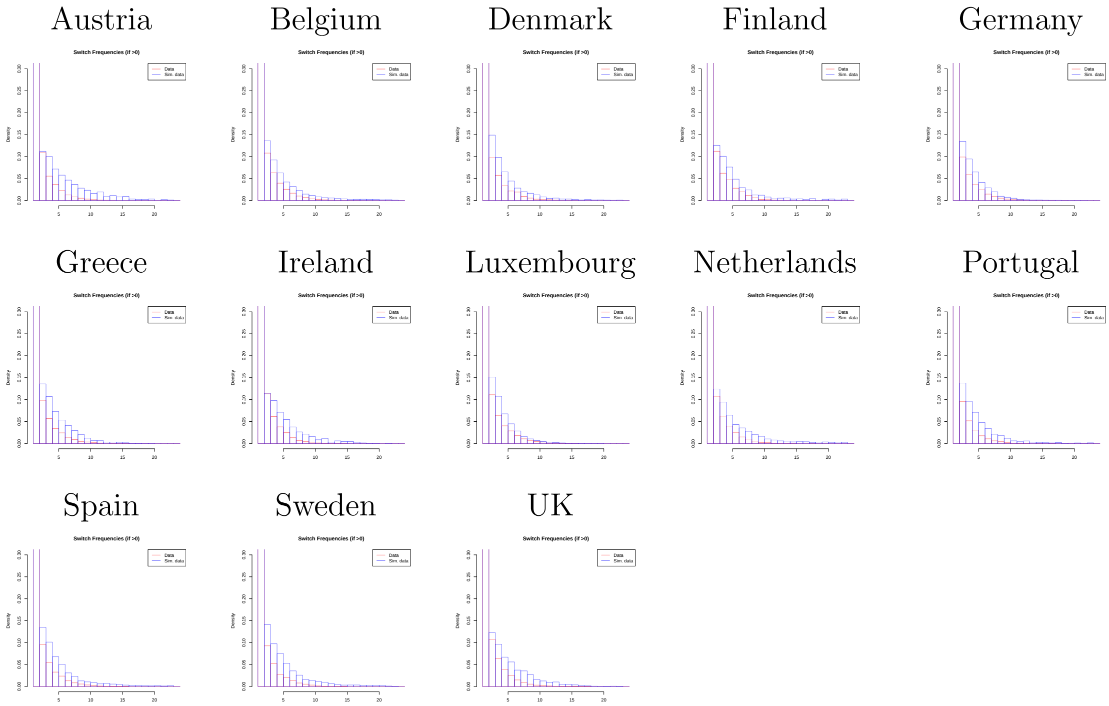
where $\hat{FE}_{spi}$ is the fixed effect recovered from the estimation of equation (1), that we interpret as a proxy for the seller's quality-adjusted price in market i, FE_{bp} is a buyer-product fixed effect and $\mathbb{1}(Partner_{bps} = l)$ is a dummy if seller s is the lth partner of buyer b when sellers are ranked sequentially based on their history of transactions with buyer b. In this equation α_l measures how $\hat{FE}_{spi}$ improves when a buyer switches from its l-1th to its lth French supplier.

Figure A8: Goodness of fit: Transaction frequencies



Note: Figure shows how we fit the transaction frequencies, by comparing the actual and simulated data.

