Master EPP, International Macroeconomics
Lecture 5-6
The intertemporal approach to the current account and
New views on global imbalance issues

1 Welfare and the terms-of-trade (Obstfeld & Rogoff, 1996)

1. Write the intertemporal Euler equation.

The Euler equation is obtained by maximizing the representative consumer's lifetime utility subject to her intertemporal budget constraint:

$$\begin{cases} \max_{C_1,C_2} U(C_1, C_2) \\ s.t. \quad C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \end{cases}$$

The first-order conditions are:

$$\frac{\partial U(C_1, C_2)}{\partial C_1} = (1+r)\frac{\partial U(C_1, C_2)}{\partial C_2}$$
$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

The first equation is the Euler equation, that equalizes the marginal utility of consumption in period 1 and the marginal utility of consumption in period 2, evaluated at period 1 price.

2. Use the Euler equation together with the differentiated budget constraint to derive the welfare impact of a rise in the world interest rate. Comment.

Totally differentiating the intertemporal budget constraint gives:

$$dC_1 + \frac{1}{1+r}dC_2 = \frac{C_2 - Y_2}{(1+r)^2}dr$$

In a two-period economy, $C_2 - Y_2 = (1+r)(Y_1 - C_1)$ and

$$dC_1 + \frac{1}{1+r}dC_2 = \frac{Y_1 - C_1}{1+r}dr$$

This can be used with the Euler equation to get the welfare impact of a rise in r:

$$\frac{dU(C_1, C_2)}{dr} = \frac{\partial U(C_1, C_2)}{\partial C_2} \times \frac{dC_2}{dr} = \frac{\partial U(C_1, C_2)}{\partial C_2}(Y_1 - C_1)$$

The welfare impact of a rise in r is thus positive if $Y_1 > C_1$. Under this condition, Home is lending in period 1 to consume more than its endowment in period 2. Said otherwise, Home is exporting period 1 consumption and importing period 2 consumption. An increase in r thus improves its terms of intertemporal trade. The impact on welfare is thus positive.

3. Let $W_1 = Y_1 + \frac{Y_2}{1+r}$ the lifetime wealth in units of period 1 consumption. Show that a small percentage gross interest rate increase of $\hat{r} = dr/(1+r)$ has the same effect on lifetime utility as a lifetime wealth change of $dW_1 = \hat{r}(Y_1 - C_1)$.

The welfare gain of a rise in lifetime wealth is:

$$\frac{dU(C_1, C_2)}{dW_1} = \frac{dU(W1 - \frac{C_2}{1+r}, C_2)}{dW_1} = \frac{\partial U(C_1, C_2)}{\partial C_1} = (1+r)\frac{\partial U(C_1, C_2)}{\partial C_2}$$

Thus, if $dW_1 = \hat{r}(Y_1 - C_1)$, the utility gain is:

$$dU(C_1, C_2) = (1+r)\frac{\partial U(C_1, C_2)}{\partial C_2}\hat{r}(Y_1 - C_1)$$

which is equal to the utility gain associated with a gross interest rate increase of $\hat{r}$. If $Y_1 - C_1 > 0$, the impact is positive because the interest rate increase rises the consumer's future income.

2 Logarithmic case of the two-country endowment model (Obstfeld & Rogoff, 1996)

1. Write the Home date 1 consumption and saving as a function of endowments Y_1 and $Y_2$and the (endogenous) equilibrium interest rate.

The Home consumer solves the following program:

$$\begin{cases} \max_{C_1, C_2} U(C_1, C_2) \\ s.t: \quad C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \end{cases}$$

Substituting C_2 obtained from the intertemporal budget constraint and maximizing with respect to C_1 gives:

$$C_1(r) = \frac{1}{1+\beta}\left(Y_1 + \frac{Y_2}{1+r}\right)$$

Expenditure shares on periods 1 and 2 are thus constants when the intertemporal substitution elasticity is constant.

Saving is thus:

$$S_1(r) = Y_1 - C_1(r) = \frac{\beta}{1+\beta}Y_1 - \frac{1}{(1+\beta)(1+r)}Y_2$$

2. Compute the equilibrium world interest rate and compare it with autarky interest rates.

Using the international capital market equilibrium condition allows to solve for the equilibrium interest rate:

$$\begin{aligned} & S_1(r) + S_1^*(r) = 0 \\ \Leftrightarrow \quad & \frac{\beta}{1+\beta}Y_1 - \frac{1}{(1+\beta)(1+r)}Y_2 + \frac{\beta}{1+\beta}Y_1^* - \frac{1}{(1+\beta)(1+r)}Y_2^* = 0 \\ \Leftrightarrow \quad & 1+r = \frac{1}{\beta}\frac{Y_2 + Y_2^*}{Y_1 + Y_1^*} \end{aligned}$$

In autarky, intertemporal trade is not possible:

$$S_1(r^A) = 0 \quad \Rightarrow \quad 1 + r^A = \frac{1}{\beta}\frac{Y_2}{Y_1}$$

$$S_1^*(r^{A*}) = 0 \quad \Rightarrow \quad 1 + r^{A*} = \frac{1}{\beta}\frac{Y_2^*}{Y_1^*}$$

One easily verifies that the equilibrium interest rate is between the autarky interest rates.

