Lecture 9-10: Financial globalisation and international risk sharing

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Uncertainty in the small open endowment economy

Hypotheses

- A small open endowment economy, producing and consuming a single tradable good.
- Two periods (1 and 2)
- The size of the population is normalized to unity.
- In period 1, known income Y_1 and zero net foreign assets.
- On date 2, two "states of nature" ($s = 1, 2$) occur randomly according to a specified probability distribution ($\pi(s)$). Associated output levels $Y_2(s)$.
- The representative agent has a discount factor β. Instantaneous utility function $u(C)$ independent from the state of the nature.
- Financial markets:
  - Agents can borrow and lend in riskless bonds that pay $1 + r$ per unit on date 2, regardless of the state of nature.
  - There is a complete market of Arrow-Debreu securities allowing households to insure against risk in every state of the nature: $B_2(s)$ = net purchase of state s Arrow-Debreu securities on date 1. $p(s)/(1 + r)$ = world price, quoted in terms of date 1 consumption (exogenous from the standpoint of the small country).

Budget constraints

- The budget constraints for each period and each state of the nature are given by:

$$Y_1 = C_1 + \frac{p(1)}{1+r} B_2(1) + \frac{p(2)}{1+r} B_2(2)$$
$$Y_2(s) + B_2(s) = C_2(s)$$

- Intertemporal budget constraint:

$$C_1 + \frac{p(1) C_2(1) + p(2) C_2(2)}{1+r} = Y_1 + \frac{p(1) Y_2(1) + p(2) Y_2(2)}{1+r}$$

- Date 1 present value of the country's uncertain consumption stream must equal date 1 present value of its uncertain income, where contingent quantities are evaluated at world Arrow-Debreu prices.

$\Rightarrow$ International markets allow countries to smooth consumption not only across time but also across states of nature.

Consumption choices

- The representative household maximizes expected utility subject to her budget constraint:

$$\begin{cases} \max_{C_1, C_2(1), C_2(2)} \left[u(C_1) + \beta\pi(1)u(C_2(1)) + \beta\pi(2)u(C_2(2))\right] \\ s.t. \quad C_1 + \frac{p(1)C_2(1)+p(2)C_2(2)}{1+r} = Y_1 + \frac{p(1)Y_2(1)+p(2)Y_2(2)}{1+r} \end{cases}$$

- First-order conditions:

$$\frac{p(s)}{1+r}u'(C_1) = \pi(s)\beta u'(C_2(s)), \quad s = 1,2$$

$\Rightarrow$ Equalize the cost, in terms of date 1 marginal utility, of acquiring the Arrow-Debreu security for state 1 and the expected discounted benefit from having an additional unit of consumption in state s on date 2:

$$\frac{\pi(s)\beta u'(C_2(s))}{u'(C_1)} = \frac{p(s)}{1+r}, \quad s = 1,2$$

Non-arbitrage condition

- One can mimic the output of a riskless bond by buying quantities of each Arrow-Debreu security: Buying $(1+r)$ state 1 Arrow-Debreu security and $1+r$ state 2 Arrow-Debreu security insures an outcome of $1+r$ in period 2, as does a riskless bond $\Rightarrow$ Non-arbitrage condition:

$$\begin{aligned} & (1+r)\frac{p(1)}{1+r} + (1+r)\frac{p(2)}{1+r} = 1 \\ \Rightarrow \quad & p(1) + p(2) = 1 \end{aligned}$$

$\Rightarrow$ Stochastic Euler equation:

$$\frac{u'(C_1)}{1+r} = \beta\pi(1)u'(C_2(1)) + \beta\pi(2)u'(C_2(2)) = \beta E_1\{u'(C_2)\}$$

where E_1 is the expectation operator conditioned on information known on date 1.