Figure A9: Goodness of fit: Switch frequencies

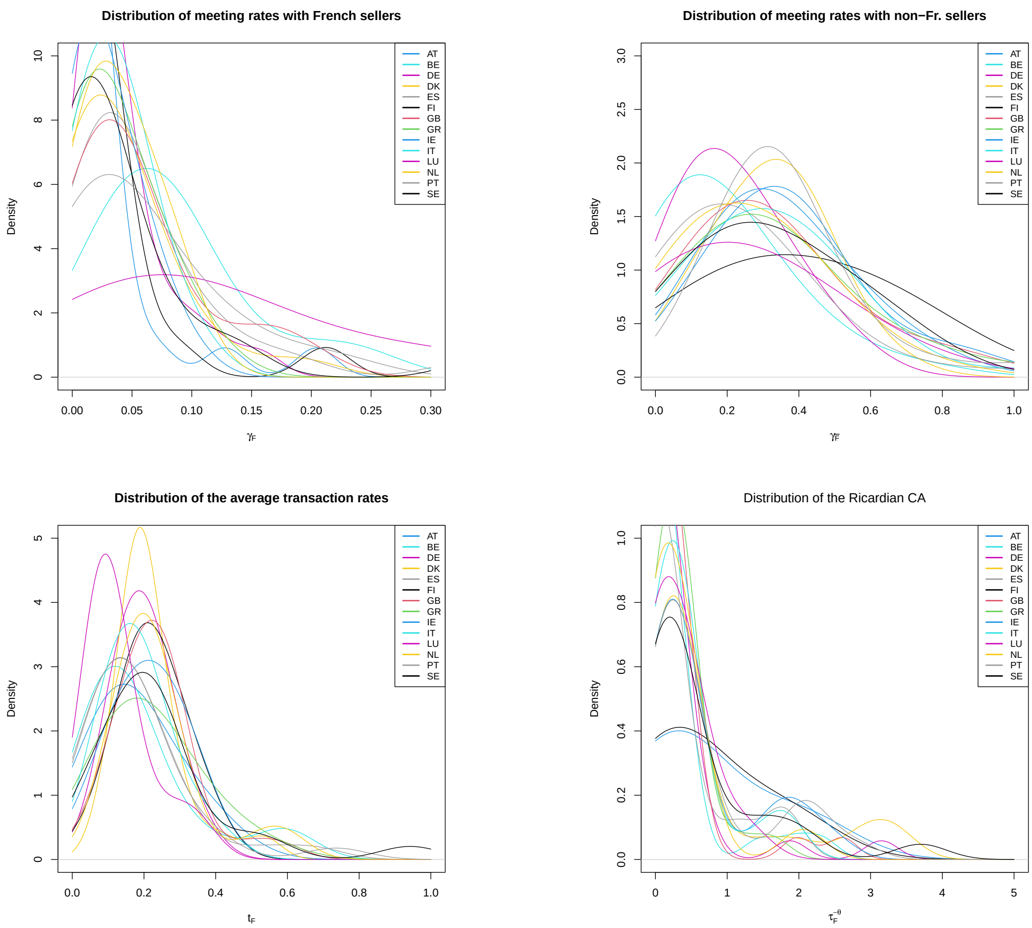


Note: Figure shows how we fit the switch frequencies, by comparing the actual and simulated data.

Table A2: 12 month volatility of the real exchange rate

Country	Mean	Median
Austria	0.0057	0.0049
Belgium	0.0138	0.0142
Bulgaria	0.0261	0.0212
Cyprus	0.0154	0.0148
Czech Republic	0.3002	0.2578
Germany	0.0065	0.0054
Denmark	0.1193	0.0867
Spain	0.0066	0.0070
Estonia	0.0103	0.0080
Finland	0.0087	0.0071
United Kingdom	0.0191	0.0169
Greece	0.0151	0.0133
Croatia	0.0790	0.0678
Hungaria	3.9977	3.2321
Ireland	0.0192	0.0164
Italy	0.0048	0.0047
Lithuania	0.0270	0.0221
Luxembourg	0.0155	0.0130
Latvia	0.0163	0.0120
Malta	0.0180	0.0164
Netherlands	0.0146	0.0120
Poland	0.0832	0.0604
Portugal	0.0136	0.0093
Romania	0.0627	0.0484
Slovakia	0.0110	0.0084
Slovenia	0.0077	0.0067
Sweden	0.1567	0.1243

Figure A10: Densities for the estimated parameters



Note: The figure shows the country-specific distributions of estimated parameters. The figure is restricted to sector×country pairs for which we observe at least 100 buyers and 2 switches.

