

# Granularity in International Markets

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Granularity and Networks  
in a Global Economy  
MIE, 2nd year

## Motivating literature

- Most of the macroeconomic literature uses dynamic GE models in which **aggregate fluctuations** are driven by **aggregate shocks**
  - See the RBC/DSGE literatures in a closed economy, Backus, Kehoe and Kydland (1995) in an open-economy context
  - Microeconomic shocks neglected on the ground of a “law of large numbers” argument(e.g. Lucas, 1977)
  - Need to feed models with quite volatile aggregate processes to match the evidence on macroeconomic volatility
- Recent works challenge this view : **Idiosyncratic shocks** to individual firms or sectors might generate significant volatility

## Motivating literature (ii)

- The **microeconomic origin** of aggregate fluctuations
  - Gabaix (2011) : When the distribution of firms' size is fat-tailed, shocks to the largest firms in the economy do not compensate with shocks to small firms
  - Acemoglu et al (2012) : When there are sufficiently strong interconnections between firms/sectors, shocks to upstream units propagate throughout the value chain (See Lecture 2 for consequences in an open-economy context)
- Supported by **empirical evidence** :
  - Gabaix (2011) : One third of fluctuations in the US GDP is accounted for by the 100 largest US firms
  - Di Giovanni et al. (2014) : Shocks to individual firms matter as much as shocks to individual sectors or countries in explaining fluctuations in French aggregate sales

## Motivating literature (iii)

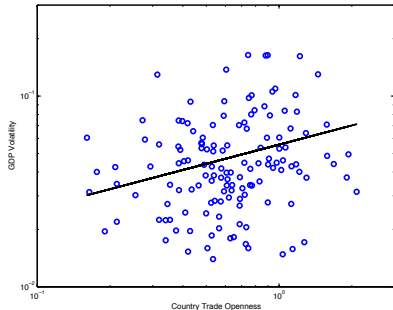
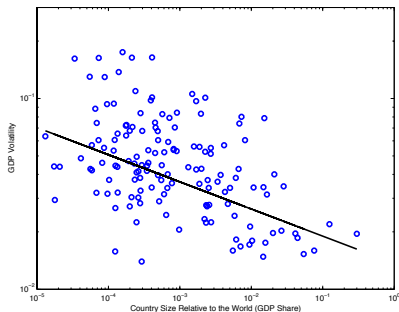
- Trade literature also makes great use of the law of large numbers
  - Melitz' type models work with a continuum of firms
  - No single firm has enough "weight" to impact aggregate outcomes
  - Inconsistent with empirical evidence of a strong degree of heterogeneity across firms
- Recent works challenge this view :
  - Models of a finite number of firms deliver new results regarding the determinants of aggregate trade (Eaton, Kortum and Sotelo, 2012)

# Granularity in international markets

- The intuitions surrounding this literature extend naturally to an **open-economy context** because
  - Firms engaged in international markets are large, on average (Bernard and Jensen, 1995, Mayer and Ottaviano, 2007)
  - Trade liberalization makes large firms even larger (Pavcnik, 2002, Bernard et al., 2003)

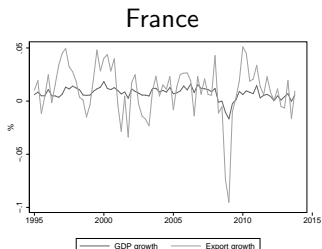
⇒ International markets characterize by their granularity

# Why do we care? Cross-Country Heterogeneity in Volatilities



Partial correlations between aggregate volatility on one side (y-axis) and country size (x-axis, left panel) and openness (x-axis, right panel). Source : di Giovanni and Levchenko (2012)

# Why do we care? The Volatility of Trade



Quarterly growth in the value of aggregate exports (grey line), aggregate GDP (black line, top panel) and industrial production (black line, bottom panel) in the US and in France. Data are seasonally adjusted. Source : IMF-IFS

# Road Map

- Granularity in a closed-economy context
- The Granularity of International Markets
- Aggregate Consequences



# Granularity in a closed-economy context

## Intuition

- When the distribution of firms' size is fat-tailed, the variance of the distribution is not finite and the central limit theorem does not apply
- Micro shocks need not average out in the aggregate : Shocks to the largest firms in the economy do not cancel out with shocks to small firms
- “Aggregate” fluctuations can be generated by a relatively low level of idiosyncratic risk (Gabaix, 2011)

## Anecdotal evidence

- In 2000, Nokia contributed 1.6 percentage points of Finland's GDP growth (OECD, 2004)
- In Korea, the top two firms (Samsung and Hyundai) together account for 22% of Korean GDP (di Giovanni and Levchenko, 2009)
- In 1970, a major strike at GM lasted 10 weeks, induced a 31% sales fall and a 13% employment decrease → Direct impact is a change in US GDP by  $-0.49\%$  that year (Gabaix, 2011)
- In December 2004, a \$24 billion one-time Microsoft dividend boosted growth in personal income from 0.6% to 3.7% (Bureau of Economic Analysis, January 31, 2005)
- “ The sales of Apples new device [iPhone5] could add as much as half a percentage point to U.S. fourth quarter GDP, according to JPMorgan” (CNBC, Sept. 17, 2012)

## A simple model : Assumptions

- Consider an economy made of  $N$  entrepreneurs, indexed by  $f$ , each one being characterized by its size at time  $t$ ,  $S_{ft}$
- The only source of volatility are idiosyncratic shocks to firms :

$$g_{S_{ft}} \equiv \frac{\Delta S_{ft}}{S_{ft-1}} = \sigma_f \varepsilon_{ft}$$

where  $\sigma_f$  is firm  $f$ 's volatility and  $\varepsilon_{ft}$  an idiosyncratic shock of mean 0 and variance 1

