Foreign Shocks as Granular Fluctuations*

Julian di Giovanni
ICREA-Universitat Pompeu Fabra
Barcelona GSE, CREI and CEPR

Andrei A. Levchenko
University of Michigan
NBER and CEPR

Isabelle Mejean
CREST-Ecole Polytechnique and CEPR

May 15, 2018

[PRELIMINARY]

Abstract

This paper uses a dataset covering the universe of French firm-level sales, imports, and exports over the period 1993-2007, and a quantitative multi-country model to study the international transmission of business cycle shocks at both the micro and the macro levels. The largest firms are both important enough to generate aggregate fluctuations (Gabaix, 2011), and most likely to be internationally connected. This implies that the largest firms are the key channel through which foreign shocks are transmitted to France. Our quantitative framework captures firm-level heterogeneity in both export and intermediate goods import markets. We first use the model to derive a theoretically-founded estimation equation relating a firm's sales growth to its exposure to foreign shocks via its intermediate input linkages. Using the model-implied estimating equation, we establish that firms that import intermediate inputs react significantly more to foreign shocks. Second, we calibrate the model to the observed firm- and country-level trade data, and perform counterfactual exercises intended to capture the response of individual French firms and macro aggregates to foreign shocks. The counterfactuals reveal the quantitative importance of "granular" firms in transmitting the foreign shocks to the French economy, due to the combination of their import and export linkages with foreign countries and their large size relative to the overall French economy.

JEL Classifications: E32; F15; F23; F44; F62; L14
Keywords: Shock transmission; Input linkages; International trade; Aggregate fluctuations

*We are grateful to Christopher Evans and William Haines for expert research assistance. Di Giovanni gratefully acknowledges the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 726168), and the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563) for financial support. Mejean gratefully acknowledges support from a public grant overseen by the French National Research Agency (ANR) as part of the "Investissements d'Avenir" program (Idex Grant Agreement No. ANR-11-IDEX-0003- 02/Labex ECODEC No. ANR-11-LABEX-0047 and Equipex reference: ANR-10-EQPX-17 Centre d'accès sécurisé aux données CASD) as well as the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 714597). E-mail (URL): julian.digiovanni@upf.edu (http://julian.digiovanni.ca), alev@umich.edu (http://www.alevchenko.com), isabelle.mejean@polytechnique.edu (http://www.isabellemejean.com).

1 Introduction

This paper studies the international transmission of business cycle shocks at the firm and the aggregate levels. After decades of globalization, the structure of production is increasingly international, with supply chains overlapping with country borders. An important feature of this internationalization of production is granularity: the largest firms are the ones responsible for the bulk of international trade linkages in a typical economy (e.g., Freund and Pierola, 2015). As a result, while only a minority of firms have direct trade linkages with foreign countries, those firms tend to account for a large share of aggregate economic activity (di Giovanni et al., 2017, 2018).

We study the consequences of this phenomenon for international shock transmission. Our main hypothesis is that foreign shocks, even if they are purely aggregate, affect firms differentially depending on the extent and nature of their international linkages. In that sense, an aggregate shock to a country's trading partners manifests itself as a set of idiosyncratic shocks to individual firms. The following simple expression conveys the impact of this heterogeneity on aggregate outcomes. Let ϵ^Y be the elasticity of GDP to a foreign shock, and let the economy be composed of a number of firms indexed by f. Then, ϵ^Y can be written as:

$$\epsilon^Y \quad = \quad \overline{\epsilon} + Cov\left(\frac{\omega_f}{\overline{\omega}}, \epsilon^f\right), \tag{1}$$

where ϵ^f is the elasticity of firm f's value added with respect to that same foreign shock, $\overline{\epsilon}$ is the unweighted average of ϵ^f across firms, and $\omega_f/\overline{\omega}$ is the share of firm f in aggregate value added relative to its unweighted average.

The response of GDP to a foreign shock is the sum of the average response of all firms to that shock, and the covariance across firms between sensitivity to that shock and relative size. In a model environment with a representative firm, the entire impact is captured by the first term, $\overline{\epsilon}$. When firms are heterogeneous in both size (ω_f) and sensitivity to foreign shocks (ϵ^f), then part of the impact of a foreign disturbance on GDP is due to the covariance term. We would expect this term to be positive, as large firms are more internationally connected, and thus disproportionately more affected by foreign shocks. Because the foreign shocks affect predominantly the largest firms in France, they lead to aggregate – granular – fluctuations.

Our analysis combines a dataset covering the universe of French firm sales and country-specific imports and exports over the period 1993-2007 with a quantitative multi-country multi-sector model with heterogeneous firms. The first step of the analysis is to establish a relationship between a firm's international linkages and its sensitivity to foreign shocks ϵ^f. We do this both econometrically and quantitatively. We use the model to derive a structural equation that relates a firm's destination-specific sales to the price of foreign inputs imported by that firm and the share of those foreign inputs in the total firm sales. The estimation exercise serves two purposes. The first is to provide econometric evidence that foreign shocks transmit to French imported input-using firms. We indeed show that internationally-connected firms react significantly more to foreign shocks than non-connected firms. The second product of this exercise is an estimate of the demand elasticity faced by firms, a key parameter in the quantitative assessment. We find this elasticity to be around 3.

The econometric estimates do not lend themselves well to aggregation, as they yield the relative impact of foreign shocks across firms, but not the overall impact. That is, the regression evidence relates the variation in ϵ^f to international linkages, but does not pin down either the level of individual ϵ^f's, nor their average $\overline{\epsilon}$. Thus, we employ the quantitative framework to simulate the effects of foreign shocks on the French economy. The model is calibrated to the observed firm-level information for France, and to the sector-level information for France's trading partners from the World Input-Output Database (WIOD). The model is general-equilibrium, and thus takes into account all the changes in wages, prices, and market shares in France and the rest of the world. As a result, this quantitative framework not only allows us to simulate the impact of a foreign shock on French GDP, but also to compute all the components of (1) and thus assess the role of granularity in the transmission of foreign shocks. Most importantly, since it is implemented on the complete data on firm imports, exports, and size, the model captures the full extent of heterogeneity across French firms in international linkages, as well as any relationship between those linkages and overall firm size. Thus, it is the right environment to quantify the impact of the $Cov\left(\omega_f/\overline{\omega}, \epsilon^f\right)$ term on aggregate outcomes.

We simulate 2 types of foreign shocks: a 10% productivity shock, and a 10% foreign demand shock for French goods. We examine both a global shock to all the countries other than France, and a shock to Germany, one of France's most important partners. We express results directly in terms of elasticities. The elasticity of French real GDP with respect of a 10% global productivity shock is 0.32 in our baseline calibration, meaning that French GDP increases by 3.2% following this shock. The impact of a German shock is predictably smaller, with an aggregate elasticity of 0.06. The elasticities of French GDP to a foreign demand shock are an order of magnitude smaller, which is expected since unlike foreign productivity, the foreign demand shock does not lower the costs of production in France.

Most importantly, the $Cov\left(\omega_f/\overline{\omega}, \epsilon^f\right)$ term accounts for 31-34% of the overall aggregate elasticity for the productivity counterfactual, and 27-32% in the demand shock counterfactual. We perform two alternative exercises to establish that this quantitatively important covariance term is a consequence of firm heterogeneity in international linkages. In the first, we simulate the economy's response to the same shocks in a model with homogeneous firms in each sector. That is, we assign to each firm within a sector the exact same export and import linkages. The aggregate impact of the foreign productivity shocks is about 20% lower than in the baseline, suggesting that heterogeneity has the potential to amplify foreign shocks. The share accounted for by the covariance term falls to zero for the productivity shock, and to 5-15% for the foreign demand shock. We also compute the covariance term at the sector instead of the firm level in the baseline model. Following a productivity shock, the covariance term across sectors is positive, whereas for the foreign demand shock the covariance term is negative, implying that the largest sectors actually have lower elasticities with respect to foreign demand shocks. Both of these alternative exercises illustrate that it is the firm, rather than sectoral, heterogeneity in the international linkages that matters.

The paper draws on the active closed-economy literature on the propagation of shocks in production networks (Carvalho, 2010; Acemoglu et al., 2012; Barrot and Sauvagnat, 2016; Baqaee, 2016; Carvalho et al., 2016; Atalay, 2017; Tintelnot et al., 2017), and the importance of large firms in aggregate fluctuations (Gabaix, 2011; di Giovanni et al., 2014; Carvalho and Grassi, 2015). We apply the insights and tools from this literature to the international transmission of shocks. The international business cycle literature is vast, but by and large has not used firm-level data in empirical and quantitative assessments of international comovement. The few recent exceptions include Kleinert et al. (2015), Boehm et al. (2017), Cravino and Levchenko (2017), Blaum (2018), and di Giovanni et al. (2018).

The rest of the paper is organized as follows. Section 2 presents a multi-country general equilibrium model of trade, featuring firm heterogeneity and input-output linkages. Section 3 describes the data and the construction of firm-level variables. It documents the extent of heterogeneity in firms' exposure to foreign shocks, whether on the demand or the supply sides. Section 4 provides empirical evidence on the importance of this heterogeneity for the transmission of foreign shocks on the domestic economy. Section 5 calibrates the model to quantify the importance of the cross-border transmission of shocks on firms and countries. Section 6 concludes.

2 Quantitative Framework

This section presents the theoretical framework used in the empirical and quantitative exercises. We build a heterogenous-firm, multi-country, multi-sector model of trade. Crucially, we allow for heterogeneity of input linkages at the firm level, as well as heterogeneity across export markets.[1] The model features endogenous factor supply so that we can analyze how domestic and foreign shocks are transmitted to aggregate fluctuations.

The world is comprised of M countries and J sectors. Time is denoted by t, countries by m, n, and k, sectors by i and j, and firms by f and g. The notation follows the convention that the first subscript always denotes exporting (source) country, and the second subscript the importing (destination) country.

[1] We only have firm-level data for France, and thus for the other countries the model collapses to an international trade model with sector-level input-output linkages that is standard in the literature (see, e.g. the Handbook chapter by Costinot and Rodríguez-Clare, 2014).

Households There are $\overline{L}_n$ households in country n. Each one consumes goods and supplies labor. Their income includes profits of domestically-owned firms. Preferences over consumption and leisure are GHH (Greenwood et al., 1988):

$$U\left(\{c_{n,t}, l_{n,t}\}_{t=0}^{\infty}\right) = \sum_{t=0}^{\infty} \delta^t \nu \left(c_{n,t} - \frac{\psi_0}{\bar{\psi}} l_{n,t}^{\bar{\psi}}\right),$$

where $c_{n,t}$ is per-capita consumption, $l_{n,t}$ the per-capita labor supply, and the function ν is increasing and concave. Note that the $l_{n,t}$ should be thought of as "equipped labor" (Alvarez and Lucas, 2007), and thus captures the supply of all the primary factors.

The final consumption aggregate is Cobb-Douglas in the j sectors, with expenditure shares ϑ_j:

$$c_{n,t} = \prod_j c_{n,j,t}^{\vartheta_j},$$

where $c_{n,j,t}$ is the per capita final consumption of sector j. Therefore, the ideal consumption price index is:

$$P_{n,t} = \prod_j \left(\frac{P_{n,j,t}}{\vartheta_j}\right)^{\vartheta_j},$$

where $P_{n,j,t}$ is the price index of sector j goods in country n at time t.

Denote by $\Pi_{n,t}$ the aggregate profits of firms owned by households in n, and by $D_{n,t}$ any trade imbalance in period t. Assume that both $\Pi_{n,t}$ and $D_{n,t}$ are divided equally among households in n (as will become clear below, under the GHH and homothetic preferences, this assumption affects neither the labor supply decision nor the allocation of consumption expenditure across j). Straightforward steps lead to the following labor supply:

$$L_{n,t} = \left(\frac{1}{\psi_0}\frac{w_{n,t}}{P_{n,t}}\right)^{\frac{1}{\bar{\psi}-1}} \overline{L}_n,$$

where $w_{n,t}$ is the wage in country n at time t.

Denote by $C_{n,t} \equiv c_{n,t}\overline{L}_n$ the aggregate final consumption in country n, and let $C_{n,j,t} \equiv c_{n,j,t}\overline{L}_n$ be the aggregate final consumption of sector j. Countries m sell (export) to country n. Origin-specific output is apportioned to consumption and intermediate input usage. Let each sector's consumption be aggregated from origin-specific components:

$$C_{n,j,t} = \left[\sum_m \mu_{mn,j}^{\frac{1}{\sigma_j}} C_{mn,j,t}^{\frac{\sigma_j - 1}{\sigma_j}}\right]^{\frac{\sigma_j}{\sigma_j - 1}},$$

where $C_{mn,j,t}$ is final consumption of imports from country m in sector j, country n. Then the price index for consumption in sector j, country n is:

$$P_{n,j,t} = \left[\sum_m \mu_{mn,j} P_{mn,j,t}^{1-\sigma_j}\right]^{\frac{1}{1-\sigma_j}},$$

where $P_{mn,j,t}$ is the price index for exports from m to n in sector j, defined below. Final demand for goods from m is:

$$P_{mn,j,t}C_{mn,j,t} = \frac{\mu_{mn,j}P_{mn,j,t}^{1-\sigma_j}}{P_{n,j,t}^{1-\sigma_j}}P_{n,j,t}C_{n,j,t} = \frac{\mu_{mn,j}P_{mn,j,t}^{1-\sigma_j}}{P_{n,j,t}^{1-\sigma_j}}\vartheta_j P_{n,t}C_{n,t}.$$

Then:

$$P_{mn,j,t}C_{mn,j,t} = \frac{\mu_{mn,j}P_{mn,j,t}^{1-\sigma_j}}{P_{n,j,t}^{1-\sigma_j}}\vartheta_j \left[w_{n,t}\left(\frac{1}{\psi_0}\frac{w_{n,t}}{P_{n,t}}\right)^{\frac{1}{\psi-1}}\overline{L}_n + \Pi_{n,t} + D_{n,t}\right].$$

Note that we use the French customs data for imports at the firm level, and thus every import transaction is associated with a French firm (which may be wholesaler or a retailer). Thus, French final consumers are never observed to import final consumption goods directly, and as a result French final consumption is composed only of domestically-supplied final goods. Formally, when $n = France$, $\mu_{mn,j} = 0\ \forall\ m \neq n$, and:

$$P_{n,j,t} = P_{nn,j,t},$$
$$P_{nn,j,t}C_{nn,j,t} = P_{n,j,t}C_{n,j,t} = \vartheta_j \left[w_{n,t}\left(\frac{1}{\psi_0}\frac{w_{n,t}}{P_{n,t}}\right)^{\frac{1}{\psi-1}}\overline{L}_n + \Pi_{n,t} + D_{n,t}\right],$$

where $P_{nn,j,t}$ is the ideal price index of output produced by French firms in France. For all the other countries, we do not have firm-level data on imports, but instead have final consumption data by source country from WIOD. Thus, we assume that foreign consumers import final goods directly.