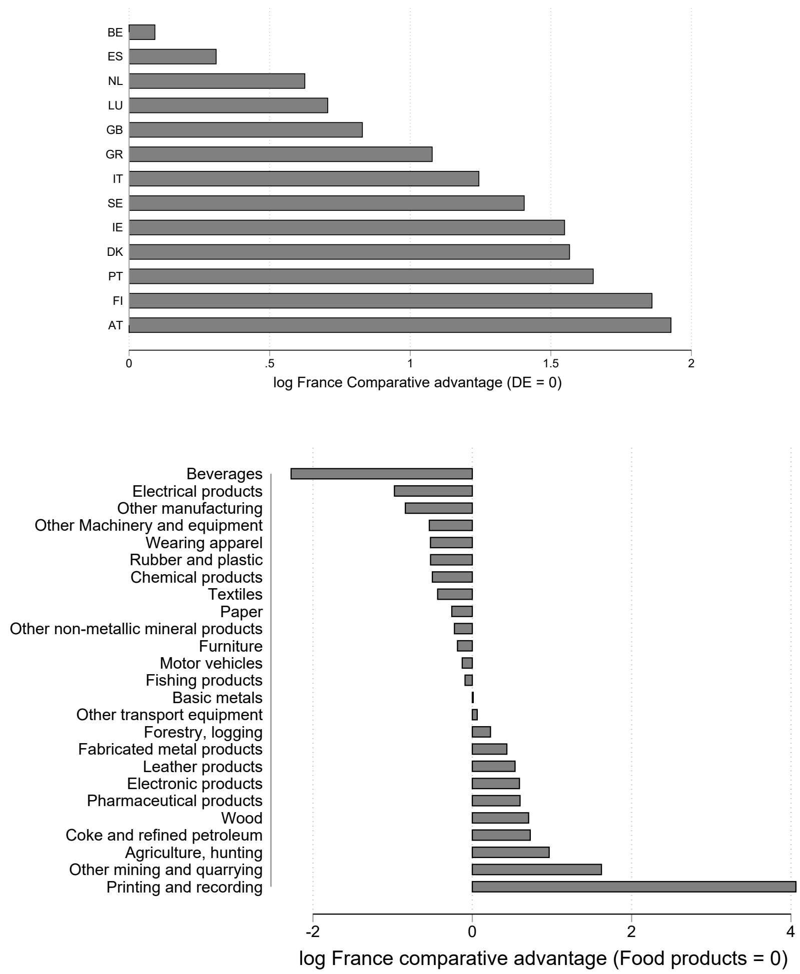
Import demand We obtain countries' import data from Eurostat (series *DS-1060915*). More precisely, we compute monthly total imports originating from countries outside the EU-28, for all countries in the sample.

B Details on the estimation procedure

As explained in the main text, our estimation of the model's parameters uses a simulated likelihood approach. For given values of the structural parameters, we simulate our model, compute the needed frequencies, and compare them with their empirical counterpart. For that purpose, we need to limit the set of possible events. In the following, we record up to 5 transactions, remembering that you need at least one transaction to be part of the sample, and up to 2 switches. Then, we compute on simulated data the probabilities needed for the likelihood, namely

$$P(\text{ transactions } = n \cap \text{ switches } = \text{s})$$

Figure A11: Dispersion in estimated Ricardian comparative advantages, across countries and sectors



Note: The figure shows the mean value of estimated Ricardian comparative advantages, by country and sector.

where $n \in \{1, ..., 5\}$ ($n = 5$ means at least 5 transactions) and $s \in \{0, 1, 2\}$ ($s = 2$ means at least 2 switches).