Impact of a shock on $p(s)/(1+r)$

- Marginal rate of substitution over states of the nature:

$$\frac{\pi(1)u'[C_2(1)]}{\pi(2)u'[C_2(2)]} = \frac{p(1)}{p(2)}$$

(Consumption is equalized across different states of the nature only to the extent that Arrow-Debreu securities are "actuarially fair", ie when $\frac{p(1)}{p(2)} = \frac{\pi(1)}{\pi(2)}$)

- Impact of a shock on relative prices:

$$d\log\left(\frac{C_2(2)}{C_2(1)}\right) = \frac{1}{\rho} d\log\left(\frac{p(1)}{p(2)}\right)$$

with $\rho(C) = \frac{-Cu''(C)}{u'(C)}$ the coefficient of relative risk aversion, assumed to be constant

⇒ The inverse of the coefficient of relative risk aversion is also the elasticity of substitution between state-contingent consumption levels with respect to relative Arrow-Debreu prices.

⇒ High risk aversion produces an inelastic response of consumption-insurance demands to relative insurance prices.

Current account

- In the special case $u(C) = log(C)$, agents optimally choose to spend a constant share $1/(1+\beta)$ of their lifetime resources on date 1 consumption:

$$C_1 = \frac{1}{1+\beta}\left[Y_1 + \frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r}\right]$$

$$\frac{p(1)}{1+r}C_2(s) = \frac{\pi(s)\beta}{1+\beta}\left[Y_1 + \frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r}\right], \quad s = 1, 2$$

$\Rightarrow$ Date 1 current account balance:

$$CA_1 = Y_1 - C_1 = \frac{\beta}{1+\beta}Y_1 - \frac{1}{1+\beta}\left[\frac{p(1)}{1+r}Y_2(1) + \frac{p(2)}{1+r}Y_2(2)\right]$$

CA adjustments allowing to smooth consumption.

A global Model with CRRA utility

Hypotheses

- 2-country economy, with $\mathcal{S}$ states of nature.
- Home and Foreign consumers share the same utility function:

$$U_1 = u(C_1) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2(s))$$

where $u(C)$ is CRRA:

$$u(C) = \begin{cases} \frac{C^{1-\rho}}{1-\rho} & (\rho > 0, \rho \neq 1) \\ \log C & (\rho = 1) \end{cases}$$

Consumption choices

- The domestic household solves the following program:

$$\begin{cases} \max \left[u(C_1) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2(s)) \right] \\ s.t. \quad Y_1 = C_1 + \sum_s \frac{p(s)}{1+r} B_2(s) \\ \qquad Y_2(s) + B_2(s) = C_2(s) \end{cases}$$

$\Rightarrow$ First order condition:

$$\frac{p(s)}{1+r} u'(C_1) = \beta \pi(s) u'(C_2(s))$$

$$\Leftrightarrow \quad C_2(s) = \left(\frac{\pi(s)\beta(1+r)}{p(s)} \right)^{1/\rho} C_1$$

- Symmetric result for the Foreign country

World market equilibria

- World market equilibrium conditions:

$$C_1 + C_1^* = Y_1 + Y_1^* \equiv Y_1^W$$
$$C_2(s) + C_2^*(s) = Y_2(s) + Y_2^*(s) \equiv Y_2^W(s)$$

- With the Euler equations for the Home and Foreign households this gives security prices:

$$\frac{p(s)}{p(s')} = \frac{\pi(s)}{\pi(s')} \left(\frac{Y_2^W(s)}{Y_2^W(s')} \right)^{-\rho}$$

⇒ Securities prices are actuarially fair if and only if total world output is the same in all states of the nature, ie if there is no output uncertainty at the aggregate level.

⇒ If, however, world output in state s' exceeds that in state s, prices must induce countries to consume relatively more in state s' ($p(s') < p(s)$).

Equilibrium prices

- Security prices:

$$
\begin{aligned}
p(s') &= 1 - \sum_{s \neq s'} p(s) \\
&= 1 - \frac{p(s')}{\pi(s')\ Y_2^W(s')^{-\rho}} \sum_{s \neq s'} \pi(s)\ Y_2^W(s)^{-\rho} \\
&= \frac{\pi(s')\ Y_2^W(s')^{-\rho}}{\sum_{s=1}^{\mathcal{S}} \pi(s)\ Y_2^W(s)^{-\rho}}
\end{aligned}
$$

- World real interest rate:

$$
1 + r = \frac{p(s)}{\beta\pi(s)\left(\frac{Y_2^W(s)}{Y_1^W}\right)^{-\rho}} = \frac{{Y_1^W}^{-\rho}}{\beta \sum_{s=1}^{\mathcal{S}} \pi(s)\ Y_2^W(s)^{-\rho}}
$$

$\Rightarrow$ Higher world output on date 1 implies a lower interest rate (ie raises the price of date2 consumption relative to date 1 consumption). Higher future output in any state lowers the real interest rate.

