Introduction

- Ricardian theories of trade (Lectures 1-4):
  - Specialization along comparative advantages
  - Inter-industry trade between countries that are different in terms of technologies (Ricardo) or resources (HOS)
  - Gains from trade due to a better allocation of resources

- Limited empirical support (Lecture 5)
  - Missing features in HOV can partially explain poor empirical performances
  - In any case, the $R^2$ of such regressions is small

- No (explicit) role for geography
  - Hardly reconcilable with the gravity equation
The gravity equation

- Robust empirical model of bilateral trade in which size and distance effects enter multiplicatively:

\[ X_{ij} = G \times S_i \times M_j \times d_{ij} \]

- Used as a workhorse for analyzing the determinants of bilateral trade flows for 50 years since being introduced by Tinbergen (1962)

- Rationalized in mainstream modeling frameworks under some (widely used) parametric restrictions (See Lecture 10)
Trade and the size of countries

Japanese exports in the EU

Correlation between the Japan-EU trade and the size of partners. The x-axis measure the GDP of each EU members, in relative terms with respect to the Greek one. The y-axis measure the size of Japanese exports in each country (left-hand side) a,d the volume of Japanese imports from each country (right-hand side), again expressed in relative terms with respect to Greece. Data are for 2006. Source : Head & Mayer (2014).
Trade and distance

Correlation between the volume of trade and the distance between partners. The x-axis is the distance from France, expressed in kilometers. The x-axis measures the size of French exports (left-hand side) and the size of French imports (right-hand side), both expressed in relative terms with respect to the destination country’s GDP. Data are for 2006. Source: Head & Mayer (2014).
Introduction

- Eaton & Kortum (ECTA, 2002): A neo-classical trade model in which
  - Comparative advantages arise randomly
  - Technological advantages interact with geography to shape comparative advantages
  - Gravity equation arises structurally

- Model is substantially used in quantitative exercises on
  - the importance/evolution of Ricardian comparative advantages
  - the gains from trade liberalization
  - the volatility of trade and GDPs
  - ...

The Eaton-Kortum model | Empirical Evidence | Quantitative applications | Conclusion
<table>
<thead>
<tr>
<th>Road Map</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Detailed presentation of the Eaton-Kortum model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimation of Eaton-Kortum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Numerical applications</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The Eaton-Kortum model

See analytical details in EatonKortumAnalytics.pdf
Main features


- Ricardian model of international trade (based on differences in technology) that incorporates a role for geography (barriers to trade)
- Model yields a gravity type equation relating bilateral trade volumes to deviations from purchasing power parity, technology and geographic barriers
- Model can be estimated structurally
Assumptions

- $l$ countries ($i = 1 \ldots l$)
- A continuum of goods $j \in [0, 1]$
- Aggregate consumption in country $i$:
  \[
  U_i = \left[ \int_0^1 Q_i(j) \frac{\sigma - 1}{\sigma} \, dj \right]^\frac{\sigma}{\sigma - 1}
  \]
- Goods produced with a bundle of inputs which price is homogenous within countries $c_i$ (first taken as exogenous)
- Iceberg trade costs $d_{ni} > 1$. Without loss of generality $d_{ii} = 1$. Cross-border arbitrage implies: $d_{ni} \leq d_{nk}d_{ki}$
Assumptions (ii)

- Country $i$’s efficiency in producing good $j$: $z_i(j)$
- CIF price of good $j$ produced in country $i$, when exported in country $n$:
  $$p_{ni}(j) = \frac{c_i}{z_i(j)} d_{ni}$$
  
  **Unit cost**
  **Trade barrier**

- Perfect competition across suppliers
- Price actually paid in country $n$ for good $j$:
  $$p_n(j) = min\{p_{ni}(j); i = 1...I\}$$

(Note that most results continue to hold with Bertrand competition)
Assumptions (iii)

- Probabilistic representation of technologies: $z_i(j)$ is the realization of a random variable $Z_i$ drawn from a country-specific probability distribution:

  $$F_i(z) = Pr[Z_i \leq z]$$

- Productivity draws assumed independent across goods and countries

- $F_i$ assumed to be Fréchet (Type II extreme value):

  $$F_i(z) = e^{-T_i z^{-\theta}}$$

  with $T_i > 0$ and $\theta > 0$

Note: Fréchet can be shown to be the outcome of a process of innovation and diffusion in which $T_i$ is a stock of ideas. See Eaton & Kortum (IER, 1999)
### Interpretation

\[ F_i(z) = e^{-T_i z^{-\theta}} \]

- \( T_i \) “state of technology” or absolute advantage: Bigger \( T_i \) means that country \( i \) is more likely to draw a high efficiency for any good \( j \)
- \( \theta \) heterogeneity across goods or extent of comparative advantages within the continuum: Bigger \( \theta \) implies less variability in productivity.
Price distribution

- Country $i$’s distribution of prices in country $n$:
  \[G_{ni}(p) \equiv Pr[P_{ni} \leq p] = 1 - e^{-T_i(c_i d_{ni})^{-\theta}} p^\theta\]

- Country $n$’s actual distribution of prices:
  \[G_n(p) \equiv Pr[P_n \leq p] = 1 - \prod_i [1 - G_{ni}(p)] = 1 - e^{-\Phi_n p^\theta}\]

where \(\Phi_n \equiv \sum_i T_i(c_i d_{ni})^{-\theta}\)
The Eaton-Kortum model

Empirical Evidence

Quantitative applications

Conclusion

Price distribution

\[ G_n(p) = 1 - e^{-\Phi_n p^\theta} , \quad \Phi_n \equiv \sum_i T_i (c_i d_{ni})^{-\theta} \]