Sectors Sectors are populated by heterogeneous, monopolistically-competitive firms. Not all firms sell to all destinations. Denote by $\Omega_{mn,j}$ the set of firms from country m, sector j that sell to country n. The CES aggregate of output in sector j of firms from m selling in country n is:

$$Q_{mn,j,t} = \left[\sum_{f\in\Omega_{mn,j}} \xi_{mn,j,t}(f)^{\frac{1}{\rho_j}} Q_{mn,j,t}(f)^{\frac{\rho_j-1}{\rho_j}}\right]^{\frac{\rho_j}{\rho_j-1}}, \tag{2}$$

where $Q_{mn,j,t}(f)$ is the quantity of firm f's good from country m and sector j selling to country n.[2] The taste shock to a firm's destination-specific sales $\xi_{mn,j,t}(f)$ is at this point left unrestricted. It could be allowed to have a firm-specific global component, and/or a source-destination-sector common component across firms. The latter would be isomorphic to $\mu_{mn,j}$ in the cross-section. The price level of the aggregate of sellers from m in n, j, t is:

$$P_{mn,j,t} = \left[\sum_{f\in\Omega_{mn,j}} \xi_{mn,j,t}(f)p_{mn,j,t}(f)^{1-\rho_j}\right]^{\frac{1}{1-\rho_j}},$$

[2] In the counterfactual experiments below, we assume that following a foreign shock, the sets of firms serving each market $\Omega_{mn,j}$ are unchanged. See di Giovanni et al. (2014, 2018) for evidence that the extensive margin adjustments are not quantitatively important at the business cycle frequency.

where $p_{mn,j,t}(f)$ is the price charged by firm f in country n.

Let X denote expenditure (at each level of aggregation). Then demand faced by firm f in country n is:

$$X_{mn,j,t}(f) = \xi_{mn,j,t}(f) \frac{p_{mn,j,t}(f)^{1-\rho_j}}{P_{mn,j,t}^{1-\rho_j}} X_{mn,j,t}.$$

Thus, $X_{mn,j,t}$ is the total value exports from m to n in sector j at t, and $X_{mn,j,t}(f)$ is the value of exports by firm f.

Firms Firms face downward-sloping demand and set price equal to a constant markup $\frac{\rho_j}{\rho_j - 1}$ over the marginal cost. Firms located in m face an iceberg cost of $\tau_{mn,j}$ to export to n. They have a unit input requirement $a_t(f)$, and the cost of the input bundle

$$b_{m,j,t}(f) = w_{m,t}^{\alpha_{m,j}(f)} \left(\prod_i \prod_k P_{km,i,t}^{\gamma_{km,ij}(f)} \right)^{1-\alpha_{m,j}(f)},$$

where $\alpha_{m,j}(f)$ is the share of expenditure on labor in total costs, $\gamma_{km,ij}(f)$ is the use of inputs sourced from country k sector i by firm f operating in country m, sector j, such that $\sum_{i,k} \gamma_{km,ij}(f) = 1$. That is, firms in m use inputs from potentially all countries k in each sector i, with firm-specific expenditure shares $\gamma_{km,ij}(f)$. Some of these will be zero, i.e. the firm does not use inputs in a particular sector from a particular country. For French firms, $\gamma_{km,ij}(f)$ are read directly from the data for imported inputs while the domestic input-output linkages are inferred using firm-level data on input usage and sector-level information on domestic IO linkages – see Section 3 for details. Sales by firm f from country m in destination n are then

$$X_{mn,j,t}(f) = \xi_{mn,j,t}(f) \frac{\left(\frac{\rho_j}{\rho_j - 1} \tau_{mn,j} b_{m,j,t}(f) a_t(f) \right)^{1-\rho_j}}{P_{mn,j,t}^{1-\rho_j}} X_{mn,j,t}.$$

Heterogeneity in firm size is thus driven by productivity, taste/quality, and differences in input sourcing across firms. To illustrate, the share of firm f's sales in total sales by domestic firms to the home market in sector j is:

$$\pi_{mm,j,t}(f) = \frac{\xi_{mm,j,t}(f) \left(w_{m,t}^{\alpha_{m,j}(f)} \left(\prod_i \prod_k P_{km,i,t}^{\gamma_{km,ij}(f)} \right)^{1-\alpha_{m,j}(f)} a_t(f) \right)^{1-\rho_j}}{\sum_{g \in \Omega_{mm,j}} \xi_{mm,j,t}(g) \left(w_{m,t}^{\alpha_{m,j}(g)} \left(\prod_i \prod_k P_{km,i,t}^{\gamma_{km,ij}(g)} \right)^{1-\alpha_{m,j}(g)} a_t(g) \right)^{1-\rho_j}}.$$

Sales dispersion across firms in the same market is generated by differences in productivity $a_t(f)$, the taste shifter $\xi_{mm,j,t}(f)$, and the fact that sourcing shares $\gamma_{km,ij}(f)$ differ across firms (even though we assume that all firms face the same input prices $P_{km,i,t}$). As will become clear below, we will not need to take a stand on the levels of $a_t(f)$ and $\xi_{mm,j,t}(f)$. Instead the counterfactual exercises will use the observed shares such as $\pi_{mn,j,t}(f)$ directly to calibrate the model at the baseline period and then use the equilibrium conditions to compute the changes in those $\pi_{mn,j,t}(f)$'s between the baseline and the counterfactual equilibrium.

Equilibrium Market clearing for exports from m to n in sector j is:

$$X_{mn,j,t} = \frac{\mu_{mn,j} P_{mn,j,t}^{1-\sigma_j}}{P_{n,j,t}^{1-\sigma_j}} \vartheta_j \left[w_{n,t} \left(\frac{1}{\psi_0} \frac{w_{n,t}}{P_{n,t}} \right)^{\frac{1}{\psi-1}} \overline{L}_n + \Pi_{n,t} + D_{n,t} \right] \tag{3}$$

$$+ \sum_i \sum_{f \in i} (1 - \alpha_{n,i}(f)) \gamma_{mn,ji}(f) \frac{\rho_i - 1}{\rho_i} \sum_k \frac{\xi_{nk,i,t}(f) \left(\frac{\rho_i}{\rho_i - 1} \tau_{nk,i} b_{n,i,t}(f) a_t(f) \right)^{1-\rho_i}}{P_{nk,i,t}^{1-\rho_i}} X_{nk,i,t}.$$

In this expression, the first line is the final demand, and the second is the intermediate demand. Note that the intermediate demand is a summation of firm-level intermediate demands, and thus captures the notion that not all firms, even within the same sector, will import inputs from a particular foreign sector-country with the same intensity. Price indices are:

$$P_{mn,j,t} = \left[\sum_{f \in \Omega_{mn,j}} \xi_{mn,j,t}(f) \left(\frac{\rho_j}{\rho_j - 1} \tau_{mn,j} b_{m,j,t}(f) a_t(f) \right)^{1-\rho_j} \right]^{\frac{1}{1-\rho_j}} \tag{4}$$

$$= \frac{\rho_j}{\rho_j - 1} \tau_{mn,j} \left[\sum_{f \in \Omega_{mn,j}} \xi_{mn,j,t}(f) \left(w_{m,t}^{\alpha_{m,j}(f)} \left(\prod_i \prod_k P_{km,i,t}^{\gamma_{km,ij}(f)} \right)^{1-\alpha_{m,j}(f)} a_t(f) \right)^{1-\rho_j} \right]^{\frac{1}{1-\rho_j}}.$$

Total labor compensation in the sector is the sum of firm-level expenditures on labor:

$$w_{n,t} L_{n,j,t} = \frac{\rho_j - 1}{\rho_j} \sum_{f \in j} \alpha_{n,j}(f) \sum_k X_{nk,j,t}(f)$$

$$= \frac{\rho_j - 1}{\rho_j} \sum_{f \in j} \alpha_{n,j}(f) \sum_k \frac{\xi_{nk,j,t}(f) \left(\frac{\rho_j}{\rho_j - 1} \tau_{nk,j} b_{n,j,t}(f) a_t(f) \right)^{1-\rho_j}}{P_{nk,j,t}^{1-\rho_j}} X_{nk,j,t}$$

Labor market clearing ensures that real wages adjust to equate the aggregate labor demand (left-hand side) with labor supply:

$$\left(\frac{1}{\psi_0} \frac{w_{n,t}}{P_{n,t}} \right)^{\frac{1}{\psi-1}} \overline{L}_n = \sum_j L_{n,j,t} \tag{5}$$

$$= \frac{1}{w_{n,t}} \sum_j \frac{\rho_j - 1}{\rho_j} \sum_{f \in j} \alpha_{n,j}(f) \sum_k \frac{\xi_{nk,j,t}(f) \left(\frac{\rho_j}{\rho_j - 1} \tau_{nk,j} b_{n,j,t}(f) a_t(f) \right)^{1-\rho_j}}{P_{nk,j,t}^{1-\rho_j}} X_{nk,j,t}.$$

Equations (3), (4), and (5) are a system of equations that define equilibrium wages, prices, and expenditures.

2.1 The Role of Heterogeneity

Let $Y_{m,t}$ denote aggregate GDP in country m, and let $Y_{m,t}(f)$ denote the value added of firm f. We are interested in evaluating the elasticity of GDP with respect to a foreign shock χ. Define $\epsilon^Y \equiv \frac{d \ln Y_{m,t}}{d \ln \chi}$ to be the elasticity of m's GDP with respect to the shock, $\epsilon^f \equiv \frac{d \ln Y_{m,t}(f)}{d \ln \chi}$ the elasticity of the value added of firm f with respect to the shock, and $\omega_{m,t}(f) \equiv \frac{Y_{m,t}(f)}{Y_{m,t}}$ the share of firm f in total value added. GDP is just the sum of firm-level value added:

$$Y_{m,t} = \sum_f Y_{m,t}(f).$$

Therefore, the aggregate elasticity with respect to the foreign shock is a weighted sum of firm-level elasticities:

$$\epsilon^Y = \sum_f \omega_{m,t}(f) \epsilon^f.$$

The aggregate elasticity can then be written as:

$$\epsilon^Y = \overline{\epsilon} + Cov\left(\frac{\omega_{m,t}(f)}{\overline{\omega}}, \epsilon^f\right),$$

where $\overline{\epsilon} \equiv \frac{1}{N}\sum_f \epsilon^f$ is the unweighted average elasticity to the shock across firms, $\overline{\omega} \equiv \frac{1}{N}\sum_f \omega_{m,t}(f) = \frac{1}{N}$ is the average share of a firm in the total GDP, and N the total number of firms. Thus, the responsiveness of GDP to a shock is determined by the average responsiveness of all firms in the economy to this shock, and the covariance between firm size with its responsiveness to the shock. Writing the aggregate elasticity this way helps illustrate the role of granularity in international shock transmission. Since the largest firms are more likely to be internationally connected, we would expect them to have higher ϵ^f, and thus $Cov\left(\frac{\omega_{m,t}(f)}{\overline{\omega}}, \epsilon^f\right) > 0$.

What are the the reasons that firms will differ in their ϵ^f? With some manipulation, we can write the approximate log change in value added of firm f as:

$$d \ln Y_{m,j,t}(f) \approx (1-\rho_j)\left[\alpha_{m,j}(f) d \ln w_{m,t} + \sum_i \sum_k (1-\alpha_{m,j}(f))\gamma_{km,ij}(f) d \ln P_{km,i,t}\right] + \sum_n \widetilde{s}_{mn,j,t}(f) d \ln \left[\xi_{mn,j,t}(f)\left(\frac{\tau_{mn,j}}{P_{mn,j,t}}\right)^{1-\rho_j} X_{mn,j,t}\right], \quad (6)$$

where the summation over n is a summation over all the markets firm f actually serves, and $\widetilde{s}_{mn,j,t}(f)$ is the share of market n in the total gross sales of firm f. Thus, a firm that only serves the domestic market has $\widetilde{s}_{mm,j,t}(f) = 1$ and $\widetilde{s}_{mn,j,t}(f) = 0 \; \forall \, n \neq m$.

The first term in (6) captures the change in the firm's costs, and the second term the change in the firm's demand following any external shock. Equation (6) highlights the sources of heterogeneity. On the cost side, following a shock in country k, only firms that import from k – $\gamma_{km,ij}(f) \neq 0$

– directly experience a change in input costs. At the same time, the change in foreign demand – be it from the price-adjusted foreign expenditure $X_{mn,j,t}/P_{mn,j,t}^{1-\rho_j}$, or from a taste ($\xi_{mn,j,t}(f)$) or trade cost shock – will to first order affect only firms that export to country n, and even among those firms will vary with the sales share to that market.

At the same time, this expression underscores the general-equilibrium channels that will in principle operate and thus should be accounted for. To the extent that the foreign shock changes domestic wages ($d\ln w_{m,t}$), all firms in m will be affected. Also, all firms sell domestically. Thus, if the foreign shock affects domestic demand $d\ln\left(X_{mm,j,t}/P_{mm,j,t}^{1-\rho_j}\right)$, it will reach all firms in m. Finally, it could be that through second-order input linkages, even the non-importing firms' input prices $d\ln P_{mm,i,t}$ change.

It is ultimately an empirical and quantitative question how much ϵ^f varies across firms, and how it covaries with firm size. In particular, the relative importance of the direct effects on the connected firms and the general equilibrium effects on all firms in the economy has not been established. This is the main question addressed in the empirical and quantitative analysis below.