3. Assume $\frac{Y_2}{Y_1} > \frac{Y_2^*}{Y_1^*}$. Which country runs a current account deficit?

When the Home country is relatively abundant in period 2 endowment, its autarky interest rate is higher than the equilibrium interest rate. As a consequence, the home country's saving is negative, ie it runs a current account deficit.

4. How does an increase in Foreign's rate of output growth affect Home's welfare?

A rise in the ratio Y_2^*/Y_1^* raises the equilibrium world interest rate. This impacts the domestic welfare as follows:

$$\begin{aligned}
\frac{dU}{dr} &= \frac{1}{C_1}\frac{dC_1}{dr} + \frac{\beta}{C_2}\frac{dC_2}{dr} \\
&= \frac{1+\beta}{Y_1 + \frac{Y_2}{1+r}}\left(\frac{dC_1}{dr} + \frac{dC_2}{dr}\right) \\
&= \frac{\beta}{1+r}\left[\frac{r - r^A}{(1+r) + \beta(1+r^A)}\right]
\end{aligned}$$

The increase in Foreign's rate of output is thus welfare improving for the Home country if its autarky interest rate is lower than the world interest rate (ie if it has a current account surplus).

3 An infinite-horizon model (Obstfeld & Rogoff, 1996)

1. Write the consumer's intertemporal budget constraint. Interpretation

The current account identity for a given period t writes :

$$CA_t = B_{t+1} - B_t = Y_t + rB_t - C_t - G_t - I_t$$

Rearranging terms gives :

$$(1+r)B_t = B_{t+1} - Y_t + C_t + G_t + I_t$$

Using a forward iteration method gives:

$$\begin{aligned}
(1+r)B_t &= C_t + G_t + I_t - Y_t + \frac{C_{t+1} + G_{t+1} + I_{t+1} - Y_{t+1}}{1+r} + \frac{B_{t+2}}{1+r} \\
&= C_t + G_t + I_t - Y_t + \frac{C_{t+1} + G_{t+1} + I_{t+1} - Y_{t+1}}{1+r} + \frac{C_{t+2} + G_{t+2} + I_{t+2} - Y_{t+2}}{(1+r)^2} \\
&= \dots \\
&= \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_s + G_s + I_s - Y_s) + \lim_{s\to\infty}\left(\frac{1}{1+r}\right)^{s} B_{t+s+1}
\end{aligned}$$

Rearranging and using the usual transversality condition leads to the intertemporal budget constraint:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_s + I_s) = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s)$$

Intuition on the transversality condition: Assume that $\lim_{s\to\infty}\left(\frac{1}{1+r}\right)^{s} B_{t+s+1} < 0$. This means that the present value of what the economy is consuming and investing exceeds the present value of its output by an amount that never converges to zero. The economy is continually borrowing to meet

the interest payments on its foreign debt rather than transferring real resources to its creditors by reducing internal demand. As a result, its debt grows at least at the rate of interest. But foreigners will never allow such a Ponzi scheme at their expense: that would amount to providing another economy with free resources. For this reason, the requirement that $\lim_{s\to\infty}\left(\frac{1}{1+r}\right)^s B_{t+s+1} \geq 0$ is called the no-Ponzi-game condition.
(remark: The scheme is named after Charles Ponzi, who became notorious for using the technique after emigrating from Italy to the United States in 1903. Ponzi was not the first to invent such a scheme, but his operation took in so much money that it was the first to become known throughout the United States. Ponzi's original scheme was in theory based on arbitraging international reply coupons for postage stamps, but soon diverted later investors' money to support payments to earlier investors and Ponzi's personal wealth.)
In the opposite case, assume that $\lim_{s\to\infty}\left(\frac{1}{1+r}\right)^s B_{t+s+1} > 0$. The present value of the resources the home economy uses never converges up to the present value of its output. In that case, domestic residents are making an unrequited gift to foreigners. Plainly they could raise their lifetime utility by consuming a little more. Only when $\lim_{s\to\infty}\left(\frac{1}{1+r}\right)^s B_{t+s+1} = 0$ is the economy asymptotically using up exactly the resources its budget constraint allows.
Rearranging the budget constraint gives:

$$-(1+r)B_t = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - C_s - I_s - G_s)$$

The right-hand side corresponds to the discounted value of future trade balances, defined as the amount of output the economy transfers to foreigners each period. The present value of an economy's resource transfers to foreigners must equal the value of the economy's initial debt to them. Thus, the intertemporal budget constraint holds if a country pays off initial foreign debt through sufficiently large future surpluses in its balance of trade.