Now we have defined the relevant set of events, the exact procedure to compute the likelihood given values of the structural parameters is as follow

i. For each dataset (country$\times$market), we simulate 100 times more buyers than there are in the dataset. If a buyer exits the market in our simulations (μ shock), it is replaced by an unmatched buyer. This ensures that the steady state assumption holds. Notice that some of these simulated buyers won't be used to compute the frequencies. This is the case when they are never matched with a French seller.

ii. We simulate first 2000 months of buyers' history to reach steady state. After this step, we sample according to the way the estimation sample is generated. Hence we simulate for 24 months and we record any buyer observed making a transaction with a French seller, as we record any buyer between January 2002 and January 2004 in the data. Then, we follow that buyer for 24 months recording any subsequent transaction or any switch. In the same way, the estimation sample follows any recorded buyer up to 24 months, that is up to January 2006 at the latest.

iii. Using simulated data, we compute the frequencies $P($ transactions $= n \cap$ switches $=$ s), $\forall (n, s)$. We denote these frequencies $P^{sim}(n, s|\omega)$ where ω is the vector of parameters' value.

iv. If there are J buyers in real data, indexed by j, the log-likelihood is as follow

$$\mathcal{L}(\omega) = \sum_{j=1}^{J} P^{sim}(n_j, s_j|\omega)$$

Finally, our estimates are obtained by maximizing the likelihood:

$$\hat{\omega} = \arg\max \mathcal{L}(\omega)$$

C The model: additional derivations

C.1 Plugging the model into a general equilibrium structure

C.1.1 The representative consumer

There is one representative consumer in each country i. She consumes a bundle of differentiated goods and her utility function reads

$$U_i = \left(\sum_{b=1}^{B_i} x_b^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

with $\sigma > 1$ the (constant) elasticity of substitution between varieties. For simplicity, final goods are assumed non-traded but it would be easy to plug trade in final goods in this set-up, following Melitz (2003). Noting R_i the consumer's income, which consists of labor income and residual profits, we obtain the demand for variety b

$$x_b = \frac{R_i}{P_i} \left(\frac{p_b}{P_i} \right)^{-\sigma} \tag{19}$$

where p_b stands for the price of variety b and P_i is the ideal price index

$$P_i = \left(\sum_{b=1}^{B_i} p_b^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

Remark that the representative consumer has no intertemporal choice. Within a certain time frame, she spends R_i to collect the different available varieties. The number of varieties depends on the number of differentiated good producers (our *buyers*) that are matched with at least one intermediate good producers (our *sellers*). The surplus generated by consumption depends on the level of final good prices, which itself is endogenous to the history of matches between final producers and their input suppliers.

C.1.2 The final good producer

Given the level of demand x_b, the final good producer chooses the quantity of each input that minimizes costs

$$\begin{cases} \min_{\{x_j\}} \sum_{j=1}^{M_b} p_j x_j \\ \text{s.t.} \\ x_b = \left(\sum_{j=1}^{M_{ib}} (q_j x_j)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \end{cases} \tag{20}$$

where η is the elasticity of substitution between inputs, that can be below one in case of complementarities, and M_b is the (endogenous) number of inputs incorporated in production.

In optimum, the nominal demand for input j writes:

$$p_j x_j = x_b \left(\frac{p_j}{q_j} \right)^{1-\eta} \left(\sum_{j=1}^{M_b} \left(\frac{p_j}{q_j} \right)^{1-\eta} \right)^{\frac{\eta}{1-\eta}}$$

and the marginal cost:

$$mc_b = \left(\sum_{j=1}^{M_b} \left(\frac{p_j}{q_j} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

Given the CES structure, it is optimal for the final good producer to choose the lowest quality-adjusted price for each input, among the set of input suppliers it has met. The love-for-variety property implies that M_b increases incrementally each type the final good producer meets with a supplier offering an input it does not already purchase.

