Master EPP, International Macroeconomics
Lecture 9-10
Financial globalisation and international risk sharing

1 Uncertainty in the small open endowment economy

Consider a small open endowment economy that exists for two periods (1 and 2), producing and consuming a single tradable good. The size of the population is normalized to unity. The representative individual has known first-period income Y_1 and starts out with zero net foreign assets.
On date 2, two "states of nature" are possible. The two states ($s = 1, 2$) occur randomly according to a specified probability distribution ($\pi(s)$) and differ only on their associated output levels ($Y_2(s)$).
The representative agent has a discount factor β. In period 1, she chooses her period 1 consumption as well as contingent plans for consumption in period 2 by maximizing expected utility. In the following, the instantaneous utility function $u(C)$ is assumed independent from the state of the nature.
Agents can borrow and lend in riskless bonds that pay $1 + r$ per unit on date 2, regardless of the state of nature. Moreover, there is a complete market of Arrow-Debreu securities allowing households to insure against risk in every state of the nature. As a consequence, the riskless bond market is redundant and only serves as a benchmark. Let $B_2(s)$ be the representative individual's net purchase of state s Arrow-Debreu securities on date 1 and $p(s)/(1+r)$ the world price, quoted in terms of date 1 consumption. This price is exogenous from the standpoint of the small country. An Arrow-Debreu security for state s pays 1 if state s occurs on date 2 and 0 otherwise.

1. Write the intertemporal budget constraint.

2. Determine the country's optimal saving and portfolio allocations.

3. Show that the non-arbitrage condition in the Arrow-Debreu and the riskless markets implies $p(1) + p(2) = 1$. Write the stochastic Euler equation.

4. What is the impact of a shock on the relative price of state 1 Arrow-Debreu security on the relative consumption in state 1? Interpretation

5. Derive date 1 current account balance in the special case $u(C) = log(C)$. Interpretation when $\beta(1 + r) = 1$.

2 A global Model with CRRA utility

Consider a world economy made of two countries, Home and Foreign, with output levels that fluctuate across $\mathcal{S}$ states of nature. Home and Foreign consumers share the same utility function given by:

$$U_1 = u(C_1) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2(s))$$

where $u(C)$ is CRRA:

$$u(C) = \begin{cases} \frac{C^{1-\rho}}{1-\rho} & (\rho > 0, \rho \neq 1) \\ \log C & (\rho = 1) \end{cases}$$

1. Derive the Euler equation for the Home and Foreign countries.

Table 1: Consumption and output: Correlations between Domestic and World Growth Rates, 1973-92

Country	$Corr(\hat{c}, \hat{c}^W)$	$Corr(\hat{y}, \hat{y}^W)$
Canada	.56	.70
France	.45	.60
Germany	.63	.70
Italy	.27	.51
Japan	.38	.46
United Kingdom	.63	.62
United States	.52	.68
OECD average	.43	.52
Developing country average	-.10	.05

$Corr(\hat{c}, \hat{c}^W)$ and $Corr(\hat{y}, \hat{y}^W)$ are the simple correlation coefficients between the annual change in the natural logarithm of a country's real per capita consumption (or output) and the annual change in the natural logarithm of the rest of the world's real per capita consumption (or output). Source: Obstfeld & Rogoff (1996)

2. Combine this with the world market equilibrium conditions to get the date 1 price of the state s contingent security as a function of world output in state s relative to date 1 world output $(Y_2^W(s)/Y_1^W)$. Under which condition are securities prices actuarially fair?

3. Solve for date 2 prices $p(s)$ and the world interest rate.

4. Discuss the predictions of the model with regards to correlations in international consumption levels across time and across states of nature.

5. Using results provided in Table 1, discuss the empirical validity of these predictions.

3 International Portfolio Diversification

Consider a world economy in which the only risky assets traded are claims to countries' uncertain outputs (shares of stock in national economies). There are two dates (1 and 2), N countries in the world, and $\mathcal{S}$ states of nature on date 2. People throughout the world have the same preferences, depicted by an intertemporal utility function:

$$U = u(C_1) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2(s))$$

where $u(C)$ is the CRRA period utility function. Let V_1^n be the date 1 market value of country n's uncertain date 2 output (price of an asset that pays $Y_2^n(s)$ in state s). Residents of different countries can exchange fractional shares of V_1^n. Together with a risk-free bond offering a real interest rate of r, the N ownership claims on national outputs are the only assets traded on date 1.

1. Calling B_2^n country n' net bond purchases in date 1 and x_m^n its net purchases of fractional shares in country m's future output, write and solve the program of the representative household in country n.

2. The model is solved taking an educated guess at the equilibrium allocation (namely, that it is Pareto efficient) and finding equilibrium portfolios and prices that support this conjecture. Under the

Table 2: Home bias in mutual fund holdings

	Market Capital Weight	Share in the domestic market
United States	46.85	85.66
United Kingdom	8.13	43.06
Canada	2.44	26.99
Germany	3.99	33.49
Italy	2.22	35.37
Sweden	1.03	46.74
France	4.32	55.27
Switzerland	2.21	21.03
Austria	0.09	6.77
Belgium	0.55	24.73
Denmark	0.31	18.41
Ireland	0.19	6.14
Finland	0.95	45.7
Greece	0.46	93.46
Luxembourg	0.1	15.08
Norway	0.19	48.81
Portugal	0.19	45.61
Spain	1.39	35.96
the Netherlands	1.97	19.49
Japan	11.29	71.82
Australia	1.18	60.5
Singapore	0.51	18.25
Hong Kong	1.82	26.44
New Zealand	0.07	74.93
Taiwan	0.91	60.88
South Africa	0.69	66.58

Source: Chan, Covrig and Ng (*Journal of Finance, 2006*). The first column is the country's average stock market capitalization weight in the world market portfolio. The second column is the share of domestic shares in the country's portfolio. All figures are in percent.

conjecture of Pareto efficiency, equilibrium allocations take the same general form as in the complete-markets case, namely:

$$C_1^N = \mu^n Y_1^W$$
$$C_2^n(s) = \mu^n Y_2^W(s), \quad s = 1, ..., \mathcal{S}$$
$$where \quad \mu^n = \frac{Y_1^n + V_1^n}{\sum_{m=1}^N (Y_1^m + V_1^m)}$$
$$and \quad Y_1^W = \sum_{m=1}^N Y_1^m$$

Find the prices and optimal shares that satisfy this allocation.

3. Discuss the implications of the model in terms of portfolio diversification. Compare them with empirical evidences in Table 2.