International risk sharing

- Under complete markets, all individuals in Home and Foreign equate their marginal rates of substitution between current consumption and state-contingent future consumption to the same state-contingent security prices ⇒ Under CRRA:

$$\frac{C_2(s)}{C_2(s')} = \frac{C_2^*(s)}{C_2^*(s')} = \frac{Y_2^W(s)}{Y_2^W(s')}$$
$$\frac{C_2(s)}{C_1} = \frac{C_2^*(s)}{C_1^*} = \frac{Y_2^W(s)}{Y_1^W}$$

⇒ Home consumption is a constant share of world date 2 output, regardless of the state:

$$\frac{C_2(s)}{Y_2^W(s)} = \frac{C_2(s')}{Y_2^W(s')}, \quad \frac{C_2^*(s)}{Y_2^W(s)} = \frac{C_2^*(s')}{Y_2^W(s')}$$

⇒ Consumption growth rates are the same across countries in every state and are equal to the growth rate of world output.

International risk sharing (2)

Table: Consumption and output: Correlations between Domestic and World Growth Rates, 1973-92

Country	$Corr(\hat{c}, \hat{c}^W)$	$Corr(\hat{y}, \hat{y}^W)$
Canada	.56	.70
France	.45	.60
Germany	.63	.70
Italy	.27	.51
Japan	.38	.46
United Kingdom	.63	.62
United States	.52	.68
OECD average	.43	.52
Developing country average	-.10	.05

$Corr(\hat{c}, \hat{c}^W)$ and $Corr(\hat{y}, \hat{y}^W)$ are the simple correlation coefficients between the annual change in the natural logarithm of a country's real per capita consumption (or output) and the annual change in the natural logarithm of the rest of the world's real per capita consumption (or output). Source: Obstfeld & Rogoff (1996)

International risk sharing (3)

- For the seven largest industrial countries, the correlation between domestic and world consumption growth is lower in almost every case than the correlation between domestic and world output growth.

$\Rightarrow$ Contradicts the prediction of the complete-markets model $\longrightarrow$ Market incompleteness?

International Portfolio Diversification

Hypotheses

- A world economy, two dates (1 and 2), N countries, and $\mathcal{S}$ states of nature on date 2.
- People throughout the world have the same preferences, depicted by an intertemporal utility function:

$$U = u(C_1) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2(s))$$

  where $u(C)$ is the CRRA period utility function.
- V_1^n the date 1 market value of country n's uncertain date 2 output (price of an asset that pays $Y_2^n(s)$ in state s). x_m^n country n's net purchases of fractional shares in country m's future output
- A risk-free bond offering a real interest rate of r. B_2^n country n' net bond purchases in date 1

Consumption choices

- In country n, the representative household solves:

$$\begin{cases} \max_{C_1^n, C_2^n(s), B_2^n, x_m^n} \left[u(C_1) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2(s)) \right] \\ s.t. \quad Y_1^n + V_1^n = C_1^n + B_2^n + \sum_{m=1}^{N} x_m^n V_1^m \\ \qquad C_2^n(s) = (1+r) B_2^n + \sum_{m=1}^{N} x_m^n Y_2^m(s) \end{cases}$$

- First-order conditions:

$$u'(C_1^n) = \beta(1+r) \sum_{s=1}^{\mathcal{S}} \pi(s) u'(C_2^n(s)) = \beta(1+r) E_1\{u'(C_2^n(s))\}$$

$$V_1^m u'(C_1^n) = \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u'(C_2^n(s)) Y_2^m(s) = \beta E_1\{Y_2^m(s) u'(C_2^n(s))\}$$

$\Rightarrow$ The stochastic Euler equation equalizes the marginal utility of period 1's consumption with the expected present discounted value of period 2's consumption

$\Rightarrow$ The second equation equalizes the marginal utility cost to a country n resident who purchases country m's risky future output on date 1 and the expected marginal utility gain.