2.2 A Shock Formulation of the Model

To perform counterfactuals that simulate the impact of foreign shocks on domestic firms and the aggregate economy, we follow the approach of Dekle et al. (2008) and express the equilibrium conditions in terms of gross changes $\widehat{x}_{t+1} = x_{t+1}/x_t$ in endogenous variables, to be solved for as a function of shocks expressed in gross changes, and initial (time-t) observables. Starting with (3), we write it as a function of observed initial expenditure shares:

$$X_{mn,j,t} = \pi_{mn,j,t}^c \pi_{n,j,t}^c \left[w_{n,t} \left(\frac{1}{\psi_0} \frac{w_{n,t}}{P_{n,t}} \right)^{\frac{1}{\psi-1}} \overline{L}_n + \Pi_{n,t} + D_{n,t} \right] + \sum_i \frac{\rho_i - 1}{\rho_i} \sum_{f\in i} (1-\alpha_{n,i}(f))\gamma_{mn,ji}(f) \sum_k \pi_{nk,i,t}(f) X_{nk,i,t}, \qquad (7)$$

where $\pi_{mn,j,t}^c$ is the share of final consumption spending on goods from m in the total consumption spending on goods in sector j, country n, $\pi_{n,j,t}^c = \vartheta_j$ is simply the share of sector j in total final consumption spending, and $\pi_{nk,i,t}(f)$ is the share of firm f in the total exports from country n to country k in sector i. All of these π's are observable when n = France. $\pi_{mn,j,t}^c$ and $\pi_{n,j,t}^c$ are observable in WIOD. $\pi_{nk,i,t}(f)$ when neither n nor k are France is not observable, so would require an assumption on which firms use imported intermediates. Since we do not have firm-level information on other countries, we assume that in those countries there is a representative firm in each sector. Writing out the shares:

$$\pi^c_{n,j,t} = \vartheta_j,$$
$$\pi^c_{mn,j,t} = \frac{\mu_{mn,j} P^{1-\sigma_j}_{mn,j,t}}{P^{1-\sigma_j}_{n,j,t}} = \frac{\mu_{mn,j} P^{1-\sigma_j}_{mn,j,t}}{\sum_k \mu_{kn,j} P^{1-\sigma_j}_{kn,j,t}},$$
$$\pi_{nk,i,t}(f) = \frac{\xi_{nk,i,t}(f)\left(\frac{\rho_i}{\rho_i-1}\tau_{nk,i} b_{n,i,t}(f) a_t(f)\right)^{1-\rho_i}}{P^{1-\rho_i}_{nk,i,t}}.$$

Then, in proportional changes, (7) can be written as:

$$\widehat{X}_{mn,j,t+1} X_{mn,j,t} = \pi^c_{mn,j,t+1}\pi^c_{n,j,t+1}\left[\widehat{w}_{n,t+1}\left(\frac{\widehat{w}_{n,t+1}}{\widehat{P}_{n,t+1}}\right)^{\frac{1}{\psi-1}} s^L_{n,t} + \widehat{\Pi}_{n,t+1} s^{\Pi}_{n,t} + \widehat{D}_{n,t+1} s^D_{n,t}\right] P_{n,t} C_{n,t} \tag{8}$$
$$+\sum_i \frac{\rho_i-1}{\rho_i}\sum_{f\in i}(1-\alpha_{n,i}(f))\gamma_{mn,ji}(f)\sum_k \pi_{nk,i,t+1}(f)\widehat{X}_{nk,i,t+1}X_{nk,i,t},$$

where $s^L_{n,t}$ is the share of labor (more generally factor payments) in the total final consumption expenditure at time t, and same for $s^{\Pi}_{n,t}$ and $s^D_{n,t}$.

Equation (5) is expressed in changes as:

$$\sum_j \sum_{f\in j}\sum_k \frac{\rho_j-1}{\rho_j}\alpha_{n,j}(f)\pi_{nk,j,t}(f)X_{nk,j,t}\left[\widehat{\pi}_{nk,j,t+1}(f)\widehat{X}_{nk,j,t+1} - \widehat{w}^{\frac{\bar{\psi}}{\bar{\psi}-1}}_{n,t+1}\widehat{P}^{\frac{1}{1-\bar{\psi}}}_{n,t+1}\right] = 0. \tag{9}$$

The prices (4) are expressed in changes as:

$$\widehat{P}_{mn,j,t+1} = \left[\sum_{f\in\Omega_{mn,j}} \pi_{mn,j,t}(f)\widehat{\xi}_{mn,j,t+1}(f)\left(\widehat{w}^{\alpha_{m,j}(f)}_{m,t+1}\left(\prod_i\prod_k \widehat{P}^{\gamma_{km,ij}(f)}_{km,i,t+1}\right)^{1-\alpha_{m,j}(f)}\widehat{a}_{t+1}(f)\right)^{1-\rho_j}\right]^{\frac{1}{1-\rho_j}}, \tag{10}$$

$$\widehat{P}_{n,j,t+1} = \left[\sum_i \widehat{P}^{1-\sigma_j}_{mn,j,t+1}\pi^c_{mn,j,t}\right]^{\frac{1}{1-\sigma_j}}, \tag{11}$$

$$\widehat{P}_{n,t+1} = \prod_j \widehat{P}^{\vartheta_j}_{n,j,t+1}. \tag{12}$$

Finally, the expressions above require knowing next period's π's. These can be expressed as:

$$\pi^c_{mn,j,t+1} = \frac{\widehat{P}^{1-\sigma_j}_{mn,j,t+1}\pi^c_{mn,j,t}}{\sum_k \widehat{P}^{1-\sigma_j}_{kn,j,t+1}\pi^c_{kn,j,t}}, \tag{13}$$

and

$$\pi_{nk,j,t+1}(f) = \frac{\widehat{\xi}_{nk,j,t+1}(f)\left(\widehat{w}^{\alpha_{n,j}(f)}_{n,t+1}\left(\prod_i\prod_m \widehat{P}^{\gamma_{mn,ij}(f)}_{mn,i,t+1}\right)^{1-\alpha_{n,j}(f)}\widehat{a}_{t+1}(f)\right)^{1-\rho_j}\pi_{nk,j,t}(f)}{\sum_{g\in\Omega_{nk,j}}\widehat{\xi}_{nk,j,t+1}(g)\left(\widehat{w}^{\alpha_{n,j}(g)}_{n,t+1}\left(\prod_i\prod_m \widehat{P}^{\gamma_{mn,ij}(g)}_{mn,i,t+1}\right)^{1-\alpha_{n,j}(g)}\widehat{a}_{t+1}(g)\right)^{1-\rho_j}\pi_{nk,j,t}(g)}. \tag{14}$$

2.3 GDP Accounting in the Model

GDP is real value added. We follow the national accounting practices and deflate nominal value added by the GDP deflator, defined implicitly as the ratio between nominal GDP and the aggregate value added evaluated at the base period prices.[3] In our framework, nominal value added associated with firm f's sales to market n is a constant fraction of its sales there:

$$Y^{NOM}_{mn,j,t}(f) = \frac{1+\alpha_{m,j}(f)(\rho_j - 1)}{\rho_j} X_{mn,j,t}(f),$$

and thus total firm value added is given by:

$$Y^{NOM}_{m,j,t}(f) = \frac{1+\alpha_{m,j}(f)(\rho_j - 1)}{\rho_j} \sum_n X_{mn,j,t}(f),$$

where the summation is over the markets the firm actually serves.

GDP is simply the sum over all firm-level value added, as in (6). Expressed in gross changes it becomes:

$$\widehat{Y}^{NOM}_{m,t+1} = \sum_f \sum_n \omega_{m,j,t}(f) \widetilde{s}_{mn,j,t}(f) \widehat{X}_{mn,j,t+1}(f),$$

where, as in Section 2.1, $\omega_{m,j,t}(f)$ is the share of firm f's value added in total GDP, and $\widetilde{s}_{mn,j,t}(f)$ is the share of sales to n in firm f's total gross sales. Finally, the aggregate outcome of interest is the real GDP change:

$$\widehat{Y}_{m,t+1} = \frac{\widehat{Y}^{NOM}_{m,t+1}}{\widehat{P}^{G}_{m,t+1}}, \tag{15}$$

where $\widehat{P}^{G}_{m,t+1}$ is the GDP deflator. Appendix A presents the formulas underlying the construction of the GDP deflator. When implementing the decomposition (1), we deflate each firm's value added growth with the GDP deflator, since doing this way ensures that aggregate real GDP is the sum of all firms' real value added.

3 Data

The data exploited for the estimation and the quantitative exercises are a combination of various sources and are used to calibrate the model's variables at different levels of aggregation. We use standard multi-country datasets to measure aggregate and sectoral variables such as the value of bilateral gross sales or the strength of input-output relationships, for each pair of sectors. This information is combined with firm-level variables obtained from French administrative data. The

[3] An alternative would be to deflate nominal value added by the CPI, which is to be precise the ideal consumption price index $P_{m,t}$. The two differences between the GDP deflator and the CPI are that (i) the CPI includes the prices of foreign final consumption imports and (ii) the CPI does not include the prices of domestically-produced intermediates. The results when deflating by the CPI are similar and available upon request.

use of micro-level data for one country allows us to capture one key element of the model, namely the heterogeneous exposure of individual firms to foreign shocks. While such heterogeneity obviously exists in all countries, firm-level information this level of detail is not available for multiple countries. As a consequence, we will study the impact of firm heterogeneity using the French firm-level data, assuming away heterogeneity within sectors in the rest of the country sample.

3.1 Aggregate and Sectoral Variables

The main source of data at the multilateral, sectoral level is the *World Input Output Database* (WIOD) (Timmer et al., 2015). This dataset combines national input-output tables and data on bilateral trade flows to reconstitute the matrix of all intra- and international flows of goods and services between sectors and final consumers as well as across sector pairs in the case of trade in intermediates. We use the 2013 release of the dataset which covers 40 countries plus a rest of the world aggregate and 35 sectors classified according to the ISIC Revision 3 nomenclature. These data are available over 1995 to 2011 and the benchmark year for the calibration is 2005.

The dataset can be used to recover: i) final consumption spending ($P_{n,t}C_{n,t}$ in the model in Section 2); ii) the value of bilateral sales by sector ($X_{mn,j,t}$); and iii) the sectoral production function parameters, which are used whenever more disaggregated data are not available. The assumption of a unitary elasticity in the aggregation of intermediate inputs and primary factors implies that one needs to measure the share of labor in country n sector j's total costs ($\alpha_{n,j}$) as well as the components of the input-output matrix, as measured by the share of inputs sourced from country m sector j by firms operating in country n sector i ($\gamma_{mn,ji}$). The IO coefficients are readily available from the WIOD. Labor shares are measured by the ratio of value added over output, to be consistent with the interpretation of L as "equipped labor."

As will become clear below, the estimation exercise requires information on the prices of foreign imported inputs. We proxy those prices by the sectoral PPIs, collected and harmonized for a large sample of countries by Auer et al. (2017). Appendix B describes in detail the process to impute prices when they are unobserved (which is predominantly in the services sectors). This leaves us with a measure of input price shocks $d\ln P_{km,i,t}$. Finally, the growth in the French wage rate $d\ln w_{m,t}$ is proxied by the annual growth in French wages, sourced from the OECD.

3.2 Firm-Level Variables

To take into account the heterogeneity in individual firms' exposure to foreign shocks, we exploit detailed data covering the universe of French firms over 1993-2007.[4] The dataset contains balance sheet information collected from individual firms' tax forms, and includes sales, value added, cost

[4]These data are described in further detail in di Giovanni et al. (2018).

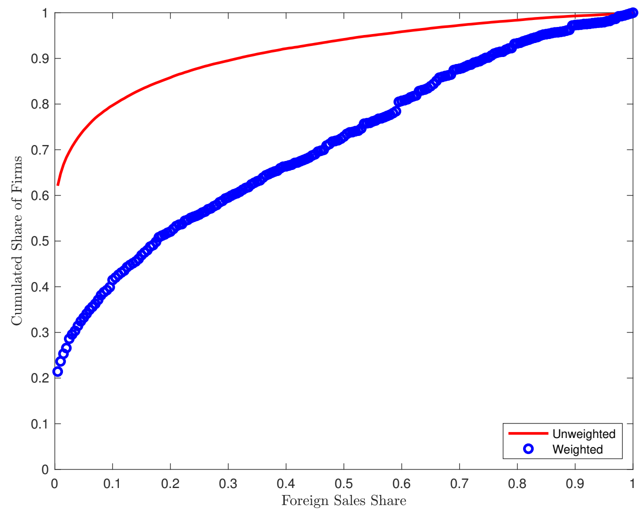
structure, as well as its sector of activity.[5] The partition of firms across sectors is described in Table A1, for 2005. Interestingly, the most important sector in terms of its contribution to aggregate value added is the one providing "Business Activities" to the rest of the economy. This underscores how important input-output relationships are to the functioning of modern economies. More generally, non-traded good sectors are a large share of the French economy, accounting for more than 80% of firms in our sample and 72% of the value added. The comparison of these two numbers indicates that non-traded sector firms tend to be relatively small. There are some exceptions, however. For instance, firms in the "Post and Telecommunications" or the "Air Transport" sectors are relatively large.

This dataset is complemented with customs data on bilateral export and import flows at the firm level. Since these are customs data, they do not include trade in services. However, goods trade by service sector firms is observed. Export data can be used to refine the definition of sales to the level of destination market ($X_{mn,j,t}(f)$ for $m = France$). Following di Giovanni et al. (2014) this is done by first allocating sales to the domestic or foreign market using the information available in the tax files on domestic and export sales. The foreign component of demand is then further decomposed by destination using the customs data.

The firm-level bilateral trade flows can also be used to obtain the allocation of bilateral sectoral sales across firms constituting a sector (namely $\pi_{mn,j,t}(f) \equiv X_{mn,j,t}(f)/X_{mn,j,t}$). Calibrating these shares is key in understanding how firms within the same sector can be differently exposed to the same foreign shock. Such heterogeneity is illustrated in Figure 1, which plots the cumulative distribution of firms according to their degree of openness. The solid (red) line depicts the unweighted distribution and the (blue) circles the distribution weighted by their share in overall value added. As already well-known from the trade literature, these cumulative distributions reveal a substantial amount of heterogeneity across firms regarding the extent of their export activity, and the fact that participation in foreign markets is heavily tilted towards larger firms. Overall, 58% of the firms producing tradable goods do not export in our data (thus having a degree of openness of zero). Among the firms who do receive part of their revenues from abroad, many of them have sales that are still strongly biased towards the domestic market. Still, about 6% of firms have openness degrees above 50%, thus being quite exposed to foreign demand shocks. And of course there is selection into exporting so that large firms are typically more exposed to such shocks (Freund and Pierola, 2015; di Giovanni et al., 2018). This is illustrated in Figure 1 by the comparison between the weighted and unweighted distributions. For instance, the 6% of firms making more than 50% of their turnover abroad represent as much as 30% of the overall value added in tradable sectors.