Distribution of prices governed by

- States of technology around the world \( \{ T_i \} \),
- Input costs around the world \( \{ c_i \} \),
- Geographic barriers \( \{ d_{ni} \} \)
  - If \( d_{ni} = 1, \forall n, i \) then \( \Phi_n = \Phi, \forall n \) (LOP)
  - If \( d_{ni} \to \infty, \forall i \) then \( \Phi_n = T_n c_n^{-\theta} \) (Autarky)

\( \Rightarrow \) \( \Phi_n \) interprets as the strength of competition that any firm will encounter in country \( n \)
Bilateral trade

- Share of goods that $n$ buys from $i = \text{Probability that } i \text{ provides the lowest price good in country } n$ :

$$\pi_{ni} = \frac{X_{ni}}{X_n} = Pr[p_{ni}(j) \leq \min\{p_{ns}(j); s \neq i\}]$$

$$= \int_{0}^{\infty} \prod_{s \neq i}[1 - G_{ns}(p)]dG_{ni}(p)$$

$$= \frac{T_i(c_id_{ni})^{-\theta}}{\Phi_n}$$

- or in log :

$$\ln X_{ni} = \ln \left( T_ic_i^{-\theta} \right) + \ln \left( X_n\Phi_n^{-1} \right) - \theta \ln d_{ni} = \text{Exporter FE } + \text{Importer FE } - \text{Gravity }$$

$\Rightarrow$ Gravity-type equation
Bilateral trade (ii)

Interpretation of the gravity equation:

- The coefficient on trade barriers relates to the distribution of productivities.
  - The more heterogenous productivities across producers of a commodity, the strongest the cost advantage of the lowest cost supplier, the more likely he remains the lowest cost supplier when trade costs increase.
  - Trade flows respond to geographic barriers at the extensive margin: As a source becomes more expensive or remote, it exports a narrower range of goods.
Bilateral trade (iii)

- Country $i$’s normalized import share in country $n$:

$$S_{ni} \equiv \frac{X_{ni}}{X_n} = \frac{\Phi_i}{\Phi_n} d_n^{-\theta} = \left( \frac{p_i d_{ni}}{p_n} \right)^{-\theta}$$

Always lower than one due to the triangle inequality (the maximum value for $p_n$ is $p_i d_{ni}$)

- As overall prices in market $n$ fall relative to prices in market $i$ ($\uparrow p_i/p_n$) or as $n$ becomes more isolated from $i$ ($\uparrow d_{ni}$), $i$’s normalized share in $n$ declines

- As the force of comparative advantages weakens (higher $\theta$), normalized import shares become more elastic

$\Rightarrow$ Structural equation that provides insight into the value of comparative advantages ($\theta$)
Suppose production is linear in labor (EK has intermediate inputs):

\[ c_i = w_i \]

\( \Rightarrow \) Price levels as a function of wages:

\[ p_n = \gamma \left[ \sum_i T_i (d_{ni}w_i)^{-\theta} \right]^{-1/\theta} \]

where \( \gamma \equiv \left[ \Gamma \left( \frac{\theta + 1 - \sigma}{\theta} \right) \right]^{1/(1 - \sigma)} \)  

\( \Rightarrow \) Trade shares as a function of wages and prices:

\[ \frac{X_{ni}}{X_n} = T_i \left( \frac{\gamma d_{ni}w_i}{p_n} \right)^{-\theta} \]
General equilibrium solution

- To close the model, one needs to solve for equilibrium wages across countries
- This is the trickier part of the exercise ⇒ Numerical solutions
- There are several simplifying assumptions that help solve the model: Exogenous labor supply, Wages determined in the nonmanufacturing sector, Trade balance
Extensions: Multiple sectors

- There are $K$ industries in each country ($k = 1...K$). Within each industry, a continuum of varieties is produced according to the technology described above.
- With multiple sectors, $T_i$ now has a sector dimension (but, crucially, $\theta$ remains common across all countries and industries...):
  \[
  F_i^k(z) = e^{-T_i^k z^{-\theta}}
  \]
Extensions: Multiple sectors (ii)

- With multiple industries, the model predicts what are the goods that a given country will specialize in.
- For any importer $j$ and any pair of exporters, $i, i' \neq j$, the ranking of relative fundamental productivities indeed determines the ranking of exports:

$$\frac{T_{i}^{1}}{T_{i'}^{1}} \leq \ldots \leq \frac{T_{i}^{K}}{T_{i'}^{K}} \iff \frac{X_{j}^{1}}{X_{j'}^{1}} \leq \ldots \leq \frac{X_{j}^{K}}{X_{j'}^{K}}$$

- Without intra-industry heterogeneity, this ranking states that country $i'$ has a comparative advantage over $i$ in the high $k$ goods.
- This does not interpret in terms of trade patterns with more than two countries, however.
Extensions: Multiple sectors (iii)

- With Fréchet, we further have:

\[
\frac{z_i^1(j)}{z_i^1(j)} \preceq ... \preceq \frac{z_i^K(j)}{z_i^K(j)}
\]

where \(\preceq\) denotes the first-order stochastic dominance order among distributions.

