Master EPP, International Macroeconomics
Lecture 5
Pricing to market and exchange rate pass through: empirics and theory

1 The exchange rate in a model of pricing-to-market (Betts & Devereux, 1996)

1. Write the aggregate price index as a function of individual prices and the exchange rate.

The domestic price level aggregates the price of the n domestic producers, the $(1-n)s$ foreign producers under LCP and the $(1-n)(1-s)$ foreign producers under PCP:

$$P = \left[\int_0^n p(i)^{1-\sigma} di + \int_n^{n+(1-n)s} p^*(i)^{1-\sigma} + \int_{n+(1-n)s}^1 (eq^*(i))^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$$

where $p(i)$ is the local price of a domestic variety, $p(i)^*$ is the domestic currency price of a foreign variety sold by a LCP firm and $q(i)^*$ is the foreign currency price chosen by a foreign firm under PCP. The exchange rate is given by e.

2. Write and solve the household's program.

Households maximize their utility under their budget constraint:

$$\begin{cases} \max_{C,h,M} \left(logC + \frac{\gamma}{1-\epsilon} \left(\frac{M}{P}\right)^{1-\epsilon} + \eta log(1-h) \right) \\ s.t.\ PC + M = Wh + \pi + M_0 + TR \end{cases}$$

The Lagrangian of the problem writes:

$$\mathcal{L} = logC + \frac{\gamma}{1-\epsilon}\left(\frac{M}{P}\right)^{1-\epsilon} + \eta log(1-h) + \lambda \left[Wh + \pi + M_0 + TR - PC - M\right]$$

and the first order conditions:

$$\begin{aligned} 0 &= \frac{1}{C} - \lambda P \\ 0 &= \gamma \left(\frac{M}{P}\right)^{1-\epsilon} \frac{1}{M} - \lambda \\ 0 &= \frac{-\eta}{1-h} + \lambda W \end{aligned}$$

This implies the following arbitrage relation:

$$\frac{1}{C} = \gamma \left(\frac{M}{P}\right)^{-\epsilon}$$
$$\frac{\eta}{1-h} = \frac{W}{PC}$$

Once households have chosen their aggregate consumption, together with their labor supply and money demand, they chose their demand of each available variety, solving:

$$\begin{cases} \max_{C(i)} \left[\int_0^1 c(i)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\ s.t.\ PC = \int_0^1 v(i)c(i)di \end{cases}$$

where $v(i)$ is the price of variety i, either $p(i)$ if $i \in [0;n]$ or $p^*(i)$ if $i \in [n; n+(1-n)s]$ or $eq(i)^*$ if $i \in [n+(1-n)s;1]$.
The solution of this problem gives the usual demand function:

$$c(i) = \left(\frac{v(i)}{P}\right)^{-\sigma} C$$

The situation of foreign households is entirely analogous.

3. Derive pricing strategies of LCP and PCP firms. Show that PPP holds in this model when prices are fully flexible.

LCP firms are able to fix different prices in their domestic and export markets. Under a constant returns to scale technology, their maximization program writes:

$$\begin{cases} \max_{p(i),q(i)} \pi^{LCP}(i) = p(i)c(i) + eq(i)c^*(i) - \frac{W}{A}(c(i)+c^*(i) \\ s.t.\ c(i) = \left(\frac{p(i)}{P}\right)^{-\sigma} nC \\ \quad c^*(i) = \left(\frac{q(i)}{P^*}\right)^{-\sigma}(1-n)C^* \end{cases}$$

The first-order conditions imply the following pricing rules:

$$\begin{aligned} p(i) &= \frac{\sigma}{\sigma-1}\frac{W}{A} \\ q(i) &= \frac{\sigma}{\sigma-1}\frac{W}{Ae} \end{aligned}$$

Under flexible prices, the law of one price holds for LCP firms as $p(i) = eq(i)$.
As for PCP firms, they are forced to set the same price in their domestic and export markets. Their maximization program writes:

$$\begin{cases} \max_{p(i)} \pi^{PCP}(i) = p(i)c(i) + p(i)c^*(i) - \frac{W}{A}(c(i)+c^*(i) \\ s.t.\ c(i) = \left(\frac{p(i)}{P}\right)^{-\sigma} nC \\ \quad c^*(i) = \left(\frac{p(i)}{eP^*}\right)^{-\sigma}(1-n)C^* \end{cases}$$

The first-order condition implies the following pricing rule:

$$p(i) = \frac{\sigma}{\sigma-1}\frac{W}{A}$$

For PCP firms, the law of one price of course holds, under flexible prices as under sticky prices.