- Total GDP is defined as  $Y_t = \sum_f S_{ft}$  thus GDP growth :

$$g_{Y_t} \equiv \frac{\Delta Y_t}{Y_{t-1}} = \sum_f \sigma_f w_{ft-1} \varepsilon_{ft}$$

with  $w_{ft-1} \equiv \frac{S_{ft-1}}{Y_{t-1}}$  the share of  $f$  in the aggregate

## A simple model : Macroeconomic Volatility

- When shocks are uncorrelated and the relative size of firms is constant, the standard deviation of GDP growth (the “macroeconomic volatility”) is :

$$\sigma_Y = \left[ \sum_f \sigma_f^2 (w_f)^2 \right]^{1/2}$$

- If the volatility of individual firms is homogenous ( $\sigma_f = \sigma \quad \forall f$ ) :

$$\sigma_Y = \sigma \left[ \sum_f (w_f)^2 \right]^{1/2} = \sigma \sqrt{Herf}$$

- Numerical exemple (di Giovanni et al, 2014) : Take  $\sigma = .2$  and  $N = 1,024,770$ ,
  - If  $Herf = 1/N$ ,  $\sigma_Y = .0002$
  - If  $Herf = .0011$ ,  $\sigma_Y = .0067$

## A simple model : General results

- If the size distribution is uniform

$$\sigma_Y = \frac{\sigma}{\sqrt{N}}$$

- If the size distribution has finite variance

$$\sigma_Y = \frac{E[S^2]^{1/2}}{E[S]} \frac{\sigma}{\sqrt{N}}$$

(Converges to 0 at rate  $1/\sqrt{N}$ )

## A simple model : General results

- If the size distribution is a power law  $P(S > x) = ax^{-\xi}$  with  $\xi \geq 1$  :

$$\begin{aligned}\sigma_Y &\sim \frac{\nu_\xi}{\ln N} \sigma && \text{for } \xi = 1 \\ \sigma_Y &\sim \frac{\nu_\xi}{N^{1-1/\xi}} \sigma && \text{for } 1 < \xi < 2 \\ \sigma_Y &\sim \frac{\nu_\xi}{N^{1/2}} \sigma && \text{for } \xi \geq 2\end{aligned}$$

where  $\nu_\xi$  is a random variable that is independent of  $N$  and  $\sigma$

⇒ Implications :

- If the size distribution has thin tails ( $\xi > 2$ ),  $\sigma_Y$  decays at rate  $1/\sqrt{N}$
- With a fat tail distribution,  $\sigma_Y$  decays much more slowly
- Zipf law ( $\xi = 1$ ) : Top  $K$  firms account for a finite (as opposed to infinitesimal) fraction of aggregate output → “Granularity”  
( $\lim_{N \rightarrow \infty} \sqrt{\text{Herf}} = a > 0$ )

## A simple model : Remarks

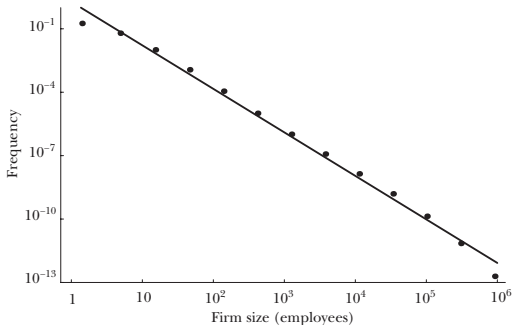
- In the data, microeconomic shocks will generate a substantial amount of aggregate volatility whenever the Herfindahl of sales is “large” enough (i.e. Zipf is not necessary, a lognormal distribution with high variance would work as well)
- When the volatility of individual firms is decreasing in their size (i.e.  $\sigma_f(S_{ft}) = kS_{ft}^{-\alpha}$ ,  $\alpha > 0$ , as observed in the data), the contribution of large firms to aggregate volatility is reduced, but still substantial under reasonable parametric value for  $\alpha$
- Results generalize to an economy with intermediate goods but the proper definition of the Herfindahl index is based on Domar weights
- Economic models of firm size distribution : Rossi-Hansberg and Wright (2007), Luttmer (2007), Gabaix (2007), Carvalho and Grassi (2015), Geerolf (2017)



# Empirical evidence : Distribution of Firm Size

## United States

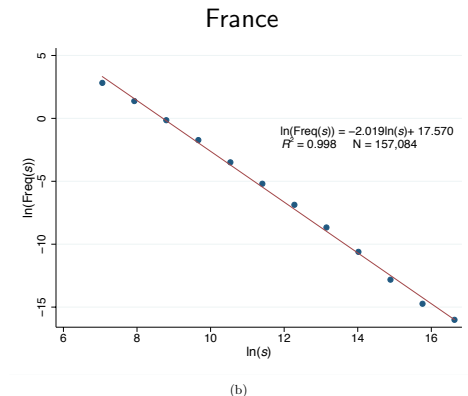
Log *Frequency* versus log *Size* of US firms (by Number of Employees) for 1997



Source: Axtell (2001).

Notes: Ordinary least squares (OLS) fit gives a slope of 2.06 (s.e. = 0.054;  $R^2 = 0.99$ ). This corresponds to a frequency  $f(S) \sim S^{-2.059}$ , which is a power law distribution with exponent 1.059. This is very close to an ideal Zipf's law, which would have an exponent  $\zeta = 1$ .

# Empirical evidence : Distribution of Firm Size



Notes: This figure reports the estimated power laws in firm size based on total sales and all firms. The power laws are estimated with two different methods, the cdf (panel a) and the pdf (panel b).