C.1.3 Closing the model in general equilibrium

C.2 The distributions of buyers across sellers

The distribution of buyers among all suppliers. The overall quality-adjusted serving cost distribution is a mixture of the country-specific ones, noted $F_i(c)$, with

$$F_i(c) = \frac{\gamma_{iF}}{\gamma_i} F_{iF}(c) + \frac{\gamma_{i\bar{F}}}{\gamma_i} F_{i\bar{F}}(c) \tag{21}$$

for all $c \in \left]0, max\left(\frac{v_F d_{iF}}{\underline{z}}, \frac{v_{\bar{F}} d_{i\bar{F}}}{\underline{z}}\right)\right]$. Notice that the two distributions (F and $\bar{F}$) are defined for all c with, for example, $F_{iF}(c) = 1$ and $f_{iF}(c) = 0$ for $c > v_F d_{iF}/\underline{z}$. This ensure that $F_i(c)$ is continuous and properly defined on the whole support.

Let us now derive the distribution of buyers across sellers irrespective of the origin of the seller. That distribution is noted $L_i(c)$ and the corresponding flow equation is simply

$$\underbrace{B_i(1-u_i)\ell_i(c)\,(\mu+\gamma_i F_i(c))}_{\text{outflows}} = \underbrace{B_i(1-u_i)\gamma_i \bar{L}_i(c) f_i(c) + B u_i \gamma_i f_i(c)}_{\text{inflows}}$$

with $\bar{L}_i(c) \equiv 1 - L_i(c)$. The outflows are the sum of buyers exiting the market (μ) and buyers switching when they meet with a lower quality-adjusted cost supplier ($\gamma_i F_i(c)$). The inflows correspond to unmatched buyers meeting a cost-c supplier ($\gamma_i f_i(c)$) and buyers previously matched with sellers of serving cost higher than c ($\gamma_i \bar{L}_i(c) f_i(c)$). Using

$$u_i = \frac{\mu}{\gamma_i + \mu}$$

and simplying one gets

$$\ell_i(c)\,(\mu + \gamma_i F_i(c)) = \gamma_i \bar{L}_i(c) f_i(c) + \mu f_i(c)$$

Then integrating by part

$$L_i(c) = \frac{\mu + \gamma_i}{\mu + \gamma_i F_i(c)} F_i(c) \tag{22}$$

The distribution of buyers among French suppliers. Consider the shares of buyers matched with a French seller, π_{iF}. Again, in equilibrium, flows in and out are balanced such that the density of buyers matched with a French-seller at cost c, noted $\ell_{iF}(c)$, satisfies

$$\underbrace{(1-u_i)\pi_{iF}\ell_{iF}(c)\,(\mu+\gamma_i F_i(c))}_{\text{outflows}} = \underbrace{u_i \gamma_{iF} f_{iF}(c) + (1-u_i)\bar{L}_i(c)\gamma_{iF} f_{iF}(c)}_{\text{inflows}} \tag{23}$$

Substituting $\bar{L}_i(c) = 1 - L_i(c)$ by its expression in equation and using $u_i = \mu/(\mu+\gamma_i)$, one gets

$$\pi_{iF}\ell_{iF}(c) = \frac{\gamma_{iF}}{\gamma_i} \frac{\mu(\mu+\gamma_i)}{(\mu+\gamma_i F_i(c))^2} f_{iF}(c) \tag{24}$$

and similarly if we consider the density of buyers matched with non-French sellers

$$(1-\pi_{iF})\ell_{i\bar{F}}(c) = \frac{\gamma_{i\bar{F}}}{\gamma_i} \frac{\mu(\mu+\gamma_i)}{(\mu+\gamma_i F_i(c))^2} f_{i\bar{F}}(c) \tag{25}$$

The trade shares when $v_F d_{iF} < v_{\bar{F}} d_{i\bar{F}}$. We denote $c_F^{max} = v_F d_{iF}/\underline{z}$ the highest serving costs among French suppliers. Consider matches with French sellers, the flows in and out satisfy

$$(1-u_i)\pi_{iF}\left(\mu+\gamma_{i\bar{F}}\int_0^{c_F^{max}} F_{i\bar{F}}(c)\ell_{iF}(c)dc\right) = u_i\gamma_{iF}$$
$$+ (1-u_i)(1-\pi_{iF})\gamma_{iF}\left(\bar{L}_{i\bar{F}}(c^{max}) + \int_0^{c^{max}} F_{iF}(c)\ell_{i\bar{F}}(c)dc\right) \tag{26}$$