- Stochastic version of the previous ordering

- Imply that country \(i'\) is not expected to only produce the high \(k\) goods but to produce and export relatively more of these goods.
Extensions: Imperfect Competition

- Bernard, Eaton, Jensen & Kortum (2003) extend EK to allow for imperfect competition between varieties.
- With imperfect competition, consumer prices are above marginal costs.
- Model predicts a distribution of mark-ups in each market, that is bounded above by the Dixit-Stiglitz constant mark-up.
- Additional predictions on within-country heterogeneity in prices, productivities, etc.
Empirical analysis

- The model can be estimated to quantitatively assess the role of Ricardian advantages in driving international trade
- Crucial parameters: those driving the distribution of productivities (and comparative advantages), namely $\theta$ and $\{T_i\}$
- Here, I am focusing on the estimation of $\theta$
- Once those coefficients are estimated, it is possible to run various counterfactuals
EK Data

- Bilateral trade in manufactures among 19 OECD countries in 1990 (342 bilateral relationships $X_{ni}$)
- Absorption of manufactures as a measure of $X_n$ (STAN, OECD)
- Proxy for trade barriers:
  - Distance and other geographic barriers
  - Retail price differentials measured at the product level (WB):
    Interpreted as a sample of $p_i(j)$, used to calculate relative prices, which are theoretically bounded above by bilateral trade costs:
    $\ln \frac{p_i d_{ni}}{p_n} \approx \frac{\max 2j \{r_{ni}(j)\}}{\text{mean}\{r_{ni}(j)\}} = D_{ni}$

$r_{ni}(j) = \ln p_n(j) - \ln p_i(j)$ relative price of commodity $j$
$\Rightarrow \exp(D_{ni})$ price index in destination $n$ that would prevail if everything was purchased from $i$, relative to the actual price index in $n$
Estimating $\theta$, EK Method 1

Normalized import shares and relative prices

Source: Eaton & Kortum, 2002. Unconditional correlation -0.4

- Model: $\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left( \frac{p_i d_{ni}}{p_n} \right)^{-\theta}$
- Estimated $\theta$ (method-of-moments)

$$\hat{\theta} = \frac{\sum_n \sum_i \ln \frac{X_{ni}/X_n}{X_{ii}/X_i}}{\sum_n \sum_i [\ln d_{ni} - \ln P_i + \ln P_n]} \Rightarrow \hat{\theta} = 8.28$$

$\rightarrow$ Standard deviation in efficiency at given $T = 15\%$
Estimating $\theta$, SW Method (i)

- Simonovska & Waugh (2011)
- EK’s estimate based on a method of moments is biased upwards: Sample size of prices used to proxy trade costs is small relative to the actual number of goods in the economy → The max price differential observed is always lower than the true cost → Systematic upward estimate
- Develop a simulated method of moments to circumvent the issue
- Apply the method to disaggregate price and trade-flow data for the year 2004
Estimating $\theta$, SW Method (ii)

- Starting point:

$$\hat{\theta} = \frac{\sum_n \sum_i \ln \frac{X_{ni}/X_n}{X_{ii}/X_i}}{\sum_n \sum_i [\ln d_{ni} - \ln P_i + \ln P_n]}$$

- Denominator is not observed $\Rightarrow$ EK use:

$$\ln \hat{d}_{ni}(L) = \max_{l \in L} \{\ln p_n(l) - \ln p_i(l)\} \quad and \quad \ln \hat{P}_i = \frac{1}{L} \sum_l \ln p_i(l)$$

- SW shows this systematically underestimates actual trade barriers: i) With a finite sample of goods, $\exists$ a positive probability that the max log price difference is less than the true log trade cost, ii) On the other hand, there is zero probability that it is larger than the true log trade cost
Estimating $\theta$, SW Method (iii)

- With an expected maximal log price difference strictly less than the true trade cost, the elasticity of trade estimated by the method of moments is systematically overestimated:

$$E(\hat{\theta}) > \theta$$

- When the sample size becomes large, the measurement bias disappears and $\hat{\theta}$ converges in probability to $\theta$

$\Rightarrow$ Since EK’s estimator has desirable asymptotic properties, a simulated method of moments is consistent (See details in SW, 2011)

- Using the same data as EK, the SMM estimator is $\hat{\theta} = 3.93$. With more recent price data, $\hat{\theta} = 4.12$
First estimate the trade equation

After integrating the general equilibrium properties:

\[
\ln \frac{X'_{ni}}{X'_{nn}} = -\theta \ln d_{ni} + \frac{1}{\beta} \ln \frac{T_i}{T_n} - \theta \ln \frac{w_i}{w_n} = -\theta \ln d_{ni} + S_i - S_n
\]

where \( \ln X'_{ni} \equiv \ln X_{ni} - \frac{1-\beta}{\beta} \ln \frac{X_i}{X_{ii}} \)

Trade costs approximated by geographic barriers:

\[
\ln d_{ni} = d \times Dist + b \times border + l \times language + e \times TradingArea + FE_m + \delta_{ni}
\]

Error term consists of two components: \( \delta_{ni} = \delta_{ni}^2 + \delta_{ni}^1 \) with
\( \delta_{ni}^2 = \delta_{in}^2 \) affecting two-way trade (variance \( \sigma_2^2 \)) and \( \delta_{ni}^1 \) affecting one-way trade (variance \( \sigma_1^2 \))