4. Incorporating optimal prices into the price indices shows that, under flexible prices, PPP holds:

$$\begin{aligned} P &= \left[n\left(\frac{\sigma}{\sigma-1}\frac{W}{A}\right)^{1-\sigma} + (1-n)s\left(e\frac{\sigma}{\sigma-1}\frac{W^*}{A^*}\right)^{1-\sigma}(1-n)(1-s)\left(e\frac{\sigma}{\sigma-1}\frac{W^*}{A^*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ P^* &= \left[ns\left(\frac{\sigma}{\sigma-1}\frac{W}{Ae}\right)^{1-\sigma} + n(1-s)\left(\frac{1}{e}\frac{\sigma}{\sigma-1}\frac{W}{A}\right)^{1-\sigma}(1-n)\left(\frac{\sigma}{\sigma-1}\frac{W^*}{A^*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ \Rightarrow \quad P &= eP^* \end{aligned}$$

Price indices are then log-linearized around a symmetric flexible-price equilibrium to get:

$$\begin{aligned} &\hat{P} = (1-n)(1-s)\hat{e} \\ &\hat{P}^* = -n(1-s)\hat{e} \\ \Rightarrow \quad &\hat{P} - \hat{P}^* = (1-s)\hat{e} \neq \hat{e} \end{aligned}$$

Under sticky prices, PPP does not holds in the short-run if some firms adopt a LCP strategy. In the limit, when $s \to 1$, relative prices are unaffected by exchange-rate movements, ie there is no exchange-rate pass-through.

5. Use the log-linearized money demand equations and the definition of price indices to get the fundamental exchange rate equation around the flexible-price equilibrium. What impact has pricing-to-market on the dynamics of the exchange rate?

Log-linearizing the money demand equations gives:

$$\begin{aligned} -\epsilon\hat{M} + \epsilon\hat{P} &= -\hat{C} \\ -\epsilon\hat{M}^* + \epsilon\hat{P}^* &= -\hat{C}^* \end{aligned}$$

Incorporating the equations for price indices derived in the previous question and combining both conditions together gives the dynamics of the exchange rate as a function of money supply differential and relative consumptions:

$$\hat{e}(1-s) = (\hat{M} - \hat{M}^*) - \frac{1}{\epsilon}(\hat{C} - \hat{C}^*)$$

The exchange rate will depreciate in response to relative national money growth, and will appreciate in response to relative national growth in real consumption. The size of s determines the magnitude of the departure from PPP and of the exchange-rate adjustment to shocks.

2 Pricing-to-Market, Trade Costs, and International Relative Prices (Atkeson & Burstein, 2008)

1. Write and solve the household program.

The household program is:

$$\begin{cases} \max_{c_{is}, l_{is}, b_{is+1}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}, 1-l_{it}) \\ u.c. \quad P_{it}c_{it} + b_{it+1} = W_{it}l_{it} + (1+r)b_{it} \end{cases}$$

The first-order conditions are:

$$\begin{aligned} u_c'(c_{it}, 1-l_{it}) &= P_{it}\lambda_{it} \\ -u_l'(c_{it}, 1-l_{it}) &= -W_{it}\lambda_{it} \\ \beta\lambda_{it+1}(1+r) &= \lambda_{it} \end{aligned}$$

where λ_{it} is the Lagrange multiplier associated with period t. From this, one gets the intratemporal arbitrage condition between consumption and leisure:

$$\frac{1-\mu}{\mu}\frac{c_{it}}{1-l_{it}} = \frac{W_{it}}{P_{it}} \tag{1}$$

and the Euler equation:

$$\beta\frac{P_{it}}{P_{it+1}}(1+r)u_c'(c_{it+1}, 1-l_{it+1}) = u_c'(c_{it}, 1-l_{it}) \tag{2}$$

Finally, under complete markets:

$$P_{1t}c_{1t} = P_{2t}c_{2t} \tag{3}$$

2. Write the optimal demand of sectoral composite goods (y_{ijt}) and of individual goods (q_{ijkt}).

The final good producer solves the following program:

$$\begin{cases} & \max_{y_{ijt}} c_{it} \\ u.c. & P_{it}c_{it} = \int_0^1 P_{ijt}y_{ijt}dj \\ & c_{it} = \left[\int_0^1 y_{ijt}^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \end{cases}$$