[5] The firm's sector is reported in the *Nomenclature d'Activités Françaises* classification, which we convert into the 35 sectors of the WIOD nomenclature. Note that the balance-sheet data do not cover Financial Activities and Private Households with Employed Persons (sectors J and P in WIOD), and thus those sectors are dropped from the analysis. We also dropped the "Public Administration" sector (sector L) which represents 23 firms and less than

Figure 1. Distribution of Export Intensity Across French Firms



Notes: This figure plots the cumulative distribution of firms according to their degree of openness, defined by the share of their sales coming from a foreign market ($\sum_{n \neq m} X_{mn,j,t}(f) / \sum_{n} X_{mn,j,t}(f)$). The solid (red) line corresponds to the unweighted distribution and the (blue) circles to the weighted distribution, where firms' weights are defined according to their share in aggregate value added. The figure is restricted to firms in traded good sectors. Source: French customs and balance-sheet data, for 2005.

3.3 Harmonizing French Firm-Level Data with Global Sectoral Data

Data on individual bilateral imports, together with information on each firm's cost structure, are used to recover the technical coefficients of each firm's production function. Firm-specific labor shares $\alpha_{n,j}(f)$ are defined as the ratio of value added over sales, both available in the balance-sheet data. In order to ensure comparability with the rest of the sample, in which labor shares are calibrated using WIOD for each country and sector, the distribution of firm-level labor shares is rescaled sector-by-sector in a way that preserves the heterogeneity but ensures that the average across firms matches the corresponding information in the WIOD, namely:

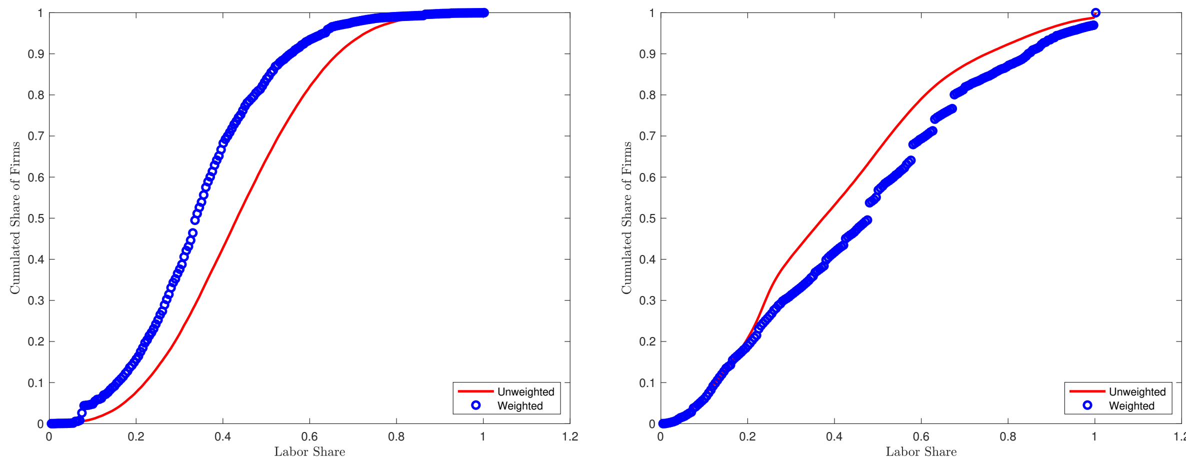
$$\alpha_{n,j}(f) = \tilde{\alpha}_{n,j}(f) \frac{\alpha_{n,j}}{\tilde{\alpha}_{n,j}}.$$

In this equation, $\alpha_{n,j}(f)$ and $\tilde{\alpha}_{n,j}(f)$ are the rescaled and original firm-level coefficients, respectively, and $\alpha_{n,j}$ is the sectoral counterpart recovered from the WIOD data. Finally, $\tilde{\alpha}_{n,j}$ is a weighted average of the original firm-level coefficients, where each firm is weighted according to its share in sectoral sales: $\tilde{\alpha}_{n,j} = \sum_{f \in (n,j)} w^S_{n,j}(f) \tilde{\alpha}_{n,j}(f)$.[6]

0.1% of overall value added in our data.

[6]The rescaling strategy implies that some rescaled firm-level coefficients end up lying outside of the range of possible values ($[0, 1]$). The corresponding coefficients are winsorized at the maximum and minimum values. This affects less than 0.02% of the firms in the total sample. The rescaling strategy is applied to all sectors but three, namely Wholesale and Retail, including Motor Vehicles and Fuel. For these three sectors, the average labor share is

Figure 2. Distribution of Labor Shares Across French Firms



(a) Tradable Sectors **(b)** Non-tradable Sectors

Notes: This figure plots the cumulative distribution of firm-level labor shares ($\alpha_{n,j}(f)$), in tradable and in non-tradable sectors. The solid (red) lines correspond to the unweighted distribution and the (blue) circles to the weighted distribution, where firms' weights are defined according to their share in aggregate value added. Calculated from French balance-sheet data together with the WIOD information on sectoral labor shares, for 2005.

Figure 2 displays the cumulative distribution of labor shares, distinguishing between tradable and non-tradable sectors. Here as well, the solid (red) line correspond to the unweighted distributions and the (blue) circles to the weighted ones. These distributions show a high degree of heterogeneity across firms, within and across broad sectors. In traded good sectors, large firms tend to be less labor intensive, although the pattern is not systematic in all individual sectors and is not very strong. On the contrary, large firms in non-traded good sectors are often more labor-intensive than smaller ones.[7]

Total input usage at the firm level equals one minus the labor share (as noted above, in our setting "labor" stands for the composite of primary factors). We further disaggregate total input usage across sectors and source countries using the information on imports, by product. This allows us to recover the $\gamma_{mn,ij}(f)$ coefficients for $n = France$. While in principle straightforward, calibrating these parameters entails two key difficulties: i) it requires the use of two sources of firm-

low in the French data compared to the WIOD. This comes from the treatment of merchandise which we categorize as intermediates while WIOD does not. Our approach is consistent with the model in the case of France, when it is assumed that consumers never interact directly with foreign firms. From that point of view, all merchandise imported from abroad is used as inputs by a French firm which ultimately sells to the final consumer. Because this is all the more important for retailing and wholesaling activities, we decided to keep the distribution of measured $\alpha_{n,j}(f)$ unchanged in these sectors.

[7]In tradable sectors, the correlation between the firm's labor share and its size varies between 0 and -0.09 (Wood products) and is often significant. In non-tradable sectors, it is positive and significant in 10 sectors out of 18 and is as high as 0.13 for Post and Telecommunication Services.

level data, which raises concerns regarding comparability; and ii) not all of these coefficients can be recovered from the firm-level data. In particular, we don't have detailed information on inputs purchased domestically and thus need to infer their sectoral breakdown using (more aggregated) information from WIOD. We proceed as follows.

For each sector i among the subset of tradable sectors and each source country $m \neq n$, we first compute a technical coefficient as the ratio of bilateral imports of goods produced by country m, sector i over the firm's input expenses.[8] Since this ratio uses data collected from two databases, the overall import share obtained from the summation of these $\gamma_{mn,ij}(f)$ coefficients over all tradable sectors and foreign countries is larger than one in some cases (for less than 1% of firms). Whenever this happens, the import share is winsorized to one and the bilateral sectoral coefficients rescaled accordingly.

Beyond comparability issues between the two firm-level sources, the introduction of these firm-level technical coefficients into the broader multi-country model also means we must ensure consistency with the sectoral coefficients in the global data. As we did with the labor shares, this implies rescaling the overall distribution of firm-level coefficients to the mean observed in the WIOD data:

$$\gamma_{mn,ij}(f) = \tilde{\gamma}_{mn,ij}(f)\frac{\gamma_{mn,ij}}{\tilde{\gamma}_{mn,ij}},$$

where $\gamma_{mn,ij}(f)$ and $\tilde{\gamma}_{mn,ij}(f)$ denote the rescaled and original firm-level coefficients, respectively, $\gamma_{mn,ij}$ is the sectoral counterpart measured with the WIOD data, and $\tilde{\gamma}_{mn,ij}$ is the weighted average of the firm-level original coefficients, where each firm is weighted according to its share in sectoral input purchases: $\tilde{\gamma}_{mn,ij} = \sum_{f\in(n,j)} \omega^I_{n,j}(f)\tilde{\gamma}_{mn,ij}(f)$. The normalization preserves as much heterogeneity across firms as possible, while avoiding overestimates of the international transmission of shocks through foreign input purchases via an exaggeration of the degree to which French firms actually rely on foreign inputs. From that point of view, our calibration is conservative.

By definition, the remaining input purchases, those not sourced abroad, include tradable goods purchased in France and all expenses on non-tradable inputs. While we do not have any information on how these domestic expenses are spread across sectors, we can recover the firm-level share of individual input purchases as $\sum_i \gamma_{nn,ij}(f) = 1 - \sum_{m\neq n}\sum_{i\in T}\gamma_{mn,ij}(f)$. This domestic input share

[8]This requires the conversion of product-level import data expressed in the highly disaggregated *Harmonized System* into broader sectoral categories. Since the customs data do not allow us to distinguish between the import of intermediates and merchandise (goods that are not further processed before being sold by the firm), we measure the firm's input expenses accordingly as the sum of raw materials *and* merchandise purchases (taking into account changes in inventories). See Blaum et al. (2016) for a similar treatment of the data.

is then assigned to domestic input sectors using information in the WIOD:[9]

$$\gamma_{nn,ij}(f) = \frac{\gamma_{nn,ij}}{\sum_i \gamma_{nn,ij}} \times \sum_i \gamma_{nn,ij}(f).$$

We have tested an alternative calibration strategy in which the input coefficients for non-traded sectors are all set exactly to their values in the WIOD. The remaining (homogeneous) share in input purchases is then spread across tradable sectors and countries using the bilateral import shares available at the firm level. The residual which corresponds to tradable inputs purchased domestically is spread across sectors using the WIOD coefficients. Note that this strategy tends to underestimate the share of tradable goods that are purchased domestically, i.e., it overestimates the participation of French firms to foreign input markets. For this reason, we have chosen to use the more conservative strategy described above as our benchmark.

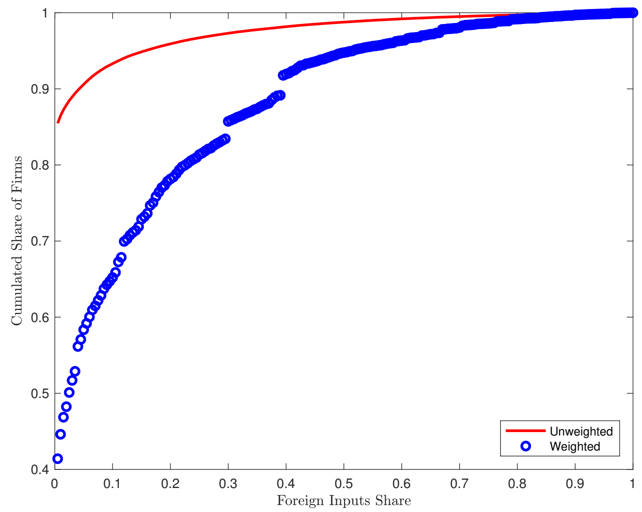
Figure 3 illustrates the within-sector heterogeneity revealed by these coefficients. Since there are as many as 1,280 (i.e. 32 sectors × 40 countries) coefficients for each firm, the information is summarized by the share of foreign inputs in firms' total input expenses ($\sum_{n\neq m}\sum_{i\in T}\gamma_{mn,ij}(f)$), a measure of how much the firm is exposed to foreign input price shocks. Here as well, the heterogeneity across firms is summarized by a cumulative distribution function, pooled across sectors. The figure reveals considerable heterogeneity across firms. More than 85% of firms source the entirety of their inputs locally, thus isolating themselves from (direct) foreign input price shocks. At the other side of the spectrum, only about 2% of firms source more than 40% of their inputs from abroad. Heterogeneity in exposure is correlated with firm size: firms that source some foreign inputs account for nearly 60% of aggregate value added, and firms sourcing more than 40% of their inputs abroad account for 10% of aggregate value added. In unreported results, we checked that the heterogeneity is not driven by cross-sector differences in overall exposure. While non-traded good sectors tend to be relatively less dependent on foreign inputs, most of the heterogeneity is actually driven by the within-sector dimension.

4 Econometric Evidence

This section sets up an estimation equation that relates firm-level sales growth to shocks in the price of inputs, which are measured at the source country and sector level. Using the model in

[9] Our definition of non-tradable sectors is somewhat unconventional since we de facto exclude from the tradable sector all services that are potentially traded but that we do not observe in the customs data. As a consequence, some of our NT sectors might display strictly positive foreign input shares in WIOD, i.e. $\gamma_{mn,ij} \neq 0$ for $j \in NT$. We adjust the WIOD data to make them consistent with our definition of non-tradable sectors by allocating all purchases from a NT sector to the same French sector, i.e.: $\gamma_{nn,ij} = \sum_m \gamma_{mn,ij}$ and $\gamma_{mn,ij} = 0, \forall i \in NT$. We apply the same adjustment to the other countries in the sample, to ensure comparability.

Figure 3. Distribution in Imported Input Use Intensity Across French Firms



Notes: This figure plots the cumulative distribution of firms according to their degree of exposure to foreign input price shocks, as defined by the share of inputs coming from other countries ($\sum_{n\neq m}\sum_{i\in T}\gamma_{mn,ij}(f)$). The solid (red) line corresponds to the unweighted distribution and the (blue) circles to the weighted distribution, where firms' weights are defined according to their share in aggregate value added. Source: French customs and balance-sheet data, for 2005.