Source : di Giovanni et al (2011)

# Empirical evidence : Distribution of Sector Size

- Carvalho and Gabaix (2013) : Investigate what drove GDP volatility over the last half century in the US
- Macroeconomic volatility is due to micro / “fundamental” volatility :

$$\sigma_{Ft} = \sqrt{\sum_i \left( \frac{S_{it}}{GDP_t} \right)^2 \sigma_i^2}$$

where  $\sigma_i^2$  is the variance of sectoral TFP

# Empirical evidence : Distribution of Sector Size

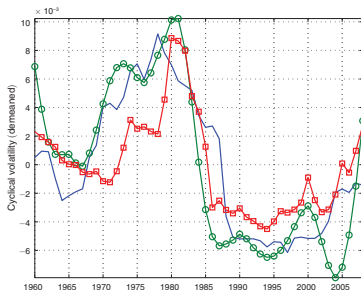


FIGURE 1. FUNDAMENTAL VOLATILITY AND GDP VOLATILITY

*Notes:* The squared line gives the fundamental volatility ( $4.5\sigma_{F_t}$ , demeaned). The solid and circle lines are annualized (and demeaned) estimates of GDP volatility, using respectively a rolling-window estimate and an HP trend of instantaneous volatility.

Source : Carvalho and Gabaix (2013)

- $\downarrow$  size of heavy-manuf sectors from 1960s to 1990s  $\rightarrow \sigma_{F_t} \downarrow$
- Growth of oil industry from mid-1970s to 1980s  $\rightarrow \sigma_{F_t} \uparrow$
- $\uparrow$  size of the financial sector in early 2000s  $\rightarrow \sigma_{F_t} \uparrow$

## Empirical evidence : Granular fluctuations

- di Giovanni et al (2014) : Use French data on individual firms' sales, by destination country
- Identification strategy : Use firm-destination specific sales to recover **microeconomic** shocks
- Start from the most disaggregated level (i.e. firm  $\times$  destination  $\times$  year) and estimate :

$$g_{fnt} = \delta_{nt}^j + \varepsilon_{fnt}$$

- Aggregate individual components using the definition of the growth rate of aggregate sales (Intensive margin) :

$$g_t = \underbrace{\sum_{f,n} w_{fnt-1} \delta_{nt}^j}_{\text{Contribution Macro}} + \underbrace{\sum_{f,n} w_{fnt-1} \varepsilon_{fnt}}_{\text{Contribution Micro}}$$

# Empirical evidence : Granular fluctuations

- Motivating model :
  - Demand-side assumptions : CD across sectors, CES across varieties

$$x_{fnt} = \omega_{fnt} \left( \frac{p_{fnt}}{P_{nt}^j} \right)^{1-\sigma^j} \alpha_{nt}^j Y_{nt}$$

- Supply-side assumptions : Monopolistic competition

$$p_{fnt} = \frac{\sigma}{\sigma - 1} \tau_n^j c_t^j a_{ft}$$

- Growth equation :

$$\begin{aligned} g_{fnt} = & \underbrace{d \ln Y_{nt}}_{\text{Macro shock } \delta_{nt}} + \underbrace{d \ln \alpha_{nt}^j + (1 - \sigma)(d \ln c_t^j - d \ln P_{nt}^j)}_{\text{Sector shock } \delta_{nt}^j} \\ & + \underbrace{d \ln \omega_{fnt} + (1 - \sigma) d \ln a_{ft}}_{\text{Micro shock } \varepsilon_{fnt}} \end{aligned}$$

Estimated year-by-year and destination-by-destination, using OLS with fixed effects

## Empirical evidence : Granular fluctuations

- Working with the aggregate decomposition is impractical if weights are treated as time-varying random variables
- Therefore work with a closely related object :

$$g_{t|\tau} = \underbrace{\sum_{f,n} w_{fn\tau-1} \delta_{nt}^j}_{\text{Contribution Macro}} + \underbrace{\sum_{f,n} w_{fn\tau-1} \varepsilon_{fnt}}_{\text{Contribution Micro}}$$

- Aggregate variance conditional on a (non-stochastic) distribution of weights :

$$\begin{aligned} \sigma_{A|\tau}^2 &= \underbrace{\sum_{j,m} \sum_{k,n} w_{m\tau-1}^j w_{n\tau-1}^k \text{Cov}(\delta_{mt}^j, \delta_{nt}^k)}_{\text{Macro component}} \\ &+ \underbrace{\sum_{g,m} \sum_{f,n} w_{gm\tau-1} w_{fn\tau-1} \text{Cov}(\varepsilon_{gmt}, \varepsilon_{fnt}) + \text{Cov}_\tau}_{\text{Micro component}} \end{aligned}$$

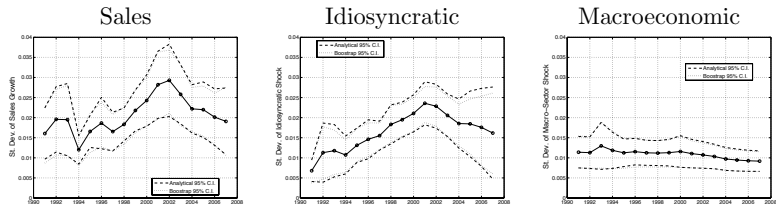
# Empirical evidence : Granular fluctuations

<b>I. Total Sales</b>				
	<i>Whole Economy</i>		<i>Manufacturing Sector</i>	
	(1)	(2)	(3)	(4)
	St. Dev.	Relative SD	St. Dev.	Relative SD
Actual	0.0206	1.0000	0.0244	1.0000
Firm-Specific	0.0165	0.8010	0.0168	0.6885
Sector-Destination	0.0109	0.5291	0.0157	0.6434
<b>II. Domestic Sales</b>				
	<i>Whole Economy</i>		<i>Manufacturing Sector</i>	
	(1)	(2)	(3)	(4)
	St. Dev.	Relative SD	St. Dev.	Relative SD
Actual	0.0196	1.0000	0.0231	1.0000
Firm-Specific	0.0154	0.7857	0.0151	0.6537
Sector-Destination	0.0112	0.5714	0.0167	0.7229
<b>III. Export Sales</b>				
	<i>Whole Economy</i>		<i>Manufacturing Sector</i>	
	(1)	(2)	(3)	(4)
	St. Dev.	Relative SD	St. Dev.	Relative SD
Actual	0.0361	1.0000	0.0374	1.0000
Firm-Specific	0.0304	0.8421	0.0287	0.7674
Sector-Destination	0.0129	0.3573	0.0153	0.4091
<b>IV. Value Added</b>				
	<i>Whole Economy</i>		<i>Manufacturing Sector</i>	
	(1)	(2)	(3)	(4)
	St. Dev.	Relative SD	St. Dev.	Relative SD
Actual	0.0210	1.0000	0.0215	1.0000
Firm-Specific	0.0190	0.9048	0.0184	0.8558
Sector-Destination	0.0107	0.5095	0.0123	0.5721