The first-order conditions are:

$$y_{ijt}^{-1/\eta}\left[\int_0^1 y_{ijt}^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}-1} = P_{ijt}, \quad \forall j \in [0,1]$$

$$P_{it}c_{it} = \int_0^1 P_{ijt}y_{ijt}dj$$

Using the first of these relations for two distinct sectors j and l gives:

$$y_{ijt} = \left(\frac{P_{ijt}}{P_{ilt}}\right)^{-\eta} y_{ilt}$$

Incorporating this in the right-hand side of the budget constraint gives:

$$\int_0^1 P_{ijt}y_{ijt}dj = P_{ilt}^{\eta}y_{ilt}\int_0^1 P_{ijt}^{1-\eta}dj$$

The left-hand side of the budget constraint can be written as:

$$P_{it}c_{it} = P_{it}\left[\int_0^1 y_{ijt}^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} = P_{it}P_{ilt}^{\eta}y_{ilt}\left[\int_0^1 P_{ijt}^{1-\eta}dj\right]^{\frac{\eta}{\eta-1}}$$

The budget constraint can then be used to get the aggregate price index:

$$P_{it}c_{it} = \int_0^1 P_{ijt}y_{ijt}dj$$

$$\Leftrightarrow P_{it} = \left[\int_0^1 P_{ijt}^{1-\eta}dj\right]^{\frac{1}{1-\eta}} \tag{4}$$

Finally, the consumption of an individual sector is: obtained as follows:

$$\int_0^1 P_{ijt}y_{ijt}dj = P_{it}c_{it} = P_{ilt}^{\eta}y_{ilt}\int_0^1 P_{ijt}^{1-\eta}dj$$

$$\Leftrightarrow y_{ilt} = \left(\frac{P_{ilt}}{P_{it}}\right)^{-\eta} c_{it} \tag{5}$$

Proceeding in a similar way at the sectoral level, one gets:

$$P_{ijt} = \left[\sum_{k=1}^{2K} P_{ijkt}^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{6}$$

$$q_{ijkt} = \left(\frac{P_{ijkt}}{P_{ijt}}\right)^{-\rho} y_{ijt} \tag{7}$$

3. Suppose that only the K domestic firms produce and sell in each country in sector j. What are the equilibrium quantities and prices? Discuss the following limiting cases: i) large number of firms ($K \leftarrow \infty$), ii) a single firm by sector, iii) $\rho = \eta$.

A vector of quantities q_{ijkt} and prices P_{ijkt} are equilibrium prices and quantities if, for each firm with productivity z_{jk}, the quantity q_{ijkt} and price P_{ijkt} solve the profit maximization problem:

$$\begin{cases} \max_{P_{ijkt},q_{ijkt}} \left[P_{ijkt}q_{ijkt} - \frac{W_{it}}{z_{jk}A_{it}}q_{ijkt}\right] \\ s.t. \quad \frac{P_{ijkt}}{P_{it}} = \left(\frac{q_{ijkt}}{y_{ijt}}\right)^{-1/\rho}\left(\frac{y_{ijt}}{c_{it}}\right)^{-1/\eta} \\ \quad\quad y_{ijt} = \left[\sum_k q_{ijkt}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \end{cases}$$

$$\Leftrightarrow \max_{q_{ijkt}} \left[P_{it} \left(\frac{q_{ijkt}}{\left[\sum_k q_{ijkt}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}} \right)^{-1/\rho} \left(\frac{\left[\sum_k q_{ijkt}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}}{c_{it}} \right)^{-1/\eta} q_{ijkt} - \frac{W_{it}}{z_{jk}A_{it}} q_{ijkt} \right]$$

The first-order condition for any good k is:

$$\frac{-1}{\rho}P_{ijkt} + \frac{1}{\rho}P_{ijkt}\frac{dy_{ijt}/y_{ijt}}{dq_{ijkt}/q_{ijkt}} - \frac{1}{\eta}P_{ijkt}\frac{dy_{ijt}/y_{ijt}}{dq_{ijkt}/q_{ijkt}} + P_{ijkt} - \frac{W_{it}}{z_{jk}A_{it}} = 0$$

