Master EPP, International Macroeconomics
Lecture 9-10
Financial globalisation and international risk sharing

1 Uncertainty in the small open endowment economy

1. Write the intertemporal budget constraint.

The budget constraints for each period and each state of the nature are given by:

$$\begin{aligned} Y_1 &= C_1 + \frac{p(1)}{1+r}B_2(1) + \frac{p(2)}{1+r}B_2(2) \\ Y_2(s) + B_2(s) &= C_2(s) \end{aligned}$$

The intertemporal budget constraint thus writes:

$$C_1 + \frac{p(1)C_2(1) + p(2)C_2(2)}{1+r} = Y_1 + \frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r}$$

It says that the date 1 present value of the country's uncertain consumption stream must equal the date 1 present value of its uncertain income, where contingent quantities are evaluated at world Arrow-Debreu prices.

International markets allow countries to smooth consumption not only across time but also across states of nature. Suppose for instance that $Y_2(1)$ is low whereas $Y_2(2)$ is high. The country can smooth consumption across states by going short in state 2 securities ($B_2(2) < 0$) and long in state 1 securities ($B_2(1) > 0$).

2. Determine the country's optimal saving and portfolio allocations.

The representative household maximizes expected utility subject to her budget constraint:

$$\begin{cases} \max_{C_1,C_2(1),C_2(2)} \left[u(C_1) + \beta\pi(1)u(C_2(1)) + \beta\pi(2)u(C_2(2))\right] \\ s.t. \quad C_1 + \frac{p(1)C_2(1)+p(2)C_2(2)}{1+r} = Y_1 + \frac{p(1)Y_2(1)+p(2)Y_2(2)}{1+r} \end{cases}$$

One can also write this program in terms of optimal saving behaviors:

$$\max_{B_2(1),B_2(2)} u\left(Y_1 - \frac{p(1)}{1+r}B_2(1) - \frac{p(2)}{1+r}B_2(2)\right) + \beta\pi(1)u\left(Y_2(1) + B_2(1)\right) + \beta\pi(2)u\left(Y_2(2) + B_2(2)\right)$$

The first-order conditions associated with each state of the nature write:

$$\frac{p(s)}{1+r}u'(C_1) = \pi(s)\beta u'(C_2(s)), \quad s = 1, 2$$

They equalize the cost, in terms of date 1 marginal utility, of acquiring the Arrow-Debreu security for state 1 and the expected discounted benefit from having an additional unit of consumption in state s on date 2. Said otherwise, households equalize the marginal rate of substitution between C_1 and $C_2(s)$ to the goods' relative price:

$$\frac{\pi(s)\beta u'(C_2(s))}{u'(C_1)} = \frac{p(s)}{1+r}, \quad s = 1, 2$$

3. Show that the non-arbitrage condition in the Arrow-Debreu and the riskless markets implies $p(1) + p(2) = 1$. Write the stochastic Euler equation.

One can mimic the output of a riskless bond by buying quantities of each Arrow-Debreu security. In particular, buying $(1 + r)$ state 1 Arrow-Debreu security and $1 + r$ state 2 Arrow-Debreu security insures an outcome of $1 + r$ in period 2, as does a riskless bond. As a consequence, the price of these Arrow-Debreu securities must equal the price of the bond:

$$(1+r)\frac{p(1)}{1+r} + (1+r)\frac{p(2)}{1+r} = 1$$
$$\Rightarrow \quad p(1) + p(2) = 1$$

Using this and combining the optimal conditions over states of the nature gives the stochastic Euler equation:

$$\frac{u'(C_1)}{1+r} = \beta\pi(1)u'(C_2(1)) + \beta\pi(2)u'(C_2(2)) = \beta E_1\{u'(C_2)\}$$

where E_1 is the expectation operator conditioned on information known on date 1. The stochastic Euler equation equalizes the exppected marginal rate of substitution of present for future consumption to the price of certain future consumption in terms of present consumption.