2. Solve the representative consumer's maximization program.

The representative consumer faces the following problem:

$$\begin{cases} \max_{C_s, B_{s+1}, K_{s+1}} \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \\ s.t. \quad B_{s+1} - B_s = A_s F(K_s) + rB_s - C_s - G_s - (K_{s+1} - K_s) \end{cases}$$

Substitute for the consumption levels derived from the budget constraint gives:

$$\max_{B_{s+1}, K_{s+1}} \sum_{s=t}^{\infty} \beta^{s-t} u\left[(1+r)B_s - B_{s+1} + A_s F(K_s) - G_s - K_{s+1} + K_s\right]$$

The first-order conditions are:

$$\begin{aligned} u'(C_s) &= \beta(1+r)u'(C_{s+1}) \\ u'(C_s) &= \beta u'(C_{s+1})[1 + A_{s+1}F'(K_{s+1})] \end{aligned}$$

The first equation corresponds to the standard Euler equation that equalizes the discounted marginal utilities of consumption in two consecutive periods. Note that, as long as $\beta > 1/(1+r)$, ie if the consumer is patient enough, the country's consumption grows forever.
After some rearrangement, the second equation becomes:

$$r = A_{s+1}F'(K_{s+1})$$

that equalizes the marginal productivity of capital with the world interest rate.

3. Assume $\beta = 1/(1+r)$, $G_s = 0, \forall s$ and $Y_s = \bar{Y}, \forall s$.What is the optimal consumption path in that case?

When $\beta = 1/(1+r)$, the Euler equation becomes $u'(C_s) = u'(C_{s+1})$ which holds if and only of $C_s = C_{s+1} = \bar{C}, \forall s$. The consumption path is thus constant over time. Using the transversality condition, one then gets:

$$\begin{aligned}
& \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T B_{t+T+1} = 0 \\
\Leftrightarrow\ & \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T \left[(1+r)B_{t+T} + \bar{Y} - \bar{C}\right] = 0 \\
\Leftrightarrow\ & \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T \left[(1+r)^2 B_{t+T-1} + (1+r)(\bar{Y} - \bar{C}) + (\bar{Y} - \bar{C})\right] = 0 \\
\Leftrightarrow\ & \dots \\
\Leftrightarrow\ & \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T \left[(1+r)^{T+1} B_t + \sum_{s=0}^{T} (1+r)^s (\bar{Y} - \bar{C})\right] = 0 \\
\Leftrightarrow\ & \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T \left[(1+r)^{T+1} B_t - \frac{1-(1+r)^{T+1}}{r}(\bar{Y} - \bar{C})\right] = 0 \\
\Leftrightarrow\ & \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T \left[B_t + (rB_t + \bar{Y} - \bar{C})\frac{(1+r)^{T+1}-1}{r}\right] = 0 \\
\Leftrightarrow\ & \bar{C} = rB_t + \bar{Y}
\end{aligned}$$

Economic actors thus choose a constant consumption level equal to the economy's income.

4. Assume now that $\beta = 1/(1+r)$, but investment, government consumption and output vary over time. What is the optimal consumption path in that case?

The condition $\beta = 1/(1+r)$ still implies that consumption is constant over time. Using this and the intertemporal budget constraint gives:

$$\begin{aligned}
& \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (\bar{C} + I_s) = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s) \\
\Leftrightarrow\ & \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \bar{C} = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s) \\
\Leftrightarrow\ & \bar{C} = \frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s)\right]
\end{aligned}$$

Again, the optimal consumption is the sum of the interest rate flow on the initial net foreign asset position plus a constant share of the present value of her lifetime revenue (the permanent level of her revenue).[1]