Using the optimality conditions, one gets:

$$\frac{dy_{ijt}/y_{ijt}}{dq_{ijkt}/q_{ijkt}} = \left(\frac{q_{ijkt}}{y_{ijt}}\right)^{\frac{\rho-1}{\rho}} = \left(\frac{P_{ijkt}}{P_{ijt}}\right)^{1-\rho} = \frac{P_{ijkt}q_{ijkt}}{\sum_k P_{ijkt}q_{ijkt}} \equiv s_{ijkt}$$

where s_{ijkt} is the firm's market share in country i. Finally, the first-order conditions can be rewritten as:

$$P_{ijkt} = \frac{\varepsilon(s_{ijkt})}{\varepsilon(s_{ijkt}) - 1}\frac{W_{it}}{z_{jk}A_{it}} \tag{8}$$

where $\varepsilon(s_{ijkt}) \equiv \left[\frac{1}{\rho}(1 - s_{ijkt}) + \frac{1}{\eta}s_{ijkt}\right]^{-1}$ is the perceived elasticity of demand. From this, one gets optimal quantities using:

$$q_{ijkt} = \left(\frac{P_{ijkt}}{P_{ijt}}\right)^{-\rho}\left(\frac{P_{ijt}}{P_{it}}\right)^{-\eta}\frac{c_{it}}{P_{it}}$$

together with the definition of the sectoral price index.
In the limiting case where $K \to \infty$, each firm has a market share s approaching zero. As a consequence, the perceived elasticity is equal to ρ, the firm only perceives the sectoral elasticity of demand ρ and chooses a markup equal to $\rho/(\rho-1)$. In the other extreme, if the firm has a market share s approaching one, it perceives the lower elasticity of demand across sectors η and sets a higher markup equal to $\eta/(\eta - 1)$. Finally, when $\eta = \rho$, then the model reduces to the standard model of monopolistic competition with a constant markup of price over marginal cost given by $\rho/(\rho - 1)$.
When $\rho > \eta$, firms with a sectoral market share between zero and one choose a markup that increases smoothly with that market share. An important consequence of those variable mark-ups is that prices and costs are not linearly related in the model. This gives the possibility that firms do not pass-through changes in cost one-for-one into prices. Specifically, if a single firm or a group of firms in a sector experiences an increase in marginal cost relative to the other firms in the sector, this firm or

group of firms lose market share and hence decrease their markup in equilibrium. As a result, the prices charged by this firm or group of firms rise by less than the increase in their costs.
The observation of incomplete pass-through of changes in costs to prices arises quite naturally in the model. However, this feature that generates incomplete pass-through is not, by itself, enough to generate pricing-to-market. To get pricing-to-market, it must be true that a change in costs for one firm or a group of firms leads to a change in markups for those firms that is different in each market in which these firms compete.

4. What is the optimal behaviour of the lowest cost foreign producer?

The lowest cost producer in country 2 solve the following optimization problem:

$$\begin{cases} \max_{P_{1jK+1t}, q_{1jK+1t}} \left[P_{1jK+1t} q_{1jK+1t} - \frac{\tau W_{2t}}{z_{jK+1} A_{2t}} q_{1jK+1t} \right] \\ s.t. \quad \frac{P_{1jK+1t}}{P_{1t}} = \left(\frac{q_{1jK+1t}}{y_{1jt}} \right)^{-1/\rho} \left(\frac{y_{1jt}}{c_{1t}} \right)^{-1/\eta} \\ \qquad y_{1jt} = \left[\sum_{k=1}^{K+1} q_{1jkt}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \end{cases}$$

Using the same reasoning as in question **3.**, one gets:

$$P_{1jK+1t} = \frac{\varepsilon(s_{1jK+1t})}{\varepsilon(s_{1jK+1t}) - 1} \frac{\tau W_{2t}}{z_{jK+1} A_{2t}} \tag{9}$$

This allows to compute the sectoral price index using (6) and the exported quantity q_{1jK+1t} using (5) and (7).
Now the lowest cost producer in country 2 enters market 1 through export if she makes a large enough operational profit to cover the fixed export cost:

$$\frac{1}{\varepsilon(s_{1jK+1t}) - 1} \frac{\tau W_{2t}}{z_{jK+1} A_{2t}} q_{1jK+1t} \geq W_{2t} F \tag{10}$$

Moreover, if her aggregate profit is strictly positive, the second lowest cost producer is likely to enter market 1 as well. Iterating over firms gives a set of equilibrium prices P_{ijkt} and a number of foreign firms supplying the domestic market in sector j, given fixed aggregate prices, wages, and quantities.

