Master EPP, International Macroeconomics
Lecture 6-7
The intertemporal approach to the current account and
New views on global imbalance issues

1 Welfare and the terms-of-trade (Obstfeld & Rogoff, 1996)

Let the representative individual in a small open economy maximize $U(C_1, C_2)$ subject to her intertemporal budget constraint. Endowments Y_1 and Y_2 are exogenous.

1. Write the intertemporal Euler equation.

2. Use the Euler equation together with the differentiated budget constraint to derive the welfare impact of a rise in the world interest rate. Comment.

3. Let $W_1 = Y_1 + \frac{Y_2}{1+r}$ the lifetime wealth in units of period 1 consumption. Show that a small percentage gross interest rate increase of $\hat{r} = dr/(1+r)$ has the same effect on lifetime utility as a lifetime wealth change of $dW_1 = \hat{r}(Y_1 - C_1)$.

2 Logarithmic case of the two-country endowment model (Obstfeld & Rogoff, 1996)

Consider the pure endowment model with two periods/two countries. The utility function is of the logarithmic form:

$$U(C_1, C_2) = \log C_1 + \beta \log C_2$$

1. Write the Home date 1 consumption and saving as a function of endowments Y_1 and Y_2 and the (endogenous) equilibrium interest rate.

2. Compute the equilibrium world interest rate and compare it with autarky interest rates.

3. Assume $\frac{Y_2}{Y_1} > \frac{Y_2^*}{Y_1^*}$. Which country runs a current account deficit?

4. How does an increase in Foreign's rate of output growth affect Home's welfare?

3 An infinite-horizon model (Obstfeld & Rogoff, 1996)

Consider a small open economy with a constant world interest rate r and perfect foresight. The representative consumer is infinitely-lived and maximizes the following intertemporal utility function:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

Output on any date s is determined by the production function $Y_s = A_s F(K_s)$. The economy starts in date t with pre-determined stocks of capital K_t and net foreign assets B_t, both accumulated on prior dates.

1. Write the consumer's intertemporal budget constraint. Interpretation

2. Solve the representative consumer's maximization program.

3. Assume $\beta = 1/(1+r)$, $G_s = 0, \forall s$ and $Y_s = \bar{Y}, \forall s$.What is the optimal consumption path in that case?

4. Assume now that $\beta = 1/(1+r)$, but investment, government consumption and output vary over time. What is the optimal consumption path in that case?

5. What happens when the utility function is isoelastic?

6. Describe the dynamics of the current account in all three special cases described in questions **3-5**.

4 Uncertain lifetimes and infinite horizons (Obstfeld & Rogoff, 1996)

One way to motivate an infinite individual planning horizon is to assume that lives, while finite in length, have an uncertain terminal date. In any given period, an individual lives on to the next period with probability $\varphi < 1$ but dies after consuming with probability $1 - \varphi$. Let $E_t U_t$ denote the individual's expected utility over all possible lengths of life, ie the weighted average over different life spans with weights equal to survival probabilities.

1. What is the expected utility function?

2. Notice that one's consumption conditional on reaching date s cannot be a function of one's eventual longevity, since the ultimate date of death is unknown when date s consumption occurs. Rewrite $E_t U_t$ by collecting all the terms involving $u(C_t)$, then all those involving $u(C_{t+1})$, and so on.