Note that, whenever $F_{iF}(c)$ and $F_{i\bar{F}}(c)$ have common support (that is up to c_F^{max}), we have $f_{iF}(c)/f_{i\bar{F}}(c) = \tau_{iF}^{-\theta}\ \forall c$. Combining equations (24) and (25),

$$(1-\pi_{iF})\tau_{iF}^{-\theta}\gamma_{iF}\ell_{i\bar{F}}(c) = \pi_{iF}\gamma_{i\bar{F}}\ell_{iF}(c) \tag{27}$$

Integrating up to c_F^{max} and simplifying, one gets

$$L_{i\bar{F}}(c_F^{max}) = \frac{\pi_{iF}}{(1-\pi_{iF})}\frac{\gamma_{i\bar{F}}}{\gamma_{iF}}\tau_{iF}^{\theta} \tag{28}$$

which can be substituted in (26) to obtain

$$(1-u_i)\pi_{iF}\left(\mu+\gamma_{i\bar{F}}\int_0^{c^{max}} F_{i\bar{F}}(c)\ell_{iF}(c)dc\right) = u_i\gamma_{iF} + (1-u_i)\left(\gamma_{iF} - \pi_{iF}(\gamma_{iF}+\gamma_{i\bar{F}}\tau_{iF}^{\theta})\right)$$
$$+ (1-u_i)(1-\pi_{iF})\gamma_{iF}\int_0^{c^{max}} F_{iF}(c)\frac{\pi_{iF}}{1-\pi_{iF}}\frac{\gamma_{i\bar{F}}}{\gamma_{iF}}\tau_{iF}^{\theta}\ell_{iF}(c)dc$$

The integrals cancel out and, after simplification, one gets

$$\pi_{iF} = \frac{\gamma_{iF}}{\gamma_{iF}+\gamma_{i\bar{F}}}\frac{\mu+\gamma_{iF}+\gamma_{i\bar{F}}}{\mu+\gamma_{iF}+\gamma_{i\bar{F}}\tau_{iF}^{\theta}} \tag{29}$$

The trade shares when $v_F d_{iF} > v_{\bar{F}} d_{i\bar{F}}$. We can derive in a similar manner the trade share, denoting $c_{\bar{F}}^{max} = v_{\bar{F}} d_{i\bar{F}}/\underline{z}$. We start with

$$(1-u_i)\pi_{iF}\left(\mu+\gamma_{i\bar{F}}\bar{L}_{iF}(c_{\bar{F}}^{max}) + \gamma_{i\bar{F}}\int_0^{c_{\bar{F}}^{max}} F_{i\bar{F}}(c)\ell_{iF}(c)dc\right) = u_i\gamma_{iF}$$
$$+ (1-u_i)(1-\pi_{iF})\gamma_{iF}\int_0^{c_{\bar{F}}^{max}} F_{iF}(c)\ell_{i\bar{F}}(c)dc \tag{30}$$

Since $(1-\pi_{iF})\gamma_{iF}\tau_{iF}^{-\theta}\ell_{i\bar{F}}(c) = \pi_{iF}\gamma_{i\bar{F}}\ell_{iF}(c)$, the integrals cancel out

$$(1-u_i)\pi_{iF}\left(\mu+\gamma_{i\bar{F}}\bar{L}_{iF}(c_{\bar{F}}^{max})\right) = u_i\gamma_{iF} \tag{31}$$

We get an expression for $\bar{L}_{iF}(c_{\bar{F}}^{max})$ by integrating (27) up to $c_{\bar{F}}^{max}$,

$$L_{iF}(c_{\bar{F}}^{max}) = \frac{1-\pi_{iF}}{\pi_{iF}}\frac{\gamma_{iF}}{\gamma_{i\bar{F}}}\tau_{iF}^{-\theta} \tag{32}$$