5. Discuss the model's predictions in terms of pricing-to-market by individual firms.

As shown in the previous two questions, optimal prices at the firm-level are of the following form:

$$P_{ijkt} = \frac{\varepsilon(s_{ijkt})}{\varepsilon(s_{ijkt}) - 1} w_{ijkt}$$

where w_{ijkt} is firm k's marginal cost when selling in country i. Totally differenciating this equation gives:

$$\dot{P}_{ijkt} = \frac{-1}{\varepsilon(s_{ijk}) - 1} \frac{d\varepsilon(s_{ijkt})/\varepsilon(s_{ijkt})}{ds_{ijkt}/s_{ijkt}} \dot{s}_{ijkt} + \dot{w}_{ijkt} = \Gamma(s_{ijkt}) \dot{s}_{ijkt} + \dot{w}_{ijkt}$$

where $\Gamma(s_{ijkt})$ is the elasticity of the markup with respect to market share and $\dot{s}_{ijkt} = (1-\rho)(\dot{P}_{ijkt} - \dot{P}_{ijt})$. In the model:

$$\Gamma(s_{ijkt}) = \frac{s_{ijkt}\left(\frac{1}{\eta} - \frac{1}{\rho}\right)}{1 - \frac{1}{\rho}(1 - s_{ijkt}) - \frac{1}{\eta} s_{ijkt}}$$

which is increasing and convex in s_{ijkt}.

Table 1: Impact of a 1% shock on relative production costs

	Complete Model	Constant Markups	Frictionless Trade
PPI-based RER (decomposition %)			
Terms-of-trade, country 1	53.4%	100%	100%
PPI/Export price, country 1	23.1%	0%	0%
Export price/PPI, country 2	23.6%	0%	0%
PPI, country 1	0.86%	1%	0.76%
Export price, country 1	0.69%	1%	0.76%
Import price, country 1	0.31%	0%	0.23%
PPI, country 2	0.14%	0%	0.23%
CPI-RER/PPI-RER	82.3%	66.9%	0%

Source: Atkeson & Burstein (2008). Benchmark calibration.

Now, consider a firm based in country 1, facing an aggregate productivity shock dA_{1t}. The change in the ratio between its export and its domestic price is:

$$\dot{P}_{2jkt} - \dot{P}_{1jkt} = \Gamma(s_{2jkt})\dot{s}_{2jkt} - \Gamma(s_{1jkt})\dot{s}_{1jkt}$$

There is pricing-to-market when the change in the markup in export prices differs from the change in the markup in domestic prices. Since the change in the markup in each market is the product of the elasticity of the markup with respect to market share and the change in market shares, this pricing-to-market arises when the elasticity of the markup varies with the firm's market share and the firm has different market shares at home and abroad, and/or when these home and export shares respond differently to a shock to aggregate costs.

6. Comment simulations of Table 1. Is the model able to reproduce the stylized facts discussed in the exercise's introduction ?

The first row of Table 1 shows that the benchmark model produces a movement in the terms of trade for country 1 that is only 53% as large as the movement in the PPI-based RER. The benchmark model thus reproduces the first fact: the terms of trade are significantly less volatile than the PPI-based RER. In the model, this arises as a result of large deviations from relative PPP at the aggregate level. Column 1, rows 4, 5, 6, and 7, give the movements in the producer price index, the export price index, and the import price index for countries 1 and 2 in response to the one percent increase in country 1's aggregate marginal costs. There we see that producer prices in country 1 rise by more than export prices. Note in rows 6 and 7 that the producer price index in country 2 and the import price index in country 1 also rise, with the latter rising more than the former. This is true despite the fact that there has been no change in costs in country 2. Thus, the model generates a positive correlation in the movements in PPI_1/EPI_1 and IPI_1/PPI_2 with PPI_1/PPI_2, as in the data.
Looking at the entries in Table 1 for the corresponding price movements for the constant markup and frictionless trade versions of the model shows that both variable markups and trade frictions are needed to deliver these implications for the terms of trade and PPI-based RER. In both of these alternative versions of our model, the movement in the terms of trade is identical to the movement in the PPI-based RER, and the ratio of export prices to producer prices is constant in both countries