Notes : The variance components do not add up to the actual variance due to unreported covariance terms. Source : di Giovanni et al. (2014)



# Empirical evidence : Granular fluctuations

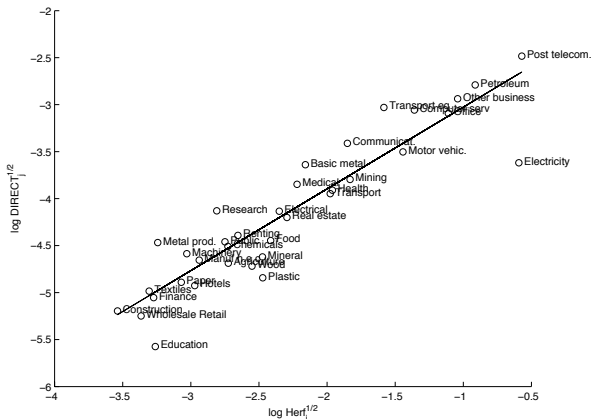


Source : di Giovanni et al. (2014). Dotted line represent the confidence intervals based on analytical and bootstrapped standard errors

- Contribution of firm-specific shocks is increasing over time

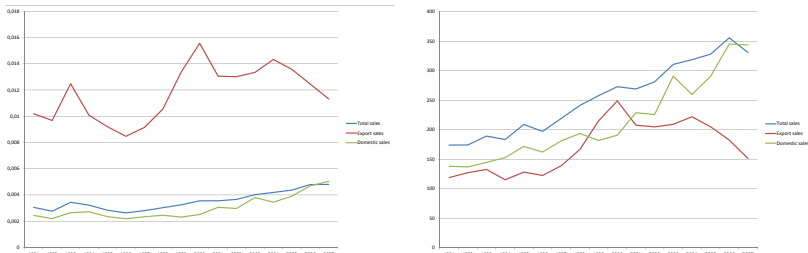
# Empirical evidence : Granular fluctuations

$$\begin{aligned}\sigma_{F|\tau}^2 &= \sum_{g,m} \sum_{f,n} w_{gm\tau-1} w_{fn\tau-1} \text{Cov}(\varepsilon_{gmt}, \varepsilon_{fnt}) \\ &= \underbrace{\sum_{f,n} w_{fn\tau-1}^2 \text{Var}(\varepsilon_{fnt})}_{\text{Direct}} + \underbrace{\sum_{g \neq f, m \neq n} \sum_{f,n} w_{gm\tau-1} w_{fn\tau-1} \text{Cov}(\varepsilon_{gmt}, \varepsilon_{fnt})}_{\text{Linkages}}\end{aligned}$$



# The “Granularity” of International Markets

# The granularity of export and domestic sales



The left panel depicts the evolution of the Herfindahl index of firms' sales, in aggregate sales, export sales and domestic sales. The right panel scales those Herfindahl indices to the value one would observe if the distribution of sales was uniform ( $Herf = 1/N$ ). Source : di Giovanni et al (2014) (unreported).

- Distribution of sales far from uniform
- Export sales are more concentrated than domestic sales
- This very much reflects selection into export markets

# The “Happy few”

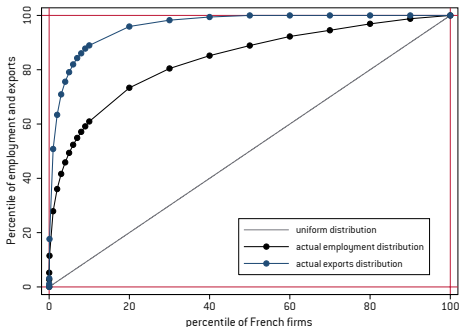
<i>NAICS industry</i>	<i>Percent of firms</i>	<i>Percent of firms that export</i>	<i>Mean exports as a percent of total shipments</i>
311 Food Manufacturing	6.8	12	15
312 Beverage and Tobacco Product	0.7	23	7
313 Textile Mills	1.0	25	13
314 Textile Product Mills	1.9	12	12
315 Apparel Manufacturing	3.2	8	14
316 Leather and Allied Product	0.4	24	13
321 Wood Product Manufacturing	5.5	8	19
322 Paper Manufacturing	1.4	24	9
323 Printing and Related Support	11.9	5	14
324 Petroleum and Coal Products	0.4	18	12
325 Chemical Manufacturing	3.1	36	14
326 Plastics and Rubber Products	4.4	28	10
327 Nonmetallic Mineral Product	4.0	9	12
331 Primary Metal Manufacturing	1.5	30	10
332 Fabricated Metal Product	19.9	14	12
333 Machinery Manufacturing	9.0	33	16
334 Computer and Electronic Product	4.5	38	21
335 Electrical Equipment, Appliance	1.7	38	13
336 Transportation Equipment	3.4	28	13
337 Furniture and Related Product	6.4	7	10
339 Miscellaneous Manufacturing	9.1	2	15
<b>Aggregate manufacturing</b>	<b>100</b>	<b>18</b>	<b>14</b>

*Sources:* Data are from the 2002 U.S. Census of Manufactures.

*Notes:* The first column of numbers summarizes the distribution of manufacturing firms across three-digit NAICS manufacturing industries. The second reports the share of firms in each industry that export. The final column reports mean exports as a percent of total shipments across all firms that export in the noted industry.