4. What is the impact of a shock on the relative price of state 1 Arrow-Debreu security on the relative consumption in state 1? Interpretation

The first-order conditions also give insight about the marginal rate of substitution over states of the nature:

$$\frac{\pi(1)u'[C_2(1)]}{\pi(2)u'[C_2(2)]} = \frac{p(1)}{p(2)}$$

Consumption is equalized across different states of the nature only to the extent that Arrow-Debreu securities are "actuarially fair", ie when:

$$\frac{p(1)}{p(2)} = \frac{\pi(1)}{\pi(2)}$$

Under this condition, the country fully insure against all future consumption fluctuations.
Totally differentiating the log version of the previous condition gives:

$$d\log\left(\frac{p(1)}{p(2)}\right) = \frac{C_2(1)u''[C_2(1)]}{u'[C_2(1)]}d\log C_2(1) + \frac{C_2(2)u''[C_2(2)]}{u'[C_2(2)]}d\log C_2(2)$$

Calling $\rho(C) = \frac{-Cu''(C)}{u'(C)}$ the coefficient of relative risk aversion and assuming it to be constant, this implies:

$$d\log\left(\frac{C_2(2)}{C_2(1)}\right) = \frac{1}{\rho}d\log\left(\frac{p(1)}{p(2)}\right)$$

The inverse of the coefficient of relative risk aversion is also the elasticity of substitution between state-contingent consumption levels with respect to relative Arrow-Debreu prices. High risk aversion produces an inelastic response of consumption-insurance demands to relative insurance prices.
Remark: A consumer is said to be risk neutral when $u''(c) = 0$.

5. Derive date 1 current account balance in the special case $u(C) = log(C)$.

Under logarithmic utility, agents optimally choose to spend a constant share $1/(1+\beta)$ of their lifetime resources on date 1 consumption. One can easily show this by solving:

$$\begin{cases} \max_{C_1, C_2(1), C_2(2)} \left[log(C_1) + \beta\pi(1)log(C_2(1)) + \beta\pi(2)log(C_2(2))\right] \\ s.t. \quad C_1 + \frac{p(1)C_2(1)+p(2)C_2(2)}{1+r} = Y_1 + \frac{p(1)Y_2(1)+p(2)Y_2(2)}{1+r} \end{cases}$$

Optimal consumptions are then given by:

$$\begin{aligned} C_1 &= \frac{1}{1+\beta}\left[Y_1 + \frac{p(1)Y_2(1)+p(2)Y_2(2)}{1+r}\right] \\ \frac{p(1)}{1+r}C_2(s) &= \frac{\pi(s)\beta}{1+\beta}\left[Y_1 + \frac{p(1)Y_2(1)+p(2)Y_2(2)}{1+r}\right], \quad s=1,2 \end{aligned}$$

Using the solution for C_1, one can easily get the date 1 current account balance as:

$$CA_1 = Y_1 - C_1 = \frac{\beta}{1+\beta}Y_1 - \frac{1}{1+\beta}\left[\frac{p(1)}{1+r}Y_2(1) + \frac{p(2)}{1+r}Y_2(2)\right]$$

The interpretation is easy to grasp when $\beta(1+r)=1$. In this case, the current account balance is zero if the value of output is the same on both dates ($Y_1 = p(1)Y_2(1)+p(2)Y_2(2)$). If on the other hand output in period 1 is lower than the value of date 2 output, then the current account balance is negative. The reason for that is that people want to smooth their consumption. Indeed, the Euler equation writes:

$$\begin{aligned} \frac{p(s)}{(1+r)C_1} &= \frac{\pi(s)\beta}{C_2(s)} \\ \Leftrightarrow\quad & p(s)C_2(s) = \beta(1+r)\pi(s)C_1 \\ \Rightarrow\quad & p(1)C_2(1)+p(2)C_2(2) = \beta(1+r)C_1 = C_1 \end{aligned}$$

If their first-period income is below (above) the constant expenditure level consistent with the intertemporal budget constraint, they will shift purchasing power to the present (future) through a current account deficit (surplus).