Estimated by GLS
The Eaton-Kortum model

Empirical Evidence

Quantitative applications

Conclusion

A quarter of the total residual variance is reciprocal

Distance inhibits trade while sharing a language increases trade

Most competitive countries: Japan and the US. Least competitive countries: Belgium and Greece

Most open countries: Japan, the US and Belgium

Estimating θ, EK Method 2 (ii)

<table>
<thead>
<tr>
<th>Variable</th>
<th>est.</th>
<th>se</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance [0, 375)</td>
<td>−θd1</td>
<td>−3.10 (0.16)</td>
</tr>
<tr>
<td>Distance [375, 750)</td>
<td>−θd2</td>
<td>−3.66 (0.11)</td>
</tr>
<tr>
<td>Distance [750, 1500]</td>
<td>−θd3</td>
<td>−4.03 (0.10)</td>
</tr>
<tr>
<td>Distance [1500, 3000)</td>
<td>−θd4</td>
<td>−4.22 (0.16)</td>
</tr>
<tr>
<td>Distance [3000, 6000)</td>
<td>−θd5</td>
<td>−6.06 (0.09)</td>
</tr>
<tr>
<td>Distance [6000, maximum]</td>
<td>−θd6</td>
<td>−6.56 (0.10)</td>
</tr>
<tr>
<td>Shared border</td>
<td>−θb</td>
<td>0.30 (0.14)</td>
</tr>
<tr>
<td>Shared language</td>
<td>−θl</td>
<td>0.51 (0.15)</td>
</tr>
<tr>
<td>European Community</td>
<td>−θe</td>
<td>0.04 (0.13)</td>
</tr>
<tr>
<td>EFTA</td>
<td>−θf</td>
<td>0.54 (0.19)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source Country</th>
<th>Destination Country</th>
<th>Variable</th>
<th>est.</th>
<th>se</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>S1</td>
<td>0.15 (0.15)</td>
<td>−θe1</td>
<td>0.24 (0.27)</td>
</tr>
<tr>
<td>Austria</td>
<td>S2</td>
<td>−1.16 (0.12)</td>
<td>−θe2</td>
<td>−1.68 (0.21)</td>
</tr>
<tr>
<td>Belgium</td>
<td>S3</td>
<td>−3.34 (0.11)</td>
<td>−θe3</td>
<td>1.12 (0.19)</td>
</tr>
<tr>
<td>Canada</td>
<td>S4</td>
<td>0.44 (0.14)</td>
<td>−θe4</td>
<td>0.69 (0.25)</td>
</tr>
<tr>
<td>Denmark</td>
<td>S5</td>
<td>−1.75 (0.12)</td>
<td>−θe5</td>
<td>−0.51 (0.19)</td>
</tr>
<tr>
<td>Finland</td>
<td>S6</td>
<td>−0.52 (0.12)</td>
<td>−θe6</td>
<td>−1.33 (0.22)</td>
</tr>
<tr>
<td>France</td>
<td>S7</td>
<td>1.28 (0.11)</td>
<td>−θe7</td>
<td>0.22 (0.19)</td>
</tr>
<tr>
<td>Germany</td>
<td>S8</td>
<td>2.35 (0.12)</td>
<td>−θe8</td>
<td>1.00 (0.19)</td>
</tr>
<tr>
<td>Greece</td>
<td>S9</td>
<td>−2.81 (0.12)</td>
<td>−θe9</td>
<td>−2.36 (0.20)</td>
</tr>
<tr>
<td>Italy</td>
<td>S10</td>
<td>1.78 (0.11)</td>
<td>−θe10</td>
<td>0.07 (0.19)</td>
</tr>
<tr>
<td>Japan</td>
<td>S11</td>
<td>4.20 (0.13)</td>
<td>−θe11</td>
<td>1.59 (0.22)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>S12</td>
<td>−2.19 (0.11)</td>
<td>−θe12</td>
<td>1.00 (0.19)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>S13</td>
<td>−1.20 (0.15)</td>
<td>−θe13</td>
<td>0.07 (0.27)</td>
</tr>
<tr>
<td>Norway</td>
<td>S14</td>
<td>−1.35 (0.12)</td>
<td>−θe14</td>
<td>−1.00 (0.21)</td>
</tr>
<tr>
<td>Portugal</td>
<td>S15</td>
<td>−1.57 (0.12)</td>
<td>−θe15</td>
<td>−1.21 (0.21)</td>
</tr>
<tr>
<td>Spain</td>
<td>S16</td>
<td>0.30 (0.12)</td>
<td>−θe16</td>
<td>−1.16 (0.19)</td>
</tr>
<tr>
<td>Sweden</td>
<td>S17</td>
<td>0.01 (0.12)</td>
<td>−θe17</td>
<td>−0.02 (0.22)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>S18</td>
<td>1.37 (0.12)</td>
<td>−θe18</td>
<td>0.81 (0.19)</td>
</tr>
<tr>
<td>United States</td>
<td>S19</td>
<td>3.98 (0.14)</td>
<td>−θe19</td>
<td>2.46 (0.25)</td>
</tr>
</tbody>
</table>

Total Sum of squares 2937
Sum of squared residuals 71
Number of observations 342

Notes: Estimated by generalized least squares using 1990 data. The specification is given in equation (39) of the paper. The parameter are normalized so that \(\sum_{i=1}^{N} \theta_i = 0\) and \(\sum_{i=1}^{N} \theta_i = 0\). Standard errors are in parentheses.
Estimating $\theta$, EK Method 2 (iii)