Section 2, sales by firm f from country m in destination n are:

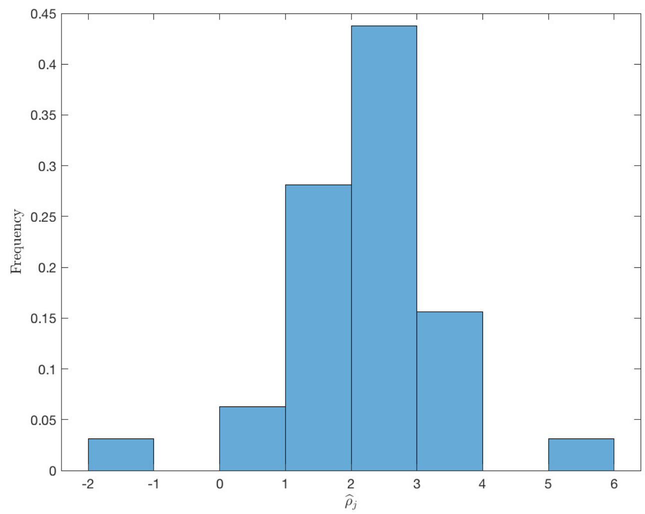
$$X_{mn,j,t}(f) = \xi_{mn,j,t}(f)\frac{\left(\frac{\rho_j}{\rho_j-1}\tau_{mn,j,t}w_{m,t}^{\alpha_{m,j}(f)}\left(\prod_i\prod_k P_{km,i,t}^{\gamma_{km,ij}(f)}\right)^{1-\alpha_{m,j}(f)}a_t(f)\right)^{1-\rho_j}}{P_{mn,j,t}^{1-\rho_j}}X_{mn,j,t}.$$

In log differences and after rearranging, we obtain the following estimable equation:

$$\begin{aligned} d\ln X_{mn,j,t}(f) = &\underbrace{(1-\rho_j)d\ln\tau_{mn,j,t} + d\ln X_{mn,j,t} - (1-\rho_j)d\ln P_{mn,j,t}}_{destination\times sector\times time\ effect} + (1-\rho_j)\underbrace{\alpha_{m,j}(f)d\ln w_{m,t}}_{data} \\ &+ (1-\rho_j)\sum_i\sum_k\underbrace{(1-\alpha_{m,j}(f))\gamma_{km,ij}(f)d\ln P_{km,i,t}}_{data} \\ &+ \underbrace{d\ln\xi_{mn,j,t}(f) + (1-\rho_j)d\ln a_t(f)}_{error\ term} \end{aligned} \quad (16)$$

In this equation, the sensitivity of a firm's market-specific sales to foreign shocks on input prices depends on i) the price elasticity of demand, $(1-\rho_j)$ and ii) the firm's exposure to this "macro" shock, as measured by $(1-\alpha_{m,j}(f))\gamma_{km,ij}(f)$. If we estimate i), we will be able to quantify how heterogeneous is the response of firms to a shock affecting the price of their inputs. Since sales and value-added at the firm-level are proportional in the model, this is also the elasticity of firm-level value added to the price shock, a partial equilibrium version of ϵ^f.

Figure 4. Histogram of Estimated Sector-Level Elasticities of Substitutions



Notes: This figure reports the histogram of the estimated elasticities of substitution by sector, which correspond to those reported in Table A2, and underlie the mean-group estimates in Table 1. There are 32 estimates of ρ_j underlying the histogram.

We estimate equation (16) sector-by-sector and using various estimators of the constrained elasticity. In all specifications, fixed effects at the destination × sector × year level control for changes in sectoral trade costs and variations in the destination country's sectoral real demand. Consistent with the model, coefficients pre-multiplying shocks to the firm's cost, whether induced by wage or price changes, are constrained to equality. In Equation (16), the left-hand side variable and the technological coefficients $\alpha_{m,j}(f)$ and $\gamma_{km,ij}(f)$ are sourced directly from the firm-level data. The prices $d \ln P_{km,i,t}$ are proxied by sectoral PPIs assembled by Auer et al. (2017) and the domestic wage $d \ln w_{m,t}$ is sourced from the OECD. (See Section 3 for more detail.)

The identification assumption required for the structural interpretation of the coefficient estimate as $(1-\rho_j)$ is that conditional on the included fixed effects, the residuals $d \ln \xi_{mn,j,t}(f) + (1-\rho_j) d \ln a_t(f)$ are uncorrelated with the regressors. That is, firm-specific idiosyncratic innovations to either demand or productivity are uncorrelated with the firm value added and input shares, and with the producer price indices at home and abroad.

Figure 4 presents the sector-by-sector $\widehat{\rho}_j$ estimates visually as a histogram. The sector-specific coefficients are reported in Table A2.[10] As can be seen, there is heterogeneity in estimated coefficients across sectors, while the tendency for them to cluster around the range of 1 to 3 is evident.

[10]The first column of Table A2 reports the substitution elasticities implied by the sectoral constrained estimates. In 22 out of 32 sectors, the elasticity is found significantly above one, consistent with the CES assumption. In another 9 sectors, the null of Cobb-Douglas ($\rho_j = 1$) cannot be rejected. Finally, 2 elasticities are significantly below one. In these sectors, increases in production costs are found to inflate destination-specific sales, which is not consistent with the model's assumptions.

Table 1. Estimated Elasticities of Substitutions, Averages across Sectors

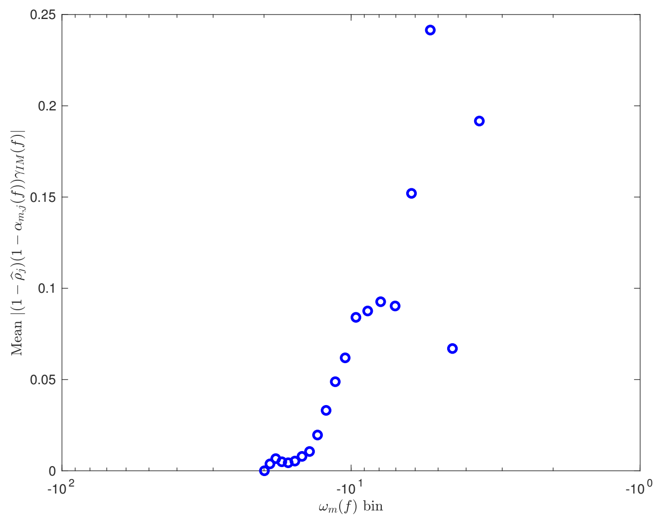
	Dep. Variable: $d \ln X_{mn,j,t}(f)$			
	(1)	(2)	(3)	(4)
	Pooled	RC	MG	MG
Input price shock	-0.961[a]	-1.112[a]	-1.233[a]	-1.725[a]
	(0.052)	(0.239)	(0.221)	(0.192)
Fixed effects	Sector×Destination×Year			
# FE	11,317	11,317	11,317	7,546
# Obs	5,067,629	5,067,629	5,067,629	3,494,717
# Firms	385,928	385,928	385,928	308,412
# Years	12	12	12	12
R^2	0.036		0.043	0.037
χ^2 statistics		726.4		
Elasticity of substitution (ρ)	1.961	2.112	2.233	2.725

Notes: This table reports the pooled, random coefficients (RC) and mean-group (MG) estimates of equation (16). Standard errors reported under parentheses with [a], [b] and [c] denoting coefficients significantly different from zero (implying a ρ coefficient significantly above one) at the 1, 5 and 10% levels, respectively. See Table A2 for the underlying sector-level elasticities for the mean-group estimator. The mean-group estimator in column (4) is restricted to sectors which display theoretically-consistent elasticities ($\rho_j > 1$). The R^2 statistics for the mean-group estimators are simple averages of the corresponding sectoral statistics.

Table 1 presents four sets of regression estimates of the elasticity of substitution based on (16). Column (1) pools all the data, so that $\rho_j = \rho \; \forall j$; column (2) presents a random coefficient estimator of the average ρ, and columns (3) and (4) present the mean-group estimator of ρ, where the column (3) uses all sector-level estimates, and column (4) uses only theoretically consistent estimates of the elasticity of substitution (i.e., those estimates that are found significantly above 1).

The first point to note is that the pooled estimate of ρ is smaller in absolute value than either the RC or MG estimators. This downward bias is to be expected given potential aggregation bias due to the positive correlation between sectors' sensitivity to foreign price shocks and their weight in the overall elasticity (Imbs and Mejean, 2015). Turning to columns (2) to (4) the estimated elasticities of substitution range between 2.1 and 2.73 on average. The random coefficient model in column (2) can be used to test the absence of heterogeneity in elasticities between sectors. The χ^2 statistics reported in the last line of column (2) unambiguously allow us to reject this null. Elasticities are significantly different across sectors, which the mean-group estimators in columns (3) and (4) better take into account. Finally, the mean-group estimators imply point estimates of an elasticity of substitution of 2.23 and 2.73, depending on whether theoretically-inconsistent sectoral elasticities ($\rho_j < 1$) are taken into account.

Figure 5. Average Partial Elasticity of Firms' Sales to Foreign Price Shocks vs. Firm Size



Notes: This figure displays the absolute value of the average of the partial elasticity of firms' sales to foreign price shocks

$$|(1-\widehat{\rho}_j)(1-\alpha_{m,j}(f))\gamma_{IM}(f)|$$

where $\gamma_{IM}(f) \equiv \sum_i \sum_{k \neq m} \gamma_{km,ij}(f)$, for each size bin. The estimated elasticities of substitution are those underlying Figure 4.

The estimation results serve two purposes. The first is to provide econometric evidence that foreign shocks transmit to French input-using firms. Consistent with the theoretical prediction (16), increases in prices of foreign inputs translate into rising sales for those firms that import those inputs, relative to firms that do not. The effect is all the stronger since the firm is more exposed to foreign input price shocks. Combining the estimated sectoral elasticities with the observed heterogeneity in firms' exposure to input price shocks, it is possible to recover the distribution of firms' partial elasticities to a given input price shock. Heterogeneity in these partial elasticities is illustrated in Figure 5 in the case of a homogeneous shock to foreign prices. Since exposure to foreign prices is positively correlated with firms' size (Figure 3), firms' sensitivity to foreign price shocks is as well. By and large, this effect is statistically significant, supporting our modeling choice to focus on heterogeneity across firms in international trade linkages.

Second, (16) gives the regression coefficients a structural interpretation by relating them to the demand elasticity faced by the firm. In the quantitative assessment below, we will set this elasticity to $\rho = 3$, which is corresponds to our preferred estimate in column 4 of Table 1. For robustness, we also perform the quantitative exercises for a low value of $\rho = 1.5$, which is closer to the mode of the distribution, as well as $\rho = 5$, a high value.

5 Quantitative Results

5.1 Calibration

The model implementation involves solving equations (8)-(14).[11] Implementing these equations requires a small number of structural parameters, and a set of initial-period values taken from the data. Table 2 summarizes the calibration. We set the elasticity of substitution between firms in the same sector selling to the same destination to $\rho = 3$. In the baseline analysis we do not assign different values to ρ across sectors. A value of elasticity of substitution across firms of 3 is implied by our estimates in Section 4. It is also a common value according to other methodologies (see e.g. Broda and Weinstein, 2006). We set the Armington elasticity of substitution between goods coming from different source countries to $\sigma = 1.5$. This is the value favored by the international business cycle literature following Backus et al. (1995), and is supported by the recent estimates by Feenstra et al. (2018). We set the labor supply parameter to $\overline{\psi} = 3$, implying the Frisch labor supply elasticity of 0.5, as advocated by Chetty et al. (2013). For the firm-specific production parameters and trade shares, we use our combined French and WIOD data, described in detail in Section 3.

Our model does not feature endogenous deficits. In all our experiments, we thus assume that the change in deficits is zero: $\widehat{D}_{n,t+1} = 0$. We adopt a similar approach to profits: $\widehat{\Pi}_{n,t+1} = 0$. In the absence of a model of multinational production, in an open economy like France changes in profits are not pinned down in our framework. This is because the aggregate profits in equation (8) refer to those used by French residents for domestic consumption spending. These are not the same as profits of firms operating in France, both because French residents own French multinationals operating abroad and thus have claims on those foreign-generated profits, and because not all firms operating in France are domestically-owned, and the profits of foreign multinational affiliates operating in France are not available to French residents for consumption spending. Since the profit share of GDP is under 10%, and for our counterfactuals what matters is not the level of profit share but the change, as an approximation we abstract from the impact of changes in profits on final consumption in our counterfactuals.

5.2 Productivity Shocks

We start by simulating the impact of two shocks on the French economy: a 10% productivity improvement in every other country in the sample other than France, and a 10% productivity improvement in Germany, one of France's most important trading partners. We report the results directly in terms of elasticities, as those lend themselves to the decomposition (1). The baseline results are reported in the top of the two panels of Table 3. French GDP increases by 3.2% when

[11] See Appendix C for details.

Table 2. Parameter values

Param.	Value	Source	Related to
ρ	3	Our estimates	substitution elasticity between firms
σ	1.5	Feenstra et al. (2018)	Armington elasticity
$\overline{\psi}$	3	Chetty et al. (2013)	Frisch elasticity
$\alpha_{n,i}(f))$, $\gamma_{mn,ji}(f)$	}	Our calculations based on French data and WIOD	labor and intermediate shares
ϑ_j			final consumption shares
$\pi^c_{mn,j,t+1}$			final trade shares
$\pi_{nk,j,t+1}(f)$			intermediate use trade shares

Notes: This table summarizes the parameter values used in the calibration.

world productivity grows by 10%. This is a sizeable elasticity, considering that France itself does not experience the productivity shock, and thus the entire change is due to it being transmitted to France via goods trade linkages. The response to a German shock (bottom panel) is understandably much smaller at 0.55%, since that shock affects only one of France's trading partners.

Our central result concerns not so much the overall magnitude, but the role of heterogeneity. Decomposing the aggregate elasticity into the mean and the covariance term, we find that the covariance term is positive as expected, and quite large. It is responsible for 31% of the overall effect of a world shock, and 34% of the German shock. Thus, our results reveal a quantitatively large role of the heterogeneity in firm-level international linkages in business cycle transmission across countries.

To provide a graphic illustration of this result, Figure 6 plots the histogram of ϵ^f across firms in the baseline model for the world shock. It is evident that firm-level elasticities have a non-trivial distribution. While most of them are positive, there is substantial density below zero as well – some firms shrink in response to a positive shock in the rest of the world. At the same time, there is an upper tail as well, as the density of ϵ^f above 1 is visible. Next, Figure 7 presents the average ϵ^f for firms of different sizes $\omega_{m,t}(f)$. We break firm shares in aggregate value added into size bins, and plot the mean ϵ^f in each size bin. This figure provides a graphical illustration of the positive $Cov\left(\frac{\omega_{m,t}(f)}{\overline{\omega}}, \epsilon^f\right)$ term. The horizontal line plots the aggregate elasticity ϵ^Y. It is notable that it is towards the top of the plot, coinciding with the ϵ^f of the largest firms.