Source : Bernard et al (2007).

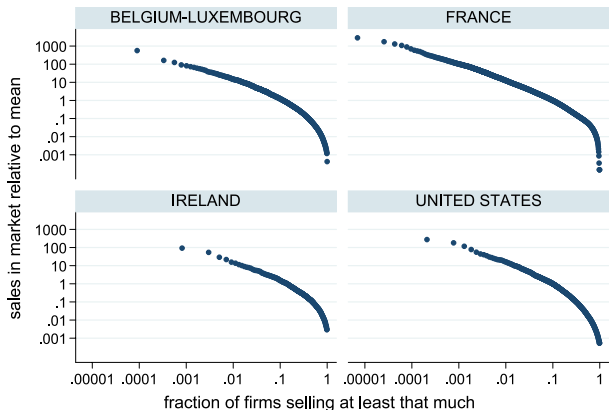
## The “Happy few”



Source : Mayer and Ottaviano (2007).

- The top one percent of French firms is responsible for 68% of aggregate exports (44% in the sample of EAE firms)
- In the manufacturing sector only 17.4% of firms exports and 34% of exporters serve a single market
- The distribution of exports is even more skewed than the distribution of employment

# Sales Distribution of French Firms



Source: Eaton et al. (2011). Plot of the sales of each firm in a particular market (relative to mean sales there) against the fraction of firms selling in the market who sell at least that much

# Exporters are different

Country of origin	Employment premia	Value added premia	Wage premia	Capital intensity premia	Skill intensity premia
Exporters premia:					
Germany	2.99 (4.39)		1.02 (0.06)		
France	2.24 (0.47)	2.68 (0.84)	1.09 (1.12)	1.49 (5.60)	
United Kingdom	1.01 (0.92)	1.29 (1.53)	1.15 (1.39)		
Italy	2.42 (2.06)	2.14 (1.78)	1.07 (1.06)	1.01 (0.45)	1.25 (1.04)
Hungary	5.31 (2.95)	13.53 (23.75)	1.44 (1.63)	0.79 (0.35)	
Belgium	9.16 (13.42)	14.80 (21.12)	1.26 (1.15)	1.04 (3.09)	
Norway	6.11 (5.59)	7.95 (7.48)	1.08 (0.68)	1.01 (0.23)	
FDI- makers premia:					
Germany	13.19 (2.86)				
France	18.45 (7.14)	22.68 (6.10)	1.13 (0.90)	1.52 (0.72)	
Belgium	16.45 (6.82)	24.65 (11.14)	1.53 (1.20)	1.03 (0.82)	
Norway	8.28 (4.48)	11.00 (5.41)	1.34 (0.76)	0.87 (0.13)	

Note : The table shows premia of the considered variable as the ratio of exporters over non-exporters (standard deviation ratio in brackets). France, Germany, Hungary, Italy and the United Kingdom have large firms only; Belgian and Norwegian data are exhaustive.

Source : Mayer and Ottaviano (2007).



# Internationalized firms account for a substantial share of aggregate GDP

<b>Panel A: Whole Economy</b>				
	No. firms	Value Added		
		Mean	Median	Share in total
All Firms	998,531	1,165	211	1.00
Importers	189,863	3,516	515	0.72
Exporters	200,775	3,219	477	0.71
Affiliates of foreign multinationals	30,654	7,061	1,335	0.25
Firms with foreign affiliates	1,786	65,829	2,279	0.14

<b>Panel B: Manufacturing Sector</b>				
	No. firms	Value Added		
		Mean	Median	Share in total
All Firms	145,575	2,367	382	1.00
Importers	60,395	4,444	872	0.93
Exporters	66,507	4,053	754	0.93
Affiliates of foreign multinationals	8,370	11,994	2,939	0.38
Firms with foreign affiliates	378	34,794	6,993	0.06

Source : di Giovanni et al, 2018

# Internationalized firms account for a substantial share of aggregate GDP

- Even more true in SOEs :
  - In New Zealand one firm (Fonterra) is responsible for one-third of global dairy exports (it is the world's single largest exporter of dairy products)
  - Fonterra accounts for 20% of New Zealand's overall exports, and 7% of its GDP
  - 95% of Fonterra's output is exported
  - The second largest producer of dairy products in New Zealand is 1.3% the size of Fonterra

# Aggregate Consequences

## dG&L : Heterogeneity in Volatilities

- di Giovanni and Levchenko (2012) : How does opening to trade impact macroeconomic volatility ?
- Trade (empirical and theoretical) literature shows that opening to trade allows the largest firms to grow even larger relative to domestic firms  $\Rightarrow$  **The economy becomes more granular**
- Consequences for aggregate fluctuations
  - Quantitative results from a multi-country, multi-sector model of trade
  - Results fit empirical evidence on volatility and size, volatility and openness to trade

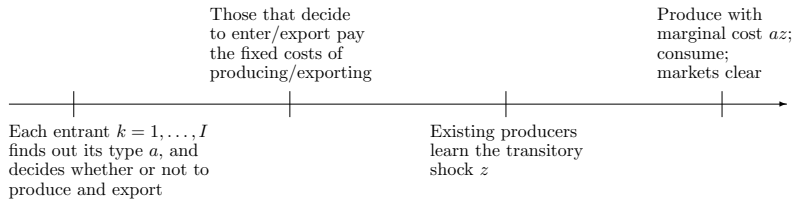
## dG&L : Heterogeneity in Volatilities

- A model with multiple granular economies
- Ingredients :
  - Melitz (2003) multi-country model with a **finite number of firms**
  - Firm productivity follows a **Pareto distribution**
  - Transitory iid productivity shock that realizes **after all fixed costs have been paid** (i.e. no extensive adjustments)
  - Complete model has non traded goods and IO linkages
- Complete model solved numerically, then simulated. Model fit assessed on trade volumes, the share of exporters and the relationship bw country size and the size of the largest firms