- Use estimated exporter fixed effects to back out $\theta$:
  \[
  S_i = \frac{1}{\beta} \ln T_i - \theta \ln w_i
  \]

- Measure relative wages from STAN, OECD
- Relate technologies to R&D and human capital (years of schooling)
  \[
  \Rightarrow \hat{S}_i = \alpha_0 + \alpha_R \ln RDStock_i - \alpha_H \frac{1}{H_i} - \theta \ln w_i + \tau_i
  \]
- Estimated by OLS and 2SLS (labor-market equilibrium implies $w_i$ is increasing in $T_i \rightarrow$ instruments using total workforce and population density)
## Estimating $\theta$, EK Method 2 (iv)

<table>
<thead>
<tr>
<th></th>
<th>Ordinary Least Squares</th>
<th>Two-Stage Least Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>est.</td>
<td>s.e.</td>
</tr>
<tr>
<td>Constant</td>
<td>3.75</td>
<td>(1.89)</td>
</tr>
<tr>
<td>Research stock, $\ln R_i$</td>
<td>$\alpha_R$</td>
<td>1.04</td>
</tr>
<tr>
<td>Human capital, $1/H_i$</td>
<td>$-\alpha_H$</td>
<td>-18.0</td>
</tr>
<tr>
<td>Wage, $\ln w_i$</td>
<td>$-\theta$</td>
<td>-2.84</td>
</tr>
<tr>
<td>Total Sum of squares</td>
<td>80.3</td>
<td></td>
</tr>
<tr>
<td>Sum of squared residuals</td>
<td>18.5</td>
<td></td>
</tr>
<tr>
<td>Number of observations</td>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Estimated using 1990 data. The dependent variable is the estimate $\hat{S}_i$ of source-country competitiveness shown in Table III. Standard errors are in parentheses.

Source: Eaton & Kortum, 2002

- Accounting for the endogeneity of $w_i$ increases the estimated $\theta$
Estimating $\theta$, EK Method 3

- Estimate:
  \[
  \ln \frac{X'_{ni}}{X'_{nn}} = \theta D_{ni} + S_i - S_n
  \]

- OLS estimate gives $\hat{\theta} = 2.44$

- 2SLS estimate using geographic trade barriers as instruments gives $\hat{\theta} = 12.86$
Estimating $\theta$, CDK Method (i)

- Estimate $\theta$ using observed measures for sectoral productivities.
- Use the additional sector dimension to simplify the trade equation:

$$
\ln \frac{X_{ni}^k X_{ni'}^k}{X_{ni'}^{k'} X_{ni}^{k'}} = \ln \frac{T_i^k T_i'^{k'}}{T_i^{k'} T_i^k} - \theta \ln \frac{d_{ni}^k d_{ni'}^k}{d_{ni'}^{k'} d_{ni}^k}
$$

(dif-in-dif strategy allows to control for wages, differences in income and differences in expenditure shares across sectors)
Estimating $\theta$, CDK Method (ii)

- Note that the fundamental productivity $z^k_i \equiv T^k_i 1/\theta = E[z^k_i(j)]$ is not observed. What is observed is $\tilde{z}^k_i = E[z^k_i(j)|\Omega^k_i]$ the conditional mean based on the set of varieties that are actually produced in country $i$.

- Since:

$$\frac{z^k_i}{z^k_i'} = \frac{\tilde{z}^k_i}{\tilde{z}^k_i'} \left( \frac{\pi^k_{ii}}{\pi^k_{i'i'}} \right)^{1/\theta}$$

we however have a link between trade flows and observed productivities:

$$\ln \frac{\tilde{X}^k_{ni} \tilde{X}^k_{ni'}}{\tilde{X}^k_{ni'} \tilde{X}^k_{ni}} = \theta \ln \frac{\tilde{z}^k_{ni} \tilde{z}^k_{ni'}}{\tilde{z}^k_{ni'} \tilde{z}^k_{ni}} - \theta \ln \frac{\tilde{d}^k_{ni} \tilde{d}^k_{ni'}}{\tilde{d}^k_{ni'} \tilde{d}^k_{ni}}$$

where $\tilde{X}^k_{ni} \equiv X^k_{ni}/\pi^k_{ii}$ measures trade, corrected for the openness of $i$ to control for trade-driven selection.
Estimating $\theta$, CDK Method (iii)

- Use sectoral data on bilateral trade and import penetration from STAN (21 countries, 13 industries)
- Proxy productivities by relative producer prices from GGDC productivity level database
- Alternative proxies: TFP, real gross output per worker
- Capture trade costs with an error term
- Estimate:
  \[
  \ln \tilde{X}_{ij}^k = \delta_{ij} + \delta_j^k + \theta \ln \tilde{z}_i^k + \varepsilon_{ij}^k
  \]
  (equivalent to the dif-in-dif equation)
Estimating $\theta$, CDK Method (iv)