The variation in firm-specific elasticities with respect to foreign shocks has the expected relationship to the intensity of intermediate input purchases from abroad and to export intensity. Figure 8 plots the average ϵ^f for each value of total imported input share, $\gamma_{IM}(f) \equiv \sum_{n\neq m}\sum_i \gamma_{mn,ji}(f)$. There is a pronounced positive relationship. Figure 9 plots the average ϵ^f against the total export intensity of each firm, defined as the ratio of total firm exports to total firm sales, $\pi_{EX}(f) \equiv$

Table 3. Responses of French Real GDP to Foreign Productivity Shocks
(10% Productivity Shocks)

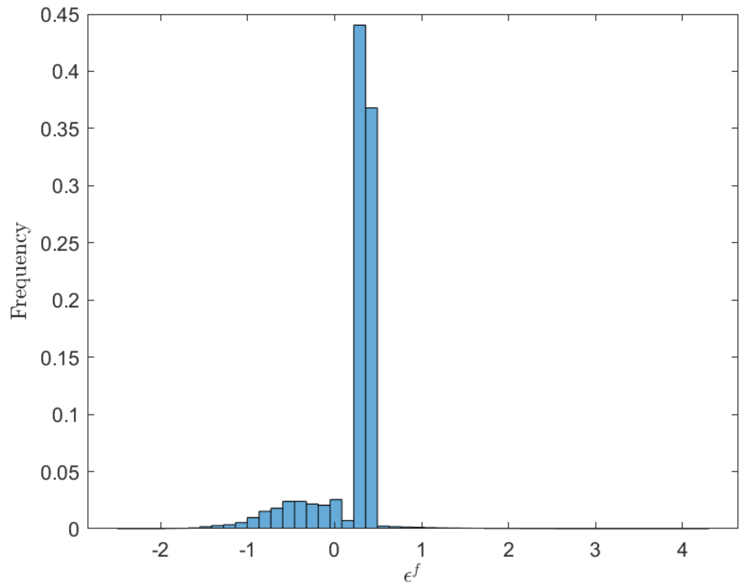
	ϵ^{Y}	$\bar{\epsilon}$	$Cov\left(\frac{\omega_{m,t}(f)}{\overline{\omega}}, \epsilon^{f}\right)$
	World Productivity Shock		
Baseline	0.317	0.217	0.100
Share:		*0.69*	*0.31*
Homogeneous $\pi_{nk,j,t}(f)$ and $\gamma_{mn,ji}(f)$	0.243	0.245	-0.002
Share:		*1.01*	*-0.01*
Homogeneous $\pi_{nk,j,t}(f)$	0.145	0.145	0.000
Share:		*1.00*	*0.00*
Homogeneous $\gamma_{mn,ji}(f)$	0.331	0.279	0.052
Share:		*0.84*	*0.16*
	German Productivity Shock		
Baseline	0.055	0.036	0.019
Share:		*0.66*	*0.34*
Homogeneous $\pi_{nk,j,t}(f)$ and $\gamma_{mn,ji}(f)$	0.046	0.048	-0.002
Share:		*1.04*	*-0.04*
Homogeneous $\pi_{nk,j,t}(f)$	0.027	0.029	-0.002
Share:		*1.07*	*-0.07*
Homogeneous $\gamma_{mn,ji}(f)$	0.061	0.052	0.009
Share:		*0.85*	*0.15*

Sector-Level Decomposition

	ϵ^{Y}	$\bar{\epsilon}_j$	$Cov\left(\frac{\omega_{j,t}(f)}{\overline{\omega_j}}, \epsilon^{j}\right)$
	World Productivity Shock		
Baseline	0.317	0.244	0.074
Share:		*0.77*	*0.23*
	German Productivity Shock		
Baseline	0.055	0.053	0.002
Share:		*0.97*	*0.03*

Notes: This table reports the elasticity of French GDP with respect to a 10% productivity shock in every other country in the world and with respect to a 10% productivity shock to Germany, both the baseline model and the alternative models that suppress sources of firm heterogeneity. The table reports the decomposition of the aggregate elasticity into the mean and the covariance terms as in (1).

Figure 6. Histogram of ϵ^f Following a 10% World Productivity Shock



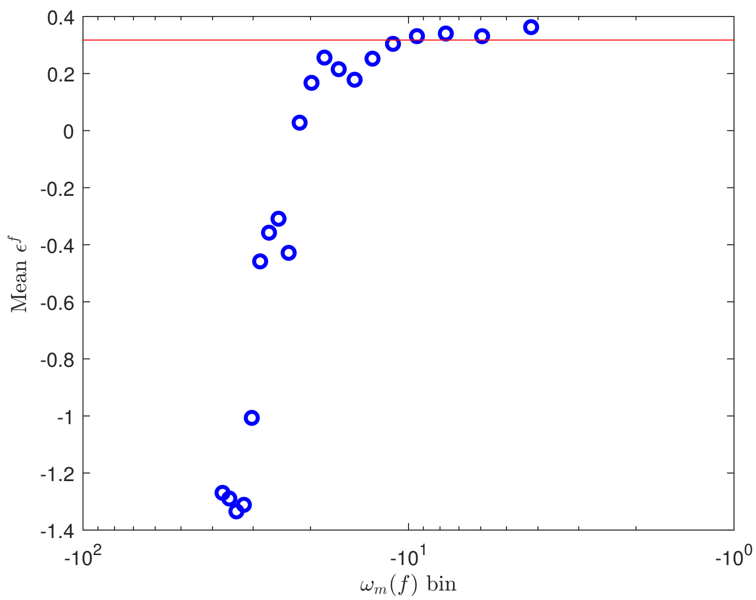
Notes: This figure displays the histogram of ϵ^f following a 10% world productivity shock in the baseline model.

$\sum_{n \neq m} \widetilde{s}_{mn,j,t}(f)$. Once again there is a pronounced positive relationship, with more export-oriented firms having higher elasticities to the foreign shock. Note that unlike the relationship with $\gamma_{IM}(f)$, this scatter shows more variability. This is expected, as the impact of a foreign productivity shock on an exporter is a combination of reduced final demand (due to substitution towards more productive source countries) and increased intermediate demand.

We highlight the role of heterogeneity by comparing the baseline model to alternative implementations that suppress heterogeneity along the importing and exporting dimensions. The line labeled "Homogeneous $\pi_{nk,j,t}(f)$ and $\gamma_{mn,ji}(f)$" of Table 3 reports the elasticities in an alternative model in which firm export participation (the trade shares $\pi_{nk,j,t+1}(f)$) and firm-level intermediate import usage ($\gamma_{mn,ji}(f)$) are made homogeneous within each sector. This scenario approximates a model with a sector-specific representative firm. Importantly, to preserve the overall levels of trade in this scenario, the $\gamma_{mn,ji}(f)$'s are rescaled to match the sector-level imported input coefficients. This implies that in this homogeneous-$\gamma_{mn,ji}(f)$ scenario, the imported input coefficients are lower for the firms that in the data actually import inputs, but higher for firms that in the data do not.

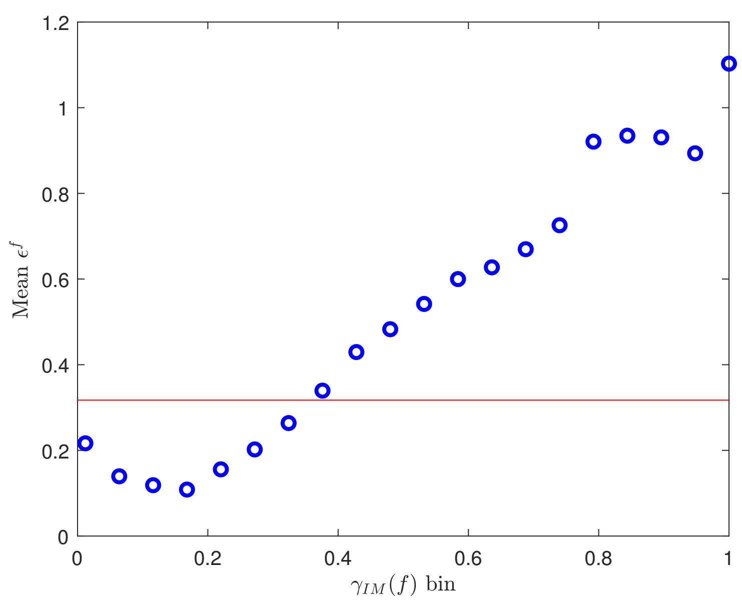
The overall elasticity to foreign shocks, at 0.243, is noticeably lower than the baseline one. This indicates that the heterogeneity across firms provides an amplification mechanism in the transmission of foreign shocks. This is made clearer by examining the decomposition. The entirety of the aggregate elasticity is explained by the average $\overline{\epsilon}$ component, with no role for the covariance term. The next two scenarios suppress the exporting ("Homogeneous $\pi_{nk,j,t}(f)$") and importing

Figure 7. Average ϵ^f for Different Values of $\omega_{m,t}(f)$ (10% World Productivity Shock)



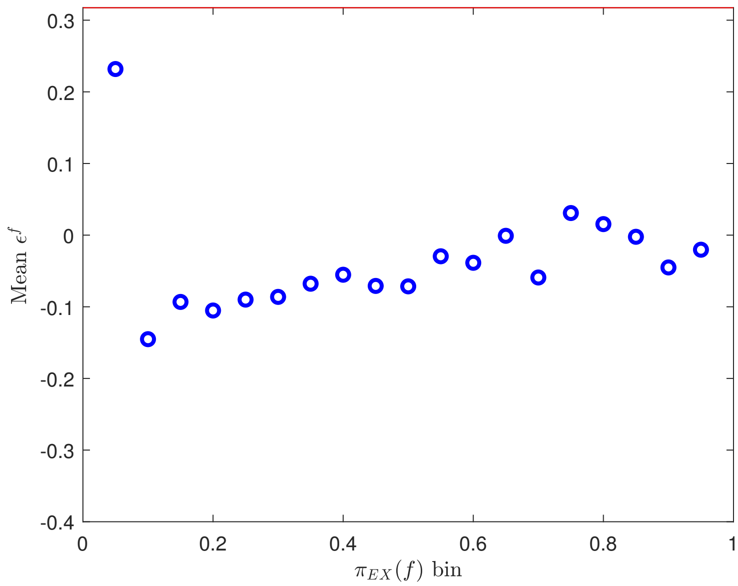
Notes: This figure displays the mean ϵ^f for each size bin following a 10% world productivity shock in the baseline model.

Figure 8. Average ϵ^f for Firms with different Intermediate Import Intensities (10% World Productivity Shock)



Notes: This figure displays the averages of ϵ^f for each value of total imported intermediate input intensity following a 10% world productivity shock in the baseline model.

Figure 9. Average ϵ^f for Firms with Different Export Intensities (10% World Productivity Shock)



Notes: This figure displays the mean ϵ^f for each value of overall export intensity following a 10% world productivity shock in the baseline model.

("Homogeneous $\gamma_{mn,ji}(f)$") heterogeneity separately. Suppressing exporting heterogeneity yields an even lower aggregate elasticity of 0.145, and once again a covariance term of zero. Suppressing the importing heterogeneity actually produces a slightly higher overall elasticity than the baseline, at 0.331. This is not surprising, as imposing homogeneity in $\gamma_{mn,ji}(f)$ involves making some non-importing firms importers, and thus engineering a direct transmission channel of foreign shocks to those firms. The covariance term in this scenario is non-negligible, as firms still differ in their export status and therefore size. However, the relative importance of the covariance term, at 16% of the total, is still much lower than in the baseline.

Finally, we evaluate whether in the baseline model, the heterogeneity that drives the high covariance term is within or across sectors. To that end, we take the results from the baseline model, and instead of writing the decomposition (1) at the firm level, write it at the sector level instead:

$$\epsilon^Y = \bar{\epsilon}_j + Cov\left(\frac{\omega_{j,t}}{\overline{\omega}_j}, \epsilon^j\right), \tag{17}$$

where ϵ^j is the elasticity of total value added in sector j to the foreign shock, $\bar{\epsilon}_j$ is its unweighted average, and $\omega_{j,t}$ is sector j share in aggregate value added. Importantly, we implement this decomposition on the baseline model featuring the full heterogeneity across firms, but we compute the sector-level shares and elasticities. The results are presented in the panel labeled "Sector-Level

Decomposition" of Table 3. By construction, the overall elasticity ϵ^Y is exactly the same as in the top panel of the table. The sector-level covariance term is 23%, indicating that for this particular shock, the impact of heterogeneity is to a large extent captured by the sectoral dimension.

5.3 Foreign Demand Shocks

Next, we evaluate the propagation of a foreign demand shock to France. To that end, we simulate an increase in the taste shock $\xi_{mn,j,t}(f)$ to all firms in $m =$ France in all foreign markets $n \neq m$, as well as only in Germany ($n =$ Germany). Examining equation (2), it is clear that an increase in the taste for all French firms abroad amounts to a $\widehat{\xi_{mn,j,t}^{\frac{1}{\rho-1}}}$ productivity increase for French exports abroad, and thus an increase in demand for French goods by foreign firms and consumers. (We assume that this is a purely external shock, such that the French domestic demand shifter $\xi_{mm,j,t}(f)$ is unchanged.) We thus simulate a 10% shift in demand for French goods, namely $\widehat{\xi_{mn,j,t}^{\frac{1}{\rho-1}}} = 0.1$.

Table 4 reports the results. It is structured in exactly the same way as Table 3. In the baseline, a 10% demand shocks for French goods abroad raises French real GDP by 0.35%. This is a smaller elascitity than that of a foreign productivity shock, but that is because the overall shock is much smaller, as it affects only the French tradeable sector. The relative importance of the covariance term is similar than for the productivity shock, as it accounts for 27% of the overall impact. Once again, when export and import shares are homogeneous, the covariance term is substantially smaller, at 5%. When the shock is to foreign demand, it is not the heterogeneity on the importing side that drives the covariance term. The importance of the covariance term in the "Homogeneous $\gamma_{mn,ji}(f)$" is 20%. This is intuitive: the foreign demand shock will generate a heterogeneous response across French firms according to their export status, since it is the exporting firms that are primarily affected by the foreign demand increase. When the foreign firms do not experience a first-order productivity increase, French firms that import inputs do not benefit disproportionately, and thus eliminating heterogeneity in imported input shares does not lead to outcomes all that different from the baseline.