2 A global Model with CRRA utility

1. Derive the Euler equation for the Home and Foreign countries.

The domestic household solves the following program:

$$\begin{cases} \max\left[u(C_1) + \beta\sum_{s=1}^{\mathcal{S}}\pi(s)u(C_2(s))\right] \\ s.t.\quad Y_1 = C_1 + \sum_s \frac{p(s)}{1+r}B_2(s) \\ \qquad\; Y_2(s) + B_2(s) = C_2(s) \end{cases}$$

The first order condition writes:

$$\begin{aligned} \frac{p(s)}{1+r}u'(C_1) &= \beta\pi(s)u'(C_2(s)) \\ \Leftrightarrow\quad C_2(s) &= \left(\frac{\pi(s)\beta(1+r)}{p(s)}\right)^{1/\rho} C_1 \end{aligned}$$

Symmetrically in the Foreign country:

$$C_2^*(s) = \left(\frac{\pi(s)\beta(1+r)}{p(s)}\right)^{1/\rho} C_1^*$$

2. Combine this with the world market equilibrium conditions to get the date 1 price of the state s contingent security as a function of world output in state s relative to date 1 world output ($Y_2^W(s)/Y_1^W$). Under which condition are securities prices actuarially fair?

World market equilibrium conditions are given by:

$$C_1 + C_1^* = Y_1 + Y_1^* \equiv Y_1^W$$
$$C_2(s) + C_2^*(s) = Y_2(s) + Y_2^*(s) \equiv Y_2^W(s)$$

Adding the Euler equations for the Home and Foreign households thus give:

$$C_2(s) + C_2^*(s) = \left(\frac{\beta\pi(s)(1+r)}{p(s)}\right)^{1/\rho} (C_1 + C_1^*)$$
$$\Leftrightarrow \quad \frac{p(s)}{1+r} = \beta\pi(s)\left(\frac{Y_2^W(s)}{Y_1^W}\right)^{-\rho}$$

or:

$$\frac{p(s)}{p(s')} = \frac{\pi(s)}{\pi(s')}\left(\frac{Y_2^W(s)}{Y_2^W(s')}\right)^{-\rho}$$

Thus, securities prices are actuarially fair if and only if total world output is the same in all states of the nature, ie if there is no output uncertainty at the aggregate level. If there is no aggregate uncertainty, it is feasible for both countries to have state-independent date 2 consumption levels. As a result, equilibrium prices need not provide an incentive for people to tilt consumption in favor of states with relatively abundant world output. If, however, world output in state s' exceeds that in state s, prices must induce countries to consume relatively more in state s' ($p(s') < p(s)$).

3. Solve for date 2 prices $p(s)$ and the world interest rate.

For any state s', the arbitrage condition $\sum_s p(s) = 1$ together with the previous relation imply:

$$\begin{aligned} p(s') &= 1 - \sum_{s \neq s'} p(s) \\ &= 1 - \frac{p(s')}{\pi(s')\, Y_2^W(s')^{-\rho}} \sum_{s \neq s'} \pi(s)\, Y_2^W(s)^{-\rho} \\ &= \frac{\pi(s')\, Y_2^W(s')^{-\rho}}{\sum_{s=1}^{\mathcal{S}} \pi(s)\, Y_2^W(s)^{-\rho}} \end{aligned}$$

The world interest rate is finally given by:

$$1 + r = \frac{p(s)}{\beta\pi(s)\left(\frac{Y_2^W(s)}{Y_1^W}\right)^{-\rho}} = \frac{Y_1^{W\,-\rho}}{\beta \sum_{s=1}^{\mathcal{S}} \pi(s)\, Y_2^W(s)^{-\rho}}$$

Higher world output on date 1 implies a lower interest rate (ie raises the price of date 2 consumption relative to date 1 consumption). Higher future output in any state lowers the real interest rate.

4. Discuss the predictions of the model with regards to correlations in international consumption levels across time and across states of nature.