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>$\ln \tilde{X}_{ij}^k$</th>
<th>$\ln \tilde{X}_{ij}^k$</th>
<th>$\ln X_{ij}^k$</th>
<th>$\ln \tilde{X}_{ij}^k$</th>
<th>$\ln \tilde{X}_{ij}^k$</th>
<th>$\ln \tilde{X}_{ij}^k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln z_i^k$</td>
<td>1.123</td>
<td>6.534</td>
<td>11.10</td>
<td>6.704</td>
<td>2.735</td>
<td>4.341</td>
</tr>
<tr>
<td></td>
<td>(.099)</td>
<td>(.708)</td>
<td>(.981)</td>
<td>(.874)</td>
<td>(.239)</td>
<td>(.521)</td>
</tr>
<tr>
<td>$\tilde{z}_i^k$</td>
<td>PP</td>
<td>PP</td>
<td>PP</td>
<td>TFP dual</td>
<td>Y/L</td>
<td>TFP primal</td>
</tr>
<tr>
<td>proxy</td>
<td>OLS</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
</tr>
<tr>
<td>Method</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$ij$ FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$ik$ FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td># obs</td>
<td>5,652</td>
<td>5,576</td>
<td>5,576</td>
<td>5,576</td>
<td>5,353</td>
<td>4,357</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.856</td>
<td>.747</td>
<td>.460</td>
<td>.587</td>
<td>.825</td>
<td>.821</td>
</tr>
</tbody>
</table>

Sd Err in parentheses with $^a$ denoting significance at the 1% level.

Instrument for observed productivities : R&D expenditures

- Strong endogeneity bias
- $\theta$ overestimated whenever trade-induced selection is not controlled for
- Robust to other controls for productivities
- Preferred specification implies $\theta = 6.5$
Estimating $\theta$, CP Method

- Caliendo & Parro (2014)
- Use a ‘dif-in-dif-in-dif’ method:

$$\ln \frac{X_{kn}^k X_{ih}^k X_{hn}^k}{X_{nh}^k X_{hi}^k X_{in}^k} = \frac{-1}{\theta^k} \ln \frac{d_{ni}^k d_{ih}^k d_{hn}^k}{d_{nh}^k d_{hi}^k d_{in}^k}$$

- Assume that trade costs follow:

$$\ln d_{ni}^k = \ln (1 + \tau_{ni}^k) + \ln e_{ni}^k$$

$$\ln e_{ni}^k = \nu_{ni}^k + \mu_n^k + \delta_i^k + \varepsilon_{ni}^k$$

where $\nu_{ni}^k = \nu_{in}^k$ captures symmetric bilateral costs (distance, language, etc), $\mu_n^k$ captures an importer sectoral fixed effect (NTB), $\delta_i^k$ is an exporter sectoral fixed effect (NTB) and $\varepsilon_{ni}^k$ is a random disturbance assumed orthogonal to tariffs.
Estimating $\theta$, CP Method (ii)

$\Rightarrow$ Estimated equation:

$$\ln \frac{X_{ni}^k X_{ih}^k X_{hn}^k}{X_{nh}^k X_{hi}^k X_{in}^k} = -\frac{1}{\theta^k} \ln \frac{\tilde{\tau}_{ni}^k \tilde{\tau}_{ih}^k \tilde{\tau}_{hn}^k}{\tilde{\tau}_{nh}^k \tilde{\tau}_{hi}^k \tilde{\tau}_{in}^k} + \tilde{\varepsilon}^k$$

where $\tilde{\tau} \equiv (1 + \tau)$ and $\tilde{\varepsilon}^k \equiv \varepsilon_{in}^k - \varepsilon_{ni}^k + \varepsilon_{hi}^k - \varepsilon_{ih}^k + \varepsilon_{nh}^k - \varepsilon_{hn}^k$

- Estimated sector-by-sector for 20 industries, the US as the importer and 1993 trade and tariff data
### Estimating $\theta$, CP Method (ii)

**Dispersion-of-productivity parameter (with importer and exporter fixed effects)**

<table>
<thead>
<tr>
<th>Sector Name</th>
<th>Full sample</th>
<th>99% sample</th>
<th>97.5% sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$1/\theta$</td>
<td>s.e.</td>
<td>N</td>
</tr>
<tr>
<td>Agriculture</td>
<td>8.59 (2.00)</td>
<td>496</td>
<td>9.54 (2.11)</td>
</tr>
<tr>
<td>Mining</td>
<td>14.83 (2.57)</td>
<td>296</td>
<td>11.96 (3.94)</td>
</tr>
<tr>
<td>Food</td>
<td>2.84 (0.57)</td>
<td>496</td>
<td>3.02 (0.57)</td>
</tr>
<tr>
<td>Textile</td>
<td>5.09 (1.24)</td>
<td>102</td>
<td>6.55 (1.38)</td>
</tr>
<tr>
<td>Wood</td>
<td>10.19 (2.24)</td>
<td>315</td>
<td>10.72 (2.63)</td>
</tr>
<tr>
<td>Paper</td>
<td>8.32 (1.66)</td>
<td>507</td>
<td>15.20 (2.69)</td>
</tr>
<tr>
<td>Petroleum</td>
<td>69.31 (19.32)</td>
<td>91</td>
<td>68.47 (19.08)</td>
</tr>
<tr>
<td>Chemicals</td>
<td>3.64 (1.75)</td>
<td>430</td>
<td>3.23 (1.76)</td>
</tr>
<tr>
<td>Plastic</td>
<td>0.68 (1.57)</td>
<td>376</td>
<td>0.50 (2.24)</td>
</tr>
<tr>
<td>Minerals</td>
<td>3.38 (1.54)</td>
<td>342</td>
<td>3.03 (1.73)</td>
</tr>
<tr>
<td>Basic metals</td>
<td>6.58 (2.28)</td>
<td>388</td>
<td>0.88 (2.58)</td>
</tr>
<tr>
<td>Metal products</td>
<td>5.03 (1.93)</td>
<td>494</td>
<td>7.30 (2.01)</td>
</tr>
<tr>
<td>Machinery n.e.c.</td>
<td>2.87 (1.85)</td>
<td>392</td>
<td>3.88 (3.14)</td>
</tr>
<tr>
<td>Office</td>
<td>13.88 (2.21)</td>
<td>306</td>
<td>9.85 (5.60)</td>
</tr>
<tr>
<td>Electrical</td>
<td>11.02 (1.46)</td>
<td>343</td>
<td>13.95 (1.66)</td>
</tr>
<tr>
<td>Comm</td>
<td>4.87 (1.76)</td>
<td>311</td>
<td>3.27 (2.07)</td>
</tr>
<tr>
<td>Medical</td>
<td>7.63 (1.23)</td>
<td>385</td>
<td>7.49 (1.46)</td>
</tr>
<tr>
<td>Auto</td>
<td>0.49 (0.91)</td>
<td>237</td>
<td>1.59 (1.04)</td>
</tr>
<tr>
<td>Other Transport</td>
<td>0.00 (1.16)</td>
<td>245</td>
<td>0.91 (1.15)</td>
</tr>
<tr>
<td>Other</td>
<td>4.95 (0.92)</td>
<td>412</td>
<td>3.52 (1.04)</td>
</tr>
</tbody>
</table>