Finally, using that expression, we obtain

$$\pi_{iF} = \frac{\gamma_{iF}}{\gamma_{iF} + \gamma_{i\bar{F}}} \frac{\mu + (\gamma_{iF} + \gamma_{i\bar{F}})\tau_{iF}^{-\theta}}{\mu + \gamma_{iF}\tau_{iF}^{-\theta} + \gamma_{i\bar{F}}} \tag{33}$$

The share of French sellers when $\mu \approx 0$. Interestingly, when μ is close to zero, (29) and (33) are approximately equal

$$\pi_{iF} = \frac{\gamma_{iF}}{\gamma_{iF} + \gamma_{i\bar{F}}\tau_{iF}^{\theta}} \tag{34}$$

The trade shares. π_{iF} is the share of French sellers among the providers but it is also the trade share when we assume $\underline{z} \to 0$ as in Eaton and Kortum (2002). To demonstrate the equivalence, first notice that the demand of input j by a buyer reads

$$p_j x_j = \alpha_b \left(\frac{p_j}{q_j}\right)^{1-\eta} \left(\sum_{s=1}^{M_b} \left(\frac{p_s}{q_s}\right)^{1-\eta}\right)^{\frac{\eta}{1-\eta}} \tag{35}$$

where, given our price setting mechanism, p_j/q_j is the quality adjusted cost to serve of the second best supplier. Consider a buyer whose best supplier is French, the expected price-to-quality is

$$\mathbb{E}\left[p/q|F\right] = \int_0^c \int_0^{+\infty} \tilde{c}\ell(c)\ell_{iF}(\tilde{c}|c)d\tilde{c}dc$$

where $\ell_{iF}(\tilde{c}|c)$ denotes the pdf of the price distribution *conditional* on being matched with a French supplier c ($L_{iF}(\tilde{c}|c)$ the cdf).

Working with the complementary cumulative distribution, $\bar{L}_{iF}(\tilde{c}|c)$ we have in steady state

$$(1-u_i)\pi_{iF}\bar{L}_{iF}(\tilde{c}|c)\left(\mu + \gamma_i F_i(\tilde{c})\right) = \left(u_i + (1-u_i)\bar{L}_i(\tilde{c})\right)\gamma_{iF} f_{iF}(c) \tag{36}$$

and the equivalent cdf conditional on being match with a non-French supplier, $\bar{L}_{i\bar{F}}(\tilde{c}|c)$,

$$(1-u_i)(1-\pi_{iF})\bar{L}_{i\bar{F}}(\tilde{c}|c)\left(\mu + \gamma_i F_i(\tilde{c})\right) = \left(u_i + (1-u_i)\bar{L}_i(\tilde{c})\right)\gamma_{i\bar{F}} f_{i\bar{F}}(c) \tag{37}$$

Remark that

$$\begin{aligned}\frac{\gamma_{iF} f_{iF}(c)}{\pi_{iF}} &= \frac{\gamma_{iF} f_{iF}(c)(\gamma_{iF} + \gamma_{i\bar{F}}\tau_{iF}^{\theta})}{\gamma_{iF}} = \frac{\gamma_{i\bar{F}}}{\gamma_{i\bar{F}}}\tau_{iF}^{-\theta} f_{i\bar{F}}(c)(\gamma_{iF} + \gamma_{i\bar{F}}\tau_{iF}^{\theta}) \\ &= \frac{\gamma_{i\bar{F}} f_{i\bar{F}}(c)}{1-\pi_{iF}}\end{aligned}$$

Hence $L_{iF}(\tilde{c}|c) = L_{i\bar{F}}(\tilde{c}|c) = L_i(\tilde{c}|c)$ and $\mathbb{E}\left[p/q|F\right] = \mathbb{E}\left[p/q|\bar{F}\right]$. For that reason, the expected quantity doesn't depend on the country of origin of the supplier and the trade shares of France simply follows the share of French sellers in the buyers' portfolio.