Under complete markets, all individuals in Home and Foreign equate their marginal rates of substitution between current consumption and state-contingent future consumption to the same state-contingent security prices. Under CRRA, this implies:

$$\frac{C_2(s)}{C_2(s')} = \frac{C_2^*(s)}{C_2^*(s')} = \frac{Y_2^W(s)}{Y_2^W(s')}$$
$$\frac{C_2(s)}{C_1} = \frac{C_2^*(s)}{C_1^*} = \frac{Y_2^W(s)}{Y_1^W}$$

Table 1: Consumption and output: Correlations between Domestic and World Growth Rates, 1973-92

Country	$Corr(\hat{c}, \hat{c}^W)$	$Corr(\hat{y}, \hat{y}^W)$
Canada	.56	.70
France	.45	.60
Germany	.63	.70
Italy	.27	.51
Japan	.38	.46
United Kingdom	.63	.62
United States	.52	.68
OECD average	.43	.52
Developing country average	-.10	.05

$Corr(\hat{c}, \hat{c}^W)$ and $Corr(\hat{y}, \hat{y}^W)$ are the simple correlation coefficients between the annual change in the natural logarithm of a country's real per capita consumption (or output) and the annual change in the natural logarithm of the rest of the world's real per capita consumption (or output). Source: Obstfeld & Rogoff (1996)

for all states. The first of these equations also implies the equalities:

$$\frac{C_2(s)}{Y_2^W(s)} = \frac{C_2(s')}{Y_2^W(s')}, \quad \frac{C_2^*(s)}{Y_2^W(s)} = \frac{C_2^*(s')}{Y_2^W(s')}$$

Home consumption is thus a constant share of world date 2 output, regardless of the state.
The second equation means that consumption growth rates are the same across countries in every state and are equal to the growth rate of world output.

5. Using results provided in Table 1, discuss the empirical validity of these predictions.

The model predicts that different countries' per capita consumption growth rates should be highly correlated even if growth rates in per capita output are not.[1] Results in Table 1 however suggest that this prediction does not match the data. For the seven largest industrial countries, the correlation between domestic and world consumption growth is lower in almost every case than the correlation between domestic and world output growth. When smaller OECD countries are included, the puzzle remains.

3 International Portfolio Diversification

1. Calling B_2^n country n's net bond purchases in date 1 and x_m^n its net purchases of fractional shares in country m's future output, write and solve the program of the representative household in country n.

In country n, the representative household solves:

$$\begin{cases} \max_{C_1^n, C_2^n(s), B_2^n, x_m^n} \left[u(C_1) + \beta \sum_{s=1}^{S} \pi(s) u(C_2(s)) \right] \\ s.t. \quad Y_1^n + V_1^n = C_1^n + B_2^n + \sum_{m=1}^{N} x_m^n V_1^m \\ \qquad C_2^n(s) = (1+r) B_2^n + \sum_{m=1}^{N} x_m^n Y_2^m(s) \end{cases}$$

[1] Note that this prediction holds even if countries have different constant coefficients of risk aversion and different subjective discount factors.

The first-order conditions associated with this problem are:

$$u'(C_1^n) = \beta(1+r)\sum_{s=1}^{\mathcal{S}} \pi(s)u'(C_2^n(s)) = \beta(1+r)E_1\{u'(C_2^n(s))\}$$

$$V_1^m u'(C_1^n) = \beta\sum_{s=1}^{\mathcal{S}} \pi(s)u'(C_2^n(s))Y_2^m(s) = \beta E_1\{Y_2^m(s)u'(C_2^n(s))\}$$

The first condition corresponds to the standard stochastic Euler equation. The second equation equalizes the marginal utility cost to a country n resident who purchases country m's risky future output on date 1 and the expected marginal utility gain.

2. The model is solved taking an educated guess at the equilibrium allocation (namely, that it is Pareto efficient) and finding equilibrium portfolios and prices that support this conjecture. Under the conjecture of Pareto efficiency, equilibrium allocations take the same general form as in the complete-markets case, namely:

$$\begin{aligned}
& C_1^N = \mu^n Y_1^W \\
& C_2^n(s) = \mu^n Y_2^W(s), \quad s = 1, ..., \mathcal{S} \\
\textit{where} \quad & \mu^n = \frac{Y_1^n + V_1^n}{\sum_{m=1}^{N}(Y_1^m + V_1^m)} \\
\textit{and} \quad & Y_1^W = \sum_{m=1}^{N} Y_1^m
\end{aligned}$$

Find the prices and optimal shares that satisfy this allocation.