Manufacturing (average) | 9.04 | 9.33 | 6.60

Note: The dependent variable is $\ln(X_{ji} X_{ih} X_{jn} / (X_{in} X_{hi} X_{nh}))$ where $X_{ji}$ are trade flows from $n$ to $i$. The independent variable is $\ln(\frac{\bar{\theta}_{ji}}{\bar{\theta}_{ih} \bar{\theta}_{jn} / (\bar{\theta}_{in} \bar{\theta}_{hi} \bar{\theta}_{nh})}$, where we also included importer and exporter fixed effects. $1/\theta$ is the negative of the estimated coefficient. We use only data from 1993 or before. Heteroskedasticity-robust standard errors are reported. The estimate for manufacturing is the mean of the sector estimations.

Source: Caliendo & Parro, 2014. Reported coefficients correspond to the $\theta$ parameters of the model

- Significant heterogeneity across sectors ⇒ Potentially bias aggregate regression (Imbs & Mejean, 2015)
### Summary on $\hat{\theta}$

<table>
<thead>
<tr>
<th>Method</th>
<th>$\hat{\theta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EK, Method of moments</td>
<td>8.28</td>
</tr>
<tr>
<td>EK, 2Stages GLS+OLS</td>
<td>2.84</td>
</tr>
<tr>
<td>EK, 2Stages GLS+2SLS</td>
<td>3.60</td>
</tr>
<tr>
<td>EK, OLS Trade Eq.</td>
<td>2.44</td>
</tr>
<tr>
<td>EK, 2SLS Trade Eq.</td>
<td>12.86</td>
</tr>
<tr>
<td>SW, SMM</td>
<td>4.12</td>
</tr>
<tr>
<td>CDK, IV</td>
<td>6.53</td>
</tr>
<tr>
<td>CP, (mean)</td>
<td>8.22</td>
</tr>
</tbody>
</table>

Counterfactuals

Once estimated, the model can be used to run counterfactuals (See Lecture 11):

- What are the welfare gains from trade? (Arkolakis et al, 2012)
- What is the impact of multilateral/unilateral tariff eliminations? (Caliendo & Parro, 2015)
- How much does trade spread the benefit of local improvements in technology?
- How does specialization affect the volatility of GDPs? (Caselli et al, 2015)
Quantitative analysis

- Quantitative analyses require to enrich the structure

  i. Multiple sectors:

  \[ U_i = \prod_{k=1}^{K} Q_i^k \alpha_i^k, \quad \sum_{k=1}^{K} \alpha_i^k = 1, \quad Q_i^k = \left[ \int_0^1 Q_i^k(j) \frac{\sigma^{k-1}}{\sigma^k} \right] \frac{\sigma^k}{\sigma^k-1} \]

  and \( \{\alpha_i^k\} \) fitted to data on sectoral absorption

  ii. Input-Output linkages:

  \[ c_i^k = w_i^k \prod_{k'=1}^{K} P_i^{k'} \gamma_i^{k,k'}, \quad \sum_{k=1}^{K} \gamma_i^{k,k'} = 1 - \gamma_i^k \]

  and \( \{\gamma_i^k\} \) and \( \{\gamma_i^{k,k'}\} \) fitted to IO tables

  iii. Non tradable sectors:

  \[ d_{ni}^k = +\infty \text{ for some } k \]
Concluding remarks

- A very elegant way of introducing Ricardo into a multi-country (eventually multi-sector) model
- Analytics strongly rely on some assumptions: Fréchet distribution, Variance of productivities homogenous across industries
- Estimation results quite sensitive to the estimation strategy → Important consequences in terms of the welfare predictions of the model
References


- Caselli F., Koren M., Lisicky M. & Tenreyro S., 2015, “Diversification through Trade”