Finally, the bottom panel reports the $\bar{\epsilon}_j$–$Cov\left(\frac{\omega_{j,t}}{\overline{\omega}_j}, \epsilon^j\right)$ decomposition at the sector level for the foreign demand shock. Not only is the covariance term not positive, it is actually strongly negative, accounting for -65% of the overall effect for the world demand shock, and -33% for the German demand shock. Evidently, sectors with the highest positive elasticities with respect to foreign demand shocks tend to actually be relatively smaller in size. This is sensible, as some of the largest sectors in our data are non-tradeable, and thus by construction not experiencing the positive foreign demand shock. The contrast between the role of the covariance at the firm vs. sector level is stark.

Table 4. Responses of French Real GDP to Foreign Demand Shocks
(10% Demand Shocks)

	ϵ^Y	$\bar{\epsilon}$	$Cov\left(\frac{\omega_{m,t}(f)}{\overline{\omega}}, \epsilon^f\right)$
	World Demand Shock		
Baseline	0.035	0.025	0.010
Share:		*0.73*	*0.27*
Homogeneous $\pi_{nk,j,t}(f)$ and $\gamma_{mn,ji}(f)$	0.036	0.034	0.002
Share:		*0.95*	*0.05*
Homogeneous $\pi_{nk,j,t}(f)$	0.030	0.029	0.001
Share:		*0.98*	*0.02*
Homogeneous $\gamma_{mn,ji}(f)$	0.036	0.029	0.007
Share:		*0.80*	*0.20*
	German Demand Shock		
Baseline	0.005	0.004	0.002
Share:		*0.68*	*0.32*
Homogeneous $\pi_{nk,j,t}(f)$ and $\gamma_{mn,ji}(f)$	0.006	0.005	0.001
Share:		*0.84*	*0.16*
Homogeneous $\pi_{nk,j,t}(f)$	0.005	0.004	0.001
Share:		*0.86*	*0.14*
Homogeneous $\gamma_{mn,ji}(f)$	0.006	0.005	0.001
Share:		*0.78*	*0.22*

Sector-Level Decomposition

	ϵ^Y	$\bar{\epsilon}_j$	$Cov\left(\frac{\omega_{j,t}(f)}{\overline{\omega_j}}, \epsilon^j\right)$
	World Demand Shock		
Baseline	0.035	0.058	-0.023
Share:		*1.65*	*-0.65*
	German Demand Shock		
Baseline	0.005	0.007	-0.002
Share:		*1.33*	*-0.33*

Notes: This table reports the elasticity of French GDP with respect to a 10% demand shock for French goods in every other country in the world and with respect to a 10% demand shock for French goods in Germany, both the baseline model and the alternative models that suppress sources of firm heterogeneity. The table reports the decomposition of the aggregate elasticity into the mean and the covariance terms as in (1).

6 Conclusion

Large firms are more likely to import and export. A natural conjecture is that this greater participation in international markets also makes the large firms more sensitive to foreign shocks. In this paper, we explored both the micro and the macro implications of this joint heterogeneity in size and international linkages. We first provided firm-level econometric evidence that firms importing intermediate inputs are significantly more responsive to foreign input price shocks. We then implemented a quantitative multi-country model in which French firms exhibit the observed joint distribution of size, importing, and exporting. The covariance between firm size and sensitivity to foreign shocks accounts for some 30% of the aggregate impact of foreign productivity and demand shocks. In addition, for some shocks this heterogeneity implies greater aggregate impact compared to a homogeneous firm model. We conclude that capturing the positive association between size and international linkages is essential for understanding the firm-level and aggregate international transmission of business cycle shocks.

References

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, "The Network Origins of Aggregate Fluctuations," *Econometrica*, September 2012, *80* (5), 1977–2016.

Alvarez, Fernando and Robert E., Jr. Lucas, "General Equilibrium Analysis of the Eaton-Kortum Model of International Trade," *Journal of Monetary Economics*, September 2007, *54* (6), 1726–1768.

Atalay, Enghin, "How Important Are Sectoral Shocks?," *American Economic Journal: Macroeconomics*, October 2017, *9* (4), 254–280.

Auer, Raphael A., Andrei A. Levchenko, and Philip Sauré, "International Inflation Spillovers Through Input Linkages," March 2017. NBER Working Paper No. 23246.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland, "International Business Cycles: Theory and Evidence," in Thomas Cooley, ed., *Frontiers of business cycle research*, Princeton: Princeton University Press, 1995, pp. 331–356.

Baqaee, David Rezza, "Cascading Failures in Production Networks," September 2016. Mimeo, LSE.

Barrot, Jean-Noël and Julien Sauvagnat, "Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks," *Quarterly Journal of Economics*, 2016, *131* (3), 1543–1592.

Blaum, Joaquin, "Importing, Exporting and Aggregate Productivity in Large Devaluations," March 2018. Mimeo, Brown University.

_ , Claire Lelarge, and Michael Peters, "The Gains from Input Trade with Heterogeneous Importers," December 2016. Forthcoming, *American Economic Journal: Macroeconomics.*

Boehm, Christoph, Aaron Flaaen, and Nitya Pandalai Nayar, "Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Tohoku Earthquake," December 2017. Forthcoming, *Review of Economics and Statistics.*

Broda, Christian and David Weinstein, "Globalization and the Gains from Variety," *Quarterly Journal of Economics*, May 2006, *121* (2), 541–85.

Bureau of Economic Analysis, "Updated Summary of NIPA Methodologies," November 2017. https://www.bea.gov/scb/pdf/2017/11-November/1117-updated-summary-of-nipa-methodologies.pdf.

Carvalho, Vasco M., "Aggregate Fluctuations and the Network Structure of Intersectoral Trade," October 2010. Mimeo, CREi and Universitat Pompeu Fabra.

_ and Basile Grassi, "Large Firm Dynamics and the Business Cycle," May 2015. CEPR Discussion Paper 10587.

_ , Makoto Nirei, Yukiko U. Saito, and Alireza Tahbaz-Salehi, "Supply Chain Disruptions: Evidence from the Great East Japan Earthquake," December 2016. mimeo, University of Cambridge, Policy Research Institute, Ministry of Finance of Japan, RIETI, and Columbia GSB.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber, "Does Indivisible Labor Explain the Difference between Micro and Macro Elasticities? A Meta-Analysis of Extensive Margin Elasticities," *NBER Macroeconomics Annual*, 2013, *27* (1), 1 – 56.

Costinot, Arnaud and Andrés Rodríguez-Clare, "Trade Theory with Numbers: Quantifying the Consequences of Globalization," in Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, eds., *Handbook of International Economics*, Vol. 4, Elsevier, 2014, chapter 4, pp. 197–261.

Cravino, Javier and Andrei A. Levchenko, "Multinational Firms and International Business Cycle Transmission," *Quarterly Journal of Economics*, May 2017, *132* (2), 921–962.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum, "Global Rebalancing with Gravity: Measuring the Burden of Adjustment," *IMF Staff Papers*, July 2008, *55* (3), 511–540.

di Giovanni, Julian, Andrei A. Levchenko, and Isabelle Mejean, "Firms, Destinations, and Aggregate Fluctuations," *Econometrica*, July 2014, *82* (4), 1303–1340.

_ , _ , and _ , "Large Firms and International Business Cycle Comovement," *American Economic Review: Papers & Proceedings*, May 2017, *107* (5), 598–602.

_ , _ , and _ , "The Micro Origins of International Business Cycle Comovement," *American Economic Review*, January 2018, *108* (1), 82–108.

Feenstra, Robert C., Philip Luck, Maurice Obstfeld, and Katheryn N. Russ, "In Search of the Armington Elasticity," *The Review of Economics and Statistics*, 2018, *100* (1), 135–150.

Freund, Caroline and Martha Denisse Pierola, "Export Superstars," *Review of Economics and Statistics*, 2015, *97* (5), 1023–1032.

Gabaix, Xavier, "The Granular Origins of Aggregate Fluctuations," *Econometrica*, May 2011, *79* (3), 733–772.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman, "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, June 1988, *78* (3), 402–17.

Imbs, Jean and Isabelle Mejean, "Elasticity Optimism," *American Economic Journal: Macroeconomics*, 2015, *7* (3), 43–83.

Kleinert, Jörn, Julien Martin, and Farid Toubal, "The Few Leading the Many: Foreign Affiliates and Business Cycle Comovement," *American Economic Journal: Macroeconomics*, October 2015, *7* (4), 134–159.

Timmer, Marcel P., Erik Dietzenbacher, Bart Los, Robert Stehrer, and Gaaitzen J. de Vries, "An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production," *Review of International Economics*, August 2015, *23* (3), 575–605.

Tintelnot, Felix, Ken Kikkawa, Magne Mogstad, and Emmanuel Dhyne, "Trade and Domestic Production Networks," 2017. Mimeo, University of Chicago.

Appendix A The GDP Deflator Construction in the Model

The GDP deflator measures the change in prices of output produced, rather than consumed. Let the base period be year t throughout. At time $t+1$, the real GDP (i.e., expressed in base period prices) is defined as the time $t+1$ quantities produced evaluated at year t prices. Denote the real GDP by $Y_{m,t+1}$. Then the gross change in the GDP deflator is defined implicitly by:

$$\widehat{P}^{G}_{m,t+1} = \frac{\widehat{Y}^{NOM}_{m,t+1}}{\widehat{Y}_{m,t+1}}.$$

In turn, the real GDP level is:

$$Y_{m,t+1} = \sum_{f}\sum_{n} \frac{1+\alpha_{m,j}(f)(\rho_j - 1)}{\rho_j} Q_{mn,j,t+1}(f) p_{mn,j,t}(f),$$

and thus its gross change is:

$$\widehat{Y}_{m,t+1} = \sum_{f}\sum_{n} \omega_{m,j,t}(f) \widetilde{s}_{mn,j,t}(f) \widehat{Q}_{mn,j,t+1}(f),$$

which is, intuitively, the value-added weighted change in quantities. In practice, national statistical agencies compute the real GDP change $\widehat{Y}_{m,t+1}$ by using sectoral quantity changes (Bureau of Economic Analysis, 2017). The sectoral quantity changes are obtained by deflating the nominal growth rates of sectoral output by either CPI or PPI price indices for the sectors. In our implementation, we stick closely to this procedure, but compute quantity changes at the firm-destination rather than sector-level.

Appendix B Sectoral Prices

The sectoral price series used in the regression in Section 4 are based on monthly PPI data provided by Auer et al. (2017). Across countries an average of 17 of the 33 sectors included in the study were completely missing data, while 6 were partially missing data and 10 were fully populated. These averages mask substantial differences between countries however, as nearly half of the countries under study had no sectors with complete time series while nearly half had over 18 sectors with complete time series. France itself had 19 complete time series, 1 partial time series, and 13 completely missing time series. Both France's and the other countries' missing time series were heavily concentrated in the services sectors. By the definition adopted for this study, tradable sectors had on average 20 complete time series and 11 partial time series, while non-tradable sectors had only 5 complete and 4 partially complete time series.

Missing prices were imputed using an iterative regression procedure which made use of three averages, each calculated from both datasets. The averages used were the country/sector's own average growth rates, the cross-country average sectoral growth rates, and the cross-sector average country growth rate. These regressors were then used in an iterative process as follows:

$$\begin{aligned} Price_{n,j,t} = MeanGrowth^{PPI}_{n,j,t} + MeanGrowth^{PPI}_{j,t} + MeanGrowth^{PPI}_{n,t} \\ + MeanGrowth^{OECD}_{n,j,t} + MeanGrowth^{OECD}_{j,t} + MeanGrowth^{OECD}_{n,t} \end{aligned} \tag{B.1}$$

$$\begin{aligned} Price_{n,j,t} = MeanGrowth^{PPI}_{n,j,t} + MeanGrowth^{PPI}_{j,t} + MeanGrowth^{PPI}_{n,t} \\ + MeanGrowth^{OECD}_{j,t} + MeanGrowth^{OECD}_{n,t} \end{aligned} \tag{B.2}$$

$$\begin{aligned} Price_{n,j,t} = MeanGrowth^{PPI}_{n,j,t} + MeanGrowth^{PPI}_{j,t} + MeanGrowth^{PPI}_{n,t} \\ + MeanGrowth^{OECD}_{j,t} \end{aligned} \tag{B.3}$$

$$Price_{n,j,t} = MeanGrowth^{PPI}_{n,j,t} + MeanGrowth^{PPI}_{j,t} + MeanGrowth^{PPI}_{n,t} \tag{B.4}$$

$$\begin{aligned} Price_{n,j,t} = MeanGrowth^{PPI}_{j,t} + MeanGrowth^{PPI}_{n,t} \\ + MeanGrowth^{OECD}_{n,j,t} + MeanGrowth^{OECD}_{j,t} + MeanGrowth^{OECD}_{n,t} \end{aligned} \tag{B.5}$$

And so on. Missing data was replaced by the predicted value of the regression which utilised the most regressors, resulting in a complete price series for all country/sector pairs.

Appendix C Model Solution and Calibration

The model is solved for the changes for all variables numerically by relying on the equilibrium equations outlined in the main text. In particular, we solve for the following equilibrium variables:

1. Changes in trade values $\widehat{X}_{mn,j,t+1}\ \forall m,n,j$
2. Changes in wages $\widehat{w}_{n,t+1}\ \forall n$
3. Changes in the price indices $\widehat{P}_{n,t+1}\ \forall n$, $\widehat{P}_{n,j,t+1}\ \forall n,j$, $\widehat{P}_{mn,j,t+1}\ \forall m,n,j$
4. Next period's trade shares $\pi^c_{mn,j,t+1}\ \forall m,n,j$, $\pi_{nk,j,t+1}(f)\ \forall k,n,j,f$

The solution of the model further requires setting parameter values for ρ_j, $\alpha_{m,j}(f)$, and ϑ_j. We base the parameter values either on those in the previous literature, or use firm-level data (for France) or sector-level information from WIOD to calculate them (see Section 3 for more details).