## dG&L : Assumptions

- $C$  countries
- Preferences are Cobb-Douglas across the  $T$  and  $NT$  sectors ( $\alpha^s$ ), CES across varieties within a sector ( $\varepsilon_N, \varepsilon_T$ )
- One factor of production supplied inelastically  $L_i$
- Production uses labor and CES composites of  $T$  and  $NT$  sectors ( $\beta_s$ )
- An endogenous, finite number of potential entrepreneurs in each sector ( $\bar{I}_i^s$ )
- Firm productivity drawn from a **Pareto distribution** ( $\theta^s$ )
- Sunk cost  $f_e$  to discover productivity type
- Fixed and variable trade costs ( $f_{ii}^s, f_{ij}^s, \tau_{ij}^s$ )
- Transitory iid productivity shock that realizes **after all fixed costs have been paid** (i.e. no extensive adjustments)

## dG&L : Timing



Source : di Giovanni and Levchenko (2012).

## dG&L : Solution

- Solution based on two additional assumptions :
  - Marginal firm ignores its impact on total expenditures and the price index
  - Marginal firm treats total expenditures and the price index as non-stochastic
- Market-specific productivity cut-off :

$$\frac{1}{\bar{z}_{ij}^s} = \frac{\varepsilon^s}{\varepsilon^s - 1} \frac{P_j^s}{\tau_{ij}^s c_i^s} \left( \frac{\alpha^s X_j}{\varepsilon^s c_i^s f_{ij}^s} \right)^{\frac{1}{\varepsilon^s - 1}}$$

- Free entry condition :

$$E \left[ \sum_j \mathbf{1}[z(k) \geq \bar{z}_{ij}^s] \left( \frac{\alpha^s X_j}{\varepsilon^s P_j^s 1^{-\varepsilon^s}} \left( \frac{\varepsilon^s}{\varepsilon^s - 1} \frac{\tau_{ij}^s c_i^s}{z(k) \bar{z}(k)} \right)^{1-\varepsilon^s} - c_i^s f_{ij}^s \right) \right] = c_i^s f_e^s$$



## dG&L : Solution

- Equilibrium price (using  $E_{\tilde{z}}(\tilde{z}(k)^{\varepsilon^s-1}) = 1$ ) :

$$P_i^s 1 - \varepsilon^s = \sum_j \left( \frac{\varepsilon^s}{\varepsilon^s - 1} \tau_{ij}^s c_j^s \right)^{1 - \varepsilon^s} \bar{l}_j^s \Pr(z(k) > z_{ij}^s) E[z(k)^{\varepsilon^s - 1} | z(k) > z_{ij}^s]$$

- Under Pareto distributions :

$$P_i^s = Cste X_i^{-\frac{\theta^s - (\varepsilon^s - 1)}{\theta^s(\varepsilon^s - 1)}} \left( \sum_j \bar{l}_j^s (\tau_{ij}^s c_j^s)^{-\theta^s} (f_{ij}^s c_j^s)^{-\frac{\theta^s - (\varepsilon^s - 1)}{\theta^s(\varepsilon^s - 1)}} \right)^{-\frac{1}{\theta^s}}$$

- Model closed assuming balanced trade

## dG&L : Intuitions

- In autarky, distribution of firms' sales is a **power law** :

$$Pr(x > q) = \delta q^{-\xi}$$

where  $\delta$  is a constant that reflects the size of domestic demand and  $\xi \equiv \frac{\theta}{\phi-1}$

⇒ **Model is granular** if the dispersion in productivities ( $\theta$ ) is sufficiently close to the price elasticity of nominal demands ( $\phi - 1$ )

- In the aggregate,  $X \equiv \sum_k x(z(k)\tilde{z}(k))$  and thus :

$$Var_{\tilde{z}} \left( \frac{\Delta X}{E_{\tilde{z}}(X)} \right) = \sigma^2 Herf$$

with  $\sigma^2$  the volatility of firm-level idiosyncratic shocks and  $Herf$  the Herfindahl of sales across firms

## dG&L : Intuitions

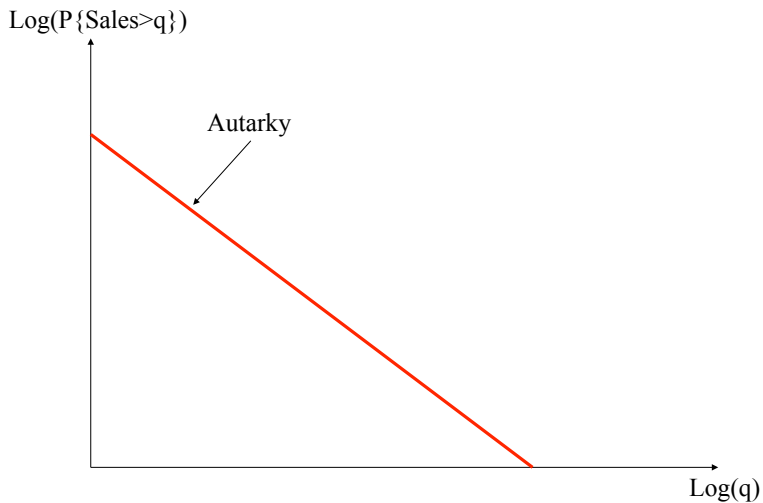
Consequences (One-sector symmetric model) :

- In autarky, the equilibrium number of firms increases in country size :

$$\bar{I}_{aut} \sim L^{\frac{1}{1-\frac{1-\beta}{\beta}} \frac{1}{\varepsilon-1}}$$