[1]For a constant interest rate r, the permanent level of variable X on date t is defined as:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \tilde{X}_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} X_s$$

ie

$$\tilde{X}_t = \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} X_s$$

5. What happens when the utility function is isoelastic?
In the isoelastic case ($u(C) = C^{1-1/\sigma}/(1-1/\sigma)$), the Euler equation becomes:

$$C_{s+1} = C_s \beta^\sigma (1+r)^\sigma$$

Introducing this in the intertemporal budget constraint gives:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} C_t \beta^{\sigma(s-t)} (1+r)^{\sigma(s-t)} = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s)$$

$$\Leftrightarrow \quad C_t = \frac{(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s)}{\sum_{s=t}^{\infty} \beta^{\sigma(s-t)} (1+r)^{(\sigma-1)(s-t)}}$$

$$\Leftrightarrow \quad C_t = \frac{W_t}{\sum_{s=t}^{\infty} \beta^{\sigma(s-t)} (1+r)^{(\sigma-1)(s-t)}}$$

where W_t is the beginning of period t wealth (including financial assets accumulated in $t-1$ and current and future incomes).
If $(1+r)^{\sigma-1}\beta^\sigma > 1$ the denominator is a nonconvergent series and the consumption function is not define (no utility maximum exists). However, if $(1+r)^{\sigma-1}\beta^\sigma < 1$, the consumption function becomes:

$$C_t = \frac{(1+r) - (1+r)^\sigma \beta^\sigma}{1+r} \left[(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s)\right]$$

6. Describe the dynamics of the current account in all three special cases described in questions **3**-**5**.
From the consumer's budget constraint, the dynamics of the current account is defined as:

$$CA_t \equiv B_{t+1} - B_t = Y_t + rB_t - C_t - G_t - I_t$$

When both output and consumption are constant over time, the current account imbalance is always equal to zero.
When $\beta = 1/(1+r)$ so that the consumption path is constant but output can vary over time, the current account becomes:

$$CA_t = (Y_t - \tilde{Y}_t) - (G_t - \tilde{G}_t) - (I_t - \tilde{I}_t)$$

Output above its permanent level contributes to a higher current account surplus because of consumption smoothing. Rather than raising consumption point for point when output rises temporarily above its long-run discounted average, individuals choose to accumulate interest-yielding foreign assets as a way of smoothing consumption over future periods. Similarly, people use foreign borrowing to cushion their consumption in the face of unusually high investment needs, rather than financing them out of domestic savings. Finally, abnormally high government spendings lead to a current account deficit spreading the impact over the entire future.
In the case of a isoelastic utility function with $(1+r)^{\sigma-1}\beta^\sigma < 1$, consumption is given by:

$$C_t = \frac{(1+r) - (1+r)^\sigma \beta^\sigma}{1+r} \left[(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s)\right] = \frac{1+r-(1+r)^\sigma \beta^\sigma}{1+r} W_t$$

where W_t is the country wealth at the beginning of period t. The current account is then:

$$CA_t = (Y_t - \tilde{Y}_t) - (G_t - \tilde{G}_t) - (I_t - \tilde{I}_t) - \frac{1-(1+r)^\sigma \beta^\sigma}{1+r} W_t$$

The current account is driven by two distinct motives: the pure smoothing motive and a tilting motive related to any discrepancy between the subjective discount factor and the world market discount factor. When the country is relatively impatient ($1-(1+r)^\sigma\beta^\sigma > 0$), the current account is reduced.

4 Uncertain lifetimes and infinite horizons (Obstfeld & Rogoff, 1996)

1. What is the expected utility function?

Consider the first period t. With probability $1-\varphi$, the individual will die after consuming. In this case, is lifetime utility is simply $u(C_t)$. But with probability φ, she consumes in t and survives until $t+1$. In this case, her lifetime utility is $u(C_t)$ plus the utility drawn from consumption in $t+1$ ($\beta u(C_{t+1}$ in present value) and possibly the following periods. Using this reasoning finally gives:

$$E_t U_t = \sum_{t=0}^{\infty} \varphi^T (1-\varphi) \left[\sum_{s=t}^{t+T} \beta^{s-t} u(C_s) \right]$$

2. Notice that one's consumption conditional on reaching date s cannot be a function of one's eventual longevity, since the ultimate date of death is unknown when date s consumption occurs. Rewrite $E_t U_t$ by collecting all the terms involving $u(C_t)$, then all those involving $u(C_{t+1})$, and so on.

$$\begin{aligned}
E_t U_t &= (1-\varphi) \sum_{T=0}^{\infty} \varphi^T u(C_t) + (1-\varphi) \sum_{T=1}^{\infty} \varphi^T \beta u(C_{t+1}) + (1-\varphi) \sum_{T=2}^{\infty} \varphi^T \beta^2 u(C_{t+2}) + ... \\
&= (1-\varphi) u(C_t) \frac{1}{1-\varphi} + (1-\varphi) \beta u(C_{t+1}) \frac{\varphi}{1-\varphi} + (1-\varphi) \beta^2 u(C_{t+2}) \frac{\varphi^2}{1-\varphi} + ... \\
&= \sum_{s=t}^{\infty} (\varphi\beta)^{s-t} u(C_s)
\end{aligned}$$

The uncertain lifetime thus gives the consumer a utility function with an infinite horizon and a subjective discount factor that equals the product of the "pure" time-preference factor and the survival probability for each date.