Equilibrium allocations

- The model is solved taking an educated guess at the equilibrium allocation (Pareto efficiency) and finding equilibrium portfolios and prices that support this conjecture.
- Under the conjecture of Pareto efficiency, equilibrium allocations take the same general form as in the complete-markets case, namely:

$$C_1^N = \mu^n Y_1^W$$
$$C_2^n(s) = \mu^n Y_2^W(s), \quad s = 1, ..., \mathcal{S}$$
$$\textit{where} \quad \mu^n = \frac{Y_1^n + V_1^n}{\sum_{m=1}^N (Y_1^m + V_1^m)}$$
$$\textit{and} \quad Y_1^W = \sum_{m=1}^N Y_1^m$$

Equilibrium allocations (2)

- For any country n, date 2 budget constraint is consistent with the previous allocation of second-period's outputs if $B_2^n = 0$ and $x_m^n = \mu^n, \forall m$ $\rightarrow$ Consumption and portfolio plans globally feasible if chosen simultaneously by every country n.
- Satisfy the Euler equation for all countries n if the equilibrium real interest rate is:

$$1 + r = \frac{Y_1^{W\ -\rho}}{\beta \sum_s \pi(s)\ Y_2^W(s)^{-\rho}}$$

- The second first-order condition for utility maximization is satisfied if:

$$V_1^m = \beta \sum_{s=1}^{\mathcal{S}} \pi(s) \left(\frac{Y_2^W(s)}{Y_1^W} \right)^{-\rho} Y_2^m(s) = \beta E_1 \left\{ \left(\frac{Y_2^W}{Y_1^W} \right)^{-\rho} Y_2^m \right\},\ \forall m = 1, ..., N$$

- Date 1's budget constraint is also satisfied under these conditions. Date 1's resources $(Y_1^n + V_1^n \equiv \mu^n(Y_1^W + V_1^W))$ are entirely consumed under the assumptions that $B_2^n = 0$, $C_1^n = \mu^n Y_1^W$ and $x_m^n = \mu^n$.

Equilibrium allocations (3)

- The Pareto efficient allocation obtained in the complete-markets case is thus a possible equilibrium of the uncomplete-markets economy.

⇒ Even when the assets traded are limited to riskless bonds and shares in each of the N countries' future outputs, the equilibrium allocation is efficient.

- The economy behaves as if there were complete markets because all agents choose to hold equities in the same proportions, regardless of their wealth levels.
- The resulting equilibrium makes everyone's date 2 consumption moves in proportion to the return on a single global mutual fund of risky assets.
- The common diversification strategy eliminates the gains from any further mutual consumption insurance between countries.

Portfolio diversification

Table: Home bias in mutual fund holdings

	Market Capital Weight	Share in the domestic market
United States	46.85	85.66
United Kingdom	8.13	43.06
Canada	2.44	26.99
Germany	3.99	33.49
Italy	2.22	35.37
Sweden	1.03	46.74
France	4.32	55.27
Switzerland	2.21	21.03
Austria	0.09	6.77
Belgium	0.55	24.73
Denmark	0.31	18.41
Ireland	0.19	6.14
Finland	0.95	45.7
Greece	0.46	93.46
Luxembourg	0.1	15.08
Norway	0.19	48.81
Portugal	0.19	45.61
Spain	1.39	35.96
the Netherlands	1.97	19.49
Japan	11.29	71.82
Australia	1.18	60.5
Singapore	0.51	18.25
Hong Kong	1.82	26.44
New Zealand	0.07	74.93
Taiwan	0.91	60.88
South Africa	0.69	66.58

Source: Chan, Covrig and Ng (*Journal of Finance, 2006*). The first column is the country's average stock market capitalization weight in the world market portfolio. The second column is the share of domestic shares in the country's portfolio. All figures are in percent.

Portfolio diversification (2)

- In the model, investors throughout the world all hold the same globally diversified portfolio of stocks.
- In practice, residents of most countries hold a large share of their equity wealth at home.

⇒ *Home bias puzzle*.

- Whatever the considered country, the share of its portfolio invested in domestic mutual funds is by much larger than the share of the country in the world capitalization.
- Several explanations: High costs connected with foreign stock acquisition or ownership, including trading costs, information costs, etc.