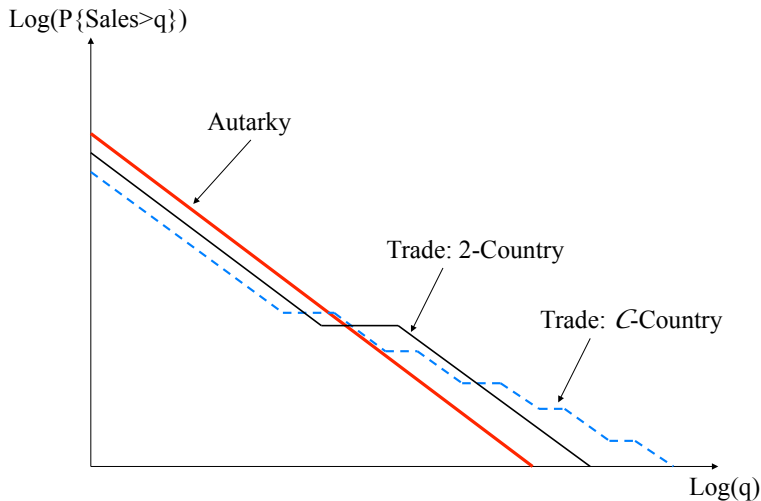
- ⇒ **Smaller countries have fewer firms and thus higher aggregate volatility**
- Trade liberalization induces net entry but increases the heterogeneity in firms' sales (domestic sales decrease but the most productive firms export) :
- ⇒ **After trade opening, aggregate volatility increases** (despite the entry of firms)

## dG&L : Intuitions



Source : di Giovanni and Levchenko (2012).

## dG&L : Intuitions



Source : di Giovanni and Levchenko (2012).

# dG&L : Intuitions

Sector	Exporting Firms				Non-Exporting Firms				
	(1) $\zeta$	(2) Std. Error	(3) $R^2$	(4) No. of firms	(5) $\zeta$	(6) Std. Error	(7) $R^2$	(8) No. of firms	(9) <i>t-stat</i>
Agriculture, Forestry, and Fishing	1.010	0.046	0.985	982	1.418	0.037	0.997	2,967	6.98**
Food Products	0.609	0.016	0.908	2,876	0.937	0.021	0.984	4,155	12.57**
Apparel and Leather Products	0.818	0.034	0.958	1,135	1.287	0.092	0.990	394	4.79**
Printing and Publishing	0.808	0.025	0.971	2,136	1.127	0.035	0.997	2,056	7.44**
Pharmaceuticals, Perfumes, and Beauty Products	0.512	0.029	0.903	605	0.604	0.071	0.975	145	1.19
Furniture, Household Goods	0.755	0.027	0.969	1,540	1.490	0.068	0.993	971	10.08**
Automotive	0.531	0.030	0.958	608	0.651	0.049	0.903	347	2.07*
Transport Equipment	0.554	0.040	0.975	393	0.877	0.084	0.991	218	3.48**
Non-electrical Machinery	0.785	0.017	0.967	4,166	1.338	0.028	0.984	4,556	16.81**
Electrical Machinery	0.710	0.027	0.979	1,394	1.437	0.059	0.991	1,179	11.18**
Mineral Products	0.656	0.031	0.948	919	0.979	0.031	0.948	2,062	7.49**
Textiles	0.844	0.038	0.919	1,008	1.194	0.091	0.992	346	3.56**
Wood and Paper Products	0.765	0.026	0.958	1,695	1.234	0.045	0.982	1,511	9.01**
Chemicals, Plastic, and Rubber	0.662	0.018	0.935	2,613	0.888	0.039	0.976	1,046	5.27**
Metals	0.793	0.017	0.976	4,574	1.241	0.031	0.993	3,244	12.79**
Electrical and Electronic Components	0.648	0.029	0.958	977	1.181	0.077	0.992	473	6.48**
Fuels	0.378	0.076	0.955	49	0.470	0.077	0.924	75	0.85
Water,Gas, Electricity	0.362	0.081	0.944	40	0.622	0.038	0.967	529	2.90**
Automotive Sales and Repair	0.737	0.016	0.947	4,516	1.029	0.012	0.981	14,648	14.84**
Wholesale Trade, Intermediaries	0.760	0.008	0.967	20,216	0.923	0.009	0.994	20,265	13.70**
Transport	0.856	0.017	0.970	5,339	1.014	0.016	0.995	8,293	6.91**
Professional Services	0.814	0.012	0.987	8,687	1.155	0.012	1.000	18,165	19.72**
Research and Development	0.751	0.072	0.983	219	0.832	0.093	0.976	159	0.69
Personal and Domestic Services	1.011	0.116	0.967	153	1.663	0.078	0.997	898	4.66**
Education	0.989	0.091	0.971	238	1.387	0.054	0.995	1,304	3.77**

Notes: This table reports the estimates of power laws in firm size (total sales) for non-exporting and exporting firms separately, for each individual sector, estimated using the log-rank-log-size estimator. The last column reports the  $t$ -statistic for the test of the difference between the coefficients in columns (1) and (5). \*\*: significant at the 1% level; \*: significant at the 5% level.

Source : di Giovanni Levchenko Ranciere (2011).

# dG&L : Calibration

Parameter	Baseline	Source
$\varepsilon^a$	6	Anderson and van Wincoop (2004)
$\theta^b$	5.3	Axtell (2001): $\frac{\theta}{\varepsilon-1} = 1.06$
$\alpha$	0.65	Yi and Zhang (2010)
$\{\beta_N, \beta_T\}$	$\{0.65, 0.35\}$	1997 U.S. Benchmark Input-Output Table
$\{\eta_N, \eta_T\}$	$\{0.77, 0.35\}$	
$\tau_{ij}^{c,d}$	2.30	Helpman et al. (2008)
$f_{ii}^c$	14.24	The World Bank (2007a); normalizing $f_{US,US}$ so that nearly all firms in the U.S. produce
$f_{ij}^c$	7.20	
$f_e$	34.0	To match 7,000,000 firms in the U.S. (U.S. Economic Census)
$\sigma^e$	0.1	Standard deviation of sales growth of the top 100 firms in COMPUSTAT

**Notes:**

<sup>a</sup> Robustness checks include  $\varepsilon = 4$  and  $\varepsilon = 8$ .