because relative PPP holds. In the constant markup version of the model, the logic for this result is quite simple: for each firm, both domestic and export prices move one-for-one with the movement in domestic costs. Hence relative PPP holds good by good. Note that since all prices charged by country 1 firms change by the same amount, and the change in the export price index is a weighted average of individual exporters' price changes, changes in firms' export participation have no impact in the change in the export price index. Thus, relative PPP holds in the aggregate as well. For the frictionless trade version of the model, the logic for this result is more subtle. In this version of the model, markups are not constant but vary with market share. Thus, it is no more the case that changes in cost are passed on fully to prices. In fact, as is shown in column 2, rows 4 and 5 of the table, there is incomplete pass-through as the firms in the country with rising wages lose market share and hence reduce their markups at home and abroad, while we see in rows 6 and 7 that the firms in country 2 with the constant costs increase their prices for domestic sales and exports. With no trade frictions, however, the set of firms and their costs competing in each sector is the same across countries, and this leads to relative PPP for each good despite imperfect competition. More specifically, each firm in a sector has the same cost for sales in each country, and hence each firm has identical market shares, identical markups, and identical prices in each country. This implies that, for each country, export prices remain constant relative to domestic producer prices and thus changes in the terms of trade are identical to changes in the PPI-based RER. Hence one can say that in the frictionless trade version of the model, there is incomplete pass-through of costs to prices, but no pricing-to-market. International trade costs are not necessary for incomplete pass-through, but they are necessary for pricing-to-market.

Now, turn to the model's implications for movements in the relative price of goods across countries when these prices are measured with consumer prices rather than producer prices. Column 1, row 8 of Table 1 shows that the benchmark model produces a movement in the CPI-based RER across countries that is 82% as large as the movement in the PPI-based RER. This finding in the benchmark model that the movement in the relative consumer price across countries is quite large stands in stark contrast to the implications of the frictionless trade version of our model. As shown in column 3, row 8 of Table 4, in the frictionless trade version of our model the CPI-based RER does not move at all. This is because, in the frictionless trade version of the model, relative PPP holds for each good and consumption baskets are identical across countries. Hence, the consumer price index is identical across countries. In this sense, the introduction of costs of international trade has a dramatic impact on the pricing implications of the model and moves the model much closer to the data not only in terms of its implications for traded quantities but also in terms of its implications for the CPI-based RER. Now consider the implications of the constant markup version of the model for movements in the CPI-based RER. In column 2, row 8 , we see that the movement in the CPI-based RER is 67% of the movement in the PPI-based RER. To understand these results, consider the following decomposition of CPI-changes in a symetric, two-country model with balanced trade:

$$
\begin{aligned}
&C\hat{P}I_1 \simeq P\hat{P}I_1 + s_M(I\hat{P}I_1 - E\hat{P}I_1)\\
&C\hat{P}I_2 \simeq P\hat{P}I_2 + s_M(E\hat{P}I_1 - I\hat{P}I_1)\\
\Leftrightarrow \quad &\frac{C\hat{P}I_1 - C\hat{P}I_2}{P\hat{P}I_1 - P\hat{P}I_2} \simeq 1 - 2s_M \frac{E\hat{P}I_1 - I\hat{P}I_1}{P\hat{P}I_1 - P\hat{P}I_2}
\end{aligned}
$$

Under frictionless trade, the import share is $s_M = 1/2$ and relative PPP holds at the aggregate level. Hence, movements in the CPI-based RER are as large as those of the PPI-based RER. Consider now the constant markup version of the model. In that model, all goods prices move one-for-one with movements in marginal costs. Thus, relative PPP holds and both the terms of trade for country 1 and the PPI-based RER move by the change in relative costs. The ratio of the percentage movement in the

CPI-based RER relative to the PPI-based RER is 67%, which follows from $s_M = 0.165$ (calibrated). Now consider the benchmark model with variable markups. There is pricing-to- market, which leads to a movement in the terms of trade in country 1 that is only 53% as large as the movement in the PPI-based RER. Using the previous decomposition with $s_M = 0.165$, the ratio of the movement in the CPI-based RER to that in the PPI-based RER is now 83%. The finding in the benchmark model that movements in the CPI-based RER are 83% as large as movements in the PPI-based RER is an improvement over the models with constant markups or frictionless trade, but still falls short of matching the US data, suggesting that movements in the CPI-based RER have roughly the same magnitudes as movements in the PPI-based RER. The solution proposed by Atkeson & Burstein is to extend the model to include non-tradeable distribution costs as a component of consumer prices.