We further require several base period data series, either at the firm or sector level. Specifically, we require information on:

1. Gross sales $X_{mn,j,t}$ $\forall m,n,j$
2. Final consumption shares within a sector across sources $\pi^c_{mn,j,t}$ $\forall m,n,j$
3. Firm-level within sector, within-destination trade shares $\pi_{nk,j,t}(f)$ $\forall k,n,j,f$
4. Final consumption spending $P_{n,t}C_{n,t}$
5. Shares of labor (factor) income, pure profits, and deficits in final consumption spending $s^L_{n,t}$, $s^\Pi_{n,t}$ and $s^D_{n,t}$ $\forall n$
6. Input coefficients $\gamma_{mn,ij}(f)$ $\forall m,n,i,j,f$

The construction of these variables and the relevant data sources are described in Section 3.

C.0.1 Satisfying market clearing

In order to proceed correctly with the hat algebra in each sector/country pair, in the pre-period the market clearing condition in levels must be satisfied:

$$X_{mn,j,t} = \pi^c_{mn,j,t}\pi^c_{n,j,t}P_{n,t}C_{n,t} + \sum_i \frac{\rho_i - 1}{\rho_i}\sum_{f\in i}(1-\alpha_{n,i}(f))\gamma_{mn,ji}(f)\sum_k \pi_{nk,i,t}(f)X_{nk,i,t} \tag{C.1}$$

In the data, this is unlikely to be the case. We therefore adopt the proposed solution: in each mn, j, t, trivially we can find a wedge $\zeta_{mn,j,t}$ such that conditional on all the other data, (C.1) does hold with equality:

$$X_{mn,j,t} = \pi^c_{mn,j,t}\pi^c_{n,j,t}P_{n,t}C_{n,t} + \sum_i \frac{\rho_i - 1}{\rho_i}\sum_{f\in i}(1-\alpha_{n,i}(f))\gamma_{mn,ji}(f)\sum_k \pi_{nk,i,t}(f)X_{nk,i,t} + \zeta_{mn,j,t}$$

Then applying the hat algebra to this equation:

$$\begin{aligned}\widehat{X}_{mn,j,t+1}X_{mn,j,t} = \pi^c_{mn,j,t+1}\pi^c_{n,j,t+1}&\left[\left(\frac{\widehat{w}_{n,t+1}}{\widehat{P}_{n,t+1}}\right)^{\frac{1}{\psi-1}} s^L_{n,t} + \widehat{\Pi}_{n,t+1}s^\Pi_{n,t} + \widehat{D}_{n,t+1}s^D_{n,t}\right]P_{n,t}C_{n,t}\\ &+\sum_i \frac{\rho_i - 1}{\rho_i}\sum_{f\in i}(1-\alpha_{n,i}(f))\gamma_{mn,ji}(f)\sum_k \pi_{nk,i,t+1}(f)\widehat{X}_{nk,i,t+1}X_{nk,i,t}\\ &+\widehat{\zeta}_{mn,j,t+1}\zeta_{mn,j,t}\end{aligned} \tag{C.2}$$

Next, we solve the entire model while feeding in a "shock" that eliminates this wedge, namely: $\widehat{\zeta}_{mn,j,t+1} = 0$. Finding the model solution will give the a set of $\widehat{X}_{mn,j,t+1}$'s that are required to arrive at a set of levels of $X_{mn,j,t+1}$ for which the market clearing condition is satisfied with equality for every mn, j. Then use these $X_{mn,j,t+1}$ as the starting values for all the real counterfactuals we want to run. The antecedent of this approach is in Costinot and Rodríguez-Clare (2014), where they use a similar device to eliminate the trade deficits.

Table A1. Summary Statistics of Firms, by Sector

WIOT sector	# firms	Share VA	Traded/ non-traded
Agriculture, Hunting, Forestry, Fishing	7,718	.0067	T
Mining, Quarrying	1,022	.0041	T
Food, Beverages, Tobacco	10,883	.0354	T
Textile Products	1,684	.0039	T
Leather, Footwear	2,501	.0058	T
Wood Products	3,045	.0044	T
Pulp, Paper, Publishing	7,721	.0202	T
Coke, Refined Petroleum, Nuclear Fuel	50	.0056	T
Chemical Products	2,051	.0358	T
Rubber and Plastics	2,992	.0155	T
Other Non-Metallic Minerals	2,607	.0127	T
Basic and Fabricated Metals	14,561	.0373	T
Machinery n.e.c.	6,442	.0243	T
Electrical, Optical Equipment	6,599	.0288	T
Transport Equipment	1,804	.0315	T
Manufacturing n.e.c.	4,946	.0086	T
Electricity, Gas, Water Supply	321	.0364	NT
Construction	54,428	.0664	NT
Wholesale and Retail Motor Vehicles and Fuel	25,975	.0218	NT
Wholesale Trade	49,166	.0867	NT
Retail Trade	76,069	.0739	NT
Hotels and restaurants	29,135	.0259	NT
Inland Transport	9,244	.0401	NT
Water Transport	171	.0017	NT
Air Transport	66	.0085	NT
Other Transport Activities	2,068	.0256	NT
Post and Telecommunications	276	.0488	NT
Real Estate	7,726	.0425	NT
Business Activities	31,605	.1849	NT
Education	1,569	.0037	NT
Health and Social Work	6,200	.0200	NT
Other Personal Services	15,283	.0324	NT
Total	385,928	1.000	

Notes: This table reports summary statistics on the number and cumulated value added of firms, by WIOT sector. The data are from INSEE-Ficus/Fare and correspond to year 2005.

Table A2. Estimated Sector-Level Elasticities of Substitution

Sector	Coefficient	Std.Err.	R^2	Observations
Agriculture, Hunting, Forestry, Fishing	0.257^a	0.219	0.038	93,577
Mining and Quarrying	2.838^b	0.715	0.059	15,482
Food, Beverages and Tobacco	2.146^a	0.268	0.041	230,056
Textiles Products	2.866^a	0.535	0.035	82,846
Leather and Footwear	1.345	0.439	0.028	119,625
Wood Products	1.765	0.465	0.044	56,825
Pulp, Paper, Publishing	1.850^b	0.374	0.030	146,748
Coke, Refined Petroleum, Nuclear Fuel	5.393^c	2.367	0.051	3,420
Chemicals Products	1.021	0.605	0.029	151,370
Rubber and Plastics	0.959	0.501	0.032	121,515
Other Non-Metallic Minerals	2.818^a	0.551	0.037	64,336
Basic and Fabricated Metal	1.978^a	0.254	0.037	291,907
Machinery n.e.c.	2.074^b	0.340	0.026	219,840
Electrical, Optical Equipment	1.678^c	0.359	0.028	211,029
Transport Equipment	1.261	0.685	0.034	50,812
Manufacturing n.e.c.	3.972^a	0.421	0.034	111,364
Electricity, Gas, Water Supply	2.994^b	0.920	0.110	2,714
Construction	3.240^a	0.107	0.014	434,568
Wholesale and Retail Motor Vehicles and Fuel	2.566^a	0.145	0.044	262,160
Wholesale Trade	1.044^a	0.163	0.030	938,636
Retail Trade	2.430^a	0.118	0.030	679,539
Hotels and Restaurants	2.043^a	0.117	0.014	213,275
Inland Transport	3.148^a	0.224	0.020	81,017
Water Transport	2.817	1.227	0.089	1,305
Air Transport	2.198	3.474	0.121	961
Other Transport Activities	2.578^a	0.463	0.050	16,847
Post and Telecommunications	3.323	1.532	0.126	1,559
Real Estate	-1.753^a	0.298	0.013	36,727
Business Activities	1.648^a	0.128	0.024	253,941
Education	2.612	0.582	0.045	10,543
Health and Social Work	2.529^a	0.232	0.019	45,407
Other Personal Services	3.823^a	0.145	0.032	117,678

Notes: This table reports the sector-level estimates of substitution elasticities obtained from equation (16). Standard errors reported under parentheses with ^{a}, ^{b} and ^{c} denoting coefficients significantly different from one at the 1, 5 and 10% levels, respectively.

Table A3. Responses of French Real GDP to Foreign Productivity Shocks, Robustness (10% Productivity Shocks)

	ϵ^Y	$\bar{\epsilon}$	$Cov\left(\frac{\omega_{m,t}(f)}{\bar{\omega}}, \epsilon^f\right)$	ϵ^Y	$\bar{\epsilon}$	$Cov\left(\frac{\omega_{m,t}(f)}{\bar{\omega}}, \epsilon^f\right)$
	World Productivity Shock, $\rho = 1.5$			World Productivity Shock, $\rho = 5$		
Baseline	0.329	0.308	0.021	0.294	0.101	0.193
Share:		*0.93*	*0.07*		*0.34*	*0.66*
Homogeneous $\pi_{nk,j,t}(f)$ and $\gamma_{mn,ji}(f)$	0.235	0.247	-0.013	0.244	0.242	0.002
Share:		*1.05*	*-0.05*		*0.99*	*0.01*
Homogeneous $\pi_{nk,j,t}(f)$	0.132	0.144	-0.012	0.151	0.147	0.004
Share:		*1.09*	*-0.09*		*0.97*	*0.03*
Homogeneous $\gamma_{mn,ji}(f)$	0.348	0.336	0.012	0.315	0.227	0.088
Share:		*0.97*	*0.03*		*0.72*	*0.28*
	German Productivity Shock, $\rho = 1.5$			German Productivity Shock, $\rho = 5$		
Baseline	0.057	0.052	0.005	0.051	0.014	0.037
Share:		*0.92*	*0.08*		*0.27*	*0.73*
Homogeneous $\pi_{nk,j,t}(f)$ and $\gamma_{mn,ji}(f)$	0.046	0.049	-0.003	0.046	0.047	-0.001
Share:		*1.07*	*-0.07*		*1.03*	*-0.03*
Homogeneous $\pi_{nk,j,t}(f)$	0.025	0.029	-0.004	0.028	0.029	-0.001
Share:		*1.15*	*-0.15*		*1.04*	*-0.04*
Homogeneous $\gamma_{mn,ji}(f)$	0.065	0.062	0.003	0.058	0.043	0.015
Share:		*0.95*	*0.05*		*0.74*	*0.26*

Sector-Level Decomposition

	ϵ^Y	$\bar{\epsilon}_j$	$Cov\left(\frac{\omega_{j,t}(f)}{\bar{\omega}_j}, \epsilon^j\right)$	ϵ^Y	$\bar{\epsilon}_j$	$Cov\left(\frac{\omega_{j,t}(f)}{\bar{\omega}_j}, \epsilon^j\right)$
	World Productivity Shock, $\rho = 1.5$			World Productivity Shock, $\rho = 5$		
Baseline	0.329	0.243	0.087	0.294	0.212	0.082
Share:		*0.74*	*0.26*		*0.72*	*0.28*
	German Productivity Shock, $\rho = 1.5$			German Productivity Shock, $\rho = 5$		
Baseline	0.057	0.055	0.002	0.051	0.048	0.003
Share:		*0.97*	*0.03*		*0.94*	*0.06*

Notes: This table reports the elasticity of French GDP with respect to a 10% productivity shock in every other country in the world and with respect to a 10% productivity shock to Germany, both the baseline model and the alternative models that suppress sources of firm heterogeneity. The table reports the decomposition of the aggregate elasticity into the mean and the covariance terms as in (1).

Table A4. Responses of French Real GDP to Foreign Demand Shocks, Robustness (10% Demand Shocks)

	ϵ^Y	$\bar{\epsilon}$	$Cov\left(\frac{\omega_{m,t}(f)}{\bar{\omega}}, \epsilon^f\right)$	ϵ^Y	$\bar{\epsilon}$	$Cov\left(\frac{\omega_{m,t}(f)}{\bar{\omega}}, \epsilon^f\right)$
	World Demand Shock, $\rho = 1.5$			World Demand Shock, $\rho = 5$		
Baseline	0.042	0.026	0.016	0.031	0.021	0.010
Share:		*0.63*	*0.37*		*0.67*	*0.33*
Homogeneous $\pi_{nk,j,t}(f)$ and $\gamma_{mn,ji}(f)$	0.041	0.036	0.006	0.034	0.033	0.001
Share:		*0.86*	*0.14*		*0.98*	*0.02*
Homogeneous $\pi_{nk,j,t}(f)$	0.036	0.031	0.005	0.028	0.028	-0.001
Share:		*0.87*	*0.13*		*1.02*	*-0.02*
Homogeneous $\gamma_{mn,ji}(f)$	0.043	0.028	0.015	0.033	0.027	0.006
Share:		*0.65*	*0.35*		*0.82*	*0.18*
	German Demand Shock, $\rho = 1.5$			German Demand Shock, $\rho = 5$		
Baseline	0.007	0.004	0.003	0.005	0.003	0.002
Share:		*0.59*	*0.41*		*0.61*	*0.39*
Homogeneous $\pi_{nk,j,t}(f)$ and $\gamma_{mn,ji}(f)$	0.006	0.005	0.002	0.005	0.005	0.001
Share:		*0.75*	*0.25*		*0.88*	*0.12*
Homogeneous $\pi_{nk,j,t}(f)$	0.005	0.004	0.001	0.004	0.004	0.000
Share:		*0.74*	*0.26*		*0.90*	*0.10*
Homogeneous $\gamma_{mn,ji}(f)$	0.007	0.004	0.003	0.005	0.004	0.001
Share:		*0.63*	*0.37*		*0.80*	*0.20*

Sector-Level Decomposition

	ϵ^Y	$\bar{\epsilon}_j$	$Cov\left(\frac{\omega_{j,t}(f)}{\bar{\omega}_j}, \epsilon^j\right)$	ϵ^Y	$\bar{\epsilon}_j$	$Cov\left(\frac{\omega_{j,t}(f)}{\bar{\omega}_j}, \epsilon^j\right)$
	World Demand Shock, $\rho = 1.5$			World Demand Shock, $\rho = 5$		
Baseline	0.042	0.069	-0.027	0.031	0.052	-0.021
Share:		*1.64*	*-0.64*		*1.68*	*-0.68*
	German Demand Shock, $\rho = 1.5$			German Demand Shock, $\rho = 5$		
Baseline	0.007	0.009	-0.002	0.005	0.007	-0.002
Share:		*1.36*	*-0.36*		*1.34*	*-0.34*

Notes: This table reports the elasticity of French GDP with respect to a 10% demand shock for French goods in every other country in the world and with respect to a 10% demand shock for French goods in Germany, both the baseline model and the alternative models that suppress sources of firm heterogeneity. The table reports the decomposition of the aggregate elasticity into the mean and the covariance terms as in (1).