<sup>b</sup> Robustness checks include  $\frac{\theta}{\varepsilon-1} = 1.5$  and  $\varepsilon = 6$ , so that  $\theta = 6.5$ .

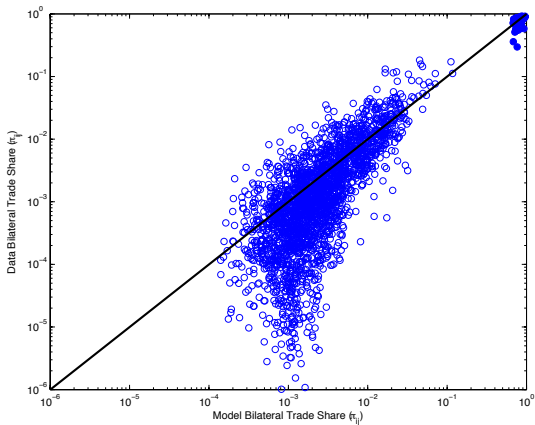
<sup>c</sup> Average in our sample of 50 countries.

<sup>d</sup>  $\tau_{ij} = \tau_{ji}$ . Adjusted by a constant ratio to match the median-level openness of the country sample.

<sup>e</sup> Robustness checks include  $\sigma$  varying with firm sales:  $\sigma = Ax^{-\xi}$ , where  $\xi = 1/6$ .

Source : di Giovanni Levchenko (2012).

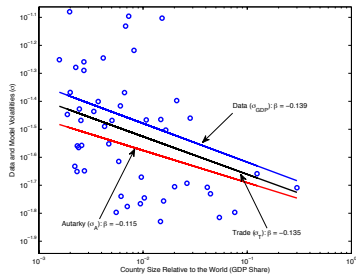
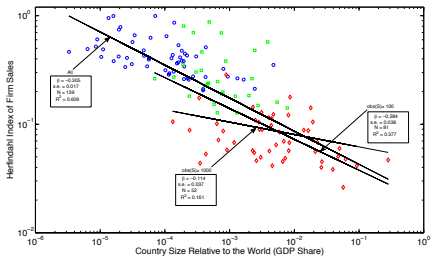
## dG&L : Model Fit (Trade shares)



Source : di Giovanni Levchenko (2012).



## dG&L : Results



The left panel depicts correlation between a country's share in world GDP and the concentration of its firms' sales. The right panel is the partial correlation between country size and the volatility of aggregate GDP. Source : di Giovanni and Levchenko (2012).

# dG&L : Results

Country	(1)	(2)	Country	(1)	(2)
	Trade/ Actual	Trade/ Autarky		Trade/ Actual	Trade/ Autarky
United States	0.377	1.035	Indonesia	0.376	1.060
Japan	0.405	1.014	South Africa	0.535	1.109
Germany	0.582	1.080	Norway	0.716	1.137
France	0.559	1.098	Poland	0.377	1.114
United Kingdom	0.476	1.076	Finland	0.437	1.109
Italy	0.463	1.098	Greece	0.414	1.116
China	0.280	1.024	Venezuela, RB	0.285	1.070
Canada	0.446	1.077	Thailand	0.337	1.099
Brazil	0.311	1.045	Portugal	0.379	1.068
Spain	0.550	1.061	Colombia	0.646	1.118
India	0.371	1.064	Nigeria	0.274	1.172
Australia	0.513	1.051	Algeria	0.271	1.156
Russian Federation	0.144	1.099	Israel	0.513	1.131
Mexico	0.329	1.052	Philippines	0.439	1.107
Netherlands	0.693	1.104	Malaysia	0.371	1.095
Korea, Rep.	0.296	1.059	Ireland	0.457	1.087
Sweden	0.634	1.099	Egypt, Arab Rep.	0.513	1.192
Switzerland	0.548	1.107	Pakistan	0.630	1.165
Belgium	0.713	1.072	Chile	0.262	1.119
Argentina	0.219	1.091	New Zealand	0.531	1.114
Saudi Arabia	0.168	1.069	Czech Republic	0.330	1.095
Austria	0.716	1.066	United Arab Emirates	0.178	1.089
Iran, Islamic Rep.	0.189	1.097	Hungary	0.399	1.114
Turkey	0.254	1.157	Romania	0.242	1.218
Denmark	0.612	1.156			

Notes: 'Trade/Actual' reports the ratio of aggregate volatility implied by the model under trade to the actual volatility of per capita GDP growth. In calculating volatility in the model, this column assumes that the firm-level volatility is equal to  $\sigma = 0.1$ . 'Trade/Autarky' reports the ratio of volatility in the model under trade to the volatility under autarky for each country.

Source : di Giovanni and Levchenko (2012).

## dG&L : Results

- A country accounting for .5% of GDP (Poland, South Africa) has granular volatility 70-100% higher than a country that accounts for 30% of world GDP (the US)
- Granular volatility accounts for 14-70% of actual observed volatility of countries (38% for the US, same as Gabaix, 2011)
- Impact of international trade on granular volatility
  - In a large economy like the US or Japan, international trade increases granular volatility by about 3.5% compared to autarky
  - In a small remote country (South Africa, New Zealand), international trade raises granular volatility by about 10%
  - In a small, close economy (Denmark, Romania), the effect is larger (15-20%)

## Conclusion

- When the distribution of size is fat-tailed, shocks to large firms can have a non-negligible impact in the aggregate
- Given granularity in international markets, this likely matters in modern open economies
  - For the magnitude of aggregate fluctuations (di Giovanni & Levchenko, 2012)
  - For the volatility of bilateral trade flows (Kramarz et al, 2017)
  - For the transmission of shocks across countries (di Giovanni et al, 2018)
- Largely unexplored : Pricing power of large firms in international markets (eg Parenti, 2018)