For any country n, date 2 budget constraint is consistent with the previous allocation of second-period's outputs if:

$$B_2^n = 0, \quad x_m^n = \mu^n, \forall m$$

These consumption and portfolio plans are globally feasible if chosen simultaneously by every country n. To establish that they characterize the economy's equilibrium, we must find prices at which they maximize any country n's intertemporal utility subject to satisfying its date 1 budget constraint.

It is simple to check that the consumption plans posited previously satisfy the Euler equation for all countries n:

$$C_1^{n\ -\rho} = \beta(1+r)\sum_s \pi(s)\ C_2^n(s)^{-\rho}$$

if the equilibrium real interest rate is:

$$1 + r = \frac{Y_1^{W\ -\rho}}{\beta\sum_s \pi(s)\ Y_2^W(s)^{-\rho}}$$

Similarly, the second first-order condition for utility maximization:

$$V_1^m\ C_1^{n\ -\rho} = \beta\sum_s \pi(s)Y_2^m(s)C_2^n(s)^{-\rho}$$

is satisfied if:

$$V_1^m = \beta\sum_{s=1}^{\mathcal{S}} \pi(s)\left(\frac{Y_2^W(s)}{Y_1^W}\right)^{-\rho} Y_2^m(s) = \beta E_1\left\{\left(\frac{Y_2^W}{Y_1^W}\right)^{-\rho} Y_2^m\right\}, \quad \forall m = 1, ..., N$$

Finally, one can check that date 1's budget constraint is also satisfied under these conditions. Date 1's resources $(Y_1^n + V_1^n \equiv \mu^n(Y_1^W + V_1^W))$ are entirely consumed under the assumptions that $B_2^n = 0$, $C_1^n = \mu^n Y_1^W$ and $x_m^n = \mu^n$.

The Pareto efficient allocation obtained in the complete-markets case is thus a possible equilibrium of the uncomplete-markets economy described in this exercise. Thus, even when the assets traded are limited to riskless bonds and shares in each of the N countries' future outputs, the equilibrium allocation is efficient. To see why, note that V_1^n, country n's stock-market value, is also the value of the country's uncertain future output evaluated at the date 1 Arrow-Debreu prices that would prevail were a complete set of contingent claims markets in operation. The economy behaves as if there were complete markets because all agents choose to hold equities in the same proportions, regardless of their wealth levels. The resulting equilibrium makes everyone's date 2 consumption moves in proportion to the return on a single global mutual fund of risky assets. The common diversification strategy eliminates the gains from any further mutual consumption insurance between countries.

3. Discuss the implications of the model in terms of portfolio diversification. Compare them with empirical evidences of Table 2

In the model, investors throughout the world all hold the same globally diversified portfolio of stocks. In practice however, residents of most countries seem to hold a very large share of their equity wealth at home. The contradiction is known as the *home bias puzzle.* Figures reported in Table 2 show that, whatever the considered country, the share of its portfolio invested in domestic mutual funds is by much larger than the share of the country in the world capitalization.

Several explanations have been presented to rationalize the home bias. One obvious explanation is relatively high costs connected with foreign stock acquisition or ownership, including trading costs, information costs, etc.

Table 2: Home bias in mutual fund holdings

	Market Capital Weight	Share in the domestic market
United States	46.85	85.66
United Kingdom	8.13	43.06
Canada	2.44	26.99
Germany	3.99	33.49
Italy	2.22	35.37
Sweden	1.03	46.74
France	4.32	55.27
Switzerland	2.21	21.03
Austria	0.09	6.77
Belgium	0.55	24.73
Denmark	0.31	18.41
Ireland	0.19	6.14
Finland	0.95	45.7
Greece	0.46	93.46
Luxembourg	0.1	15.08
Norway	0.19	48.81
Portugal	0.19	45.61
Spain	1.39	35.96
the Netherlands	1.97	19.49
Japan	11.29	71.82
Australia	1.18	60.5
Singapore	0.51	18.25
Hong Kong	1.82	26.44
New Zealand	0.07	74.93
Taiwan	0.91	60.88
South Africa	0.69	66.58

Source: Chan, Covrig and Ng (*Journal of Finance, 2006*). The first column is the country's average stock market capitalization weight in the world market portfolio. The second column is the share of domestic shares in the country's portfolio. All figures are in percent.