References


- Imbs J. & Mejean I., 2015, “Elasticity Optimism”, *AEJ : Macro*, 7(3) : 43-83


Demand functions

- Consumers solves:

\[
\begin{align*}
\max_{\{Q_i(j)\}_{j \in [0,1]}} & \left[ \int_0^1 Q_i(j) \frac{\sigma-1}{\sigma} dj \right]^{\frac{\sigma}{\sigma-1}} \\
\text{s.t.} & \int_0^1 P_i(j) Q_i(j) dj \leq R_i
\end{align*}
\]

- Solution of the maximization program is:

\[
Q_i(j) = \left( \frac{P_i(j)}{P_i} \right)^{-\sigma} \frac{R_i}{P_i}
\]

with \(P_i\) the ideal price index \((R_i / P_i = U_i, \ \forall R_i)\):

\[
P_i = \left[ \int_0^1 P_i(j)^{1-\sigma} dj \right]^\frac{1}{1-\sigma}
\]
Firms’ profit:

\[ \pi_i(j) = \sum_n \left[ p_{ni}(j) Q_{ni}(j) - \frac{c_i}{z_i(j)} d_{ni} Q_{ni}(j) \right] = \sum_n \pi_{ni}(j) \]

Under perfect competition:

\[ p_{ni}(j) = \frac{c_i}{z_i(j)} d_{ni} \]

and

\[ Q_{in}(j) = 0 \text{ if } p_{in}(j) > p_n(j)/Q_n(j) \text{ otherwise} \]
Price distribution

- \( p_{ni}(j) = \frac{c_i}{z_i(j)} d_{ni} \) is the realization of a random variable \( P_{ni} \) which cdf is:

\[
G_{ni}(p) = Pr[P_{ni} \leq p] = Pr \left[ Z_i \geq \frac{c_i d_{ni}}{p} \right] = 1 - F_i \left( \frac{c_i d_{ni}}{p} \right) = 1 - e^{-T_i \left( \frac{c_i d_{ni}}{p} \right)^{-\theta}}
\]

- \( p_n(j) = \min\{ p_{ni}(j); i = 1...I \} \) is the realization of a random variable \( P_n = \min\{ P_{ni}; i = 1...I \} \) which cdf is:

\[
G_n(p) = Pr[P_n \leq p] = 1 - \prod_{i=1}^{I} Pr[P_{ni} > p] = 1 - \prod_{i=1}^{I} \left[ 1 - G_{ni}(p) \right] = 1 - e^{-p^\theta \sum_{i=1}^{I} T_i (c_i d_{ni})^{-\theta}}
\]
<table>
<thead>
<tr>
<th>The Eaton-Kortum model</th>
<th>Empirical Evidence</th>
<th>Quantitative applications</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>PricelIndex</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Price index

- Cdf / pdf of consumption prices:
  \[ F_n(p) = 1 - e^{-\Phi_n p^\theta} \quad \text{and} \quad f_n(p) = \Phi_n \theta p^{\theta-1} e^{-\Phi_n p^\theta} \]

- Define: \( y = g(p) = p^\theta \), then
  \[ G_n(y) = F_n(g^{-1}(y)) \quad \text{and} \quad g_n(y) = f(g^{-1}(y)) \left| \frac{\partial g^{-1}(y)}{\partial y} \right| = \Phi_n e^{-\Phi_n y} \]

- Thus the price index:
  \[
P_n = \left[ \int_0^1 p_n(j)^{1-\sigma} \, dj \right]^{\frac{1}{1-\sigma}}
  = \left[ \int_0^1 y^{\frac{1-\sigma}{\theta}} \Phi_n e^{-\Phi_n y} \, dy \right]^{\frac{1}{1-\sigma}}
  = \Phi_n^{-1/\theta} \left[ \int_0^1 u^{\frac{1-\sigma}{\theta}} e^{-u} \, du \right]^{\frac{1}{1-\sigma}} \quad \text{where} \quad u = \Phi_n y
  = \Phi_n^{-1/\theta} \left[ \Gamma \left( \frac{1-\sigma}{\theta} \right) - 1 \right]^{\frac{1}{1-\sigma}}
\]
Fréchet distribution

- Generalized extreme value distribution: A family of continuous probability distributions usually used as an approximation to model the maxima of long (finite) sequences of random variables.

- CDF:

\[
F(x; \mu, \sigma, \xi) = \exp \left\{ - \left[ 1 + \xi \left( \frac{x - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}
\]

- \( \mu \) a location parameter, \( \sigma > 0 \) the scale parameter, \( \xi \) the shape parameter.
In particular:

- Gumbel or type I extreme value: $\xi = 0$
  \[ F(x; \mu, \sigma, 0) = \exp \left\{ -\exp \left[ -\frac{x - \mu}{\sigma} \right] \right\}, \quad x \in \mathbb{R} \]

- Frechet of type II extreme value: $\xi = \alpha^{-1} > 0$
  \[ F(x; \mu, \sigma, \xi) = \begin{cases} 0, & x \leq \mu \\ \exp \left\{ - \left[ \frac{x-\mu}{\sigma} \right]^{-\alpha} \right\}, & x > \mu \end{cases} \]

- Reversed Weibull or type III extreme value: $\xi = -\alpha^{-1} < 0$
  \[ F(x; \mu, \sigma, \xi) = \begin{cases} \exp \left\{ - \left[ -\frac{x-\mu}{\sigma} \right]^\alpha \right\}, & x < \mu \\ 1, & x \geq \mu \end{cases} \]