Master EPP, International Macroeconomics
Lecture 4
Heterogenous firms and international trade

1 Endogenous mark-ups and firms' heterogeneity (Melitz & Ottaviano, 2006)

Motivation: In Melitz' seminar model of firms' heterogeneity, Dixit-Stiglitz preferences imply that mark-ups are homogenous across firms and destination markets. This does not allow reproducing empirical evidence linking the extent of trade barriers to the distribution of prices and markups across firms and the pro-competitive effects of trade liberalization.
Melitz and Ottaviano (2006) instead build a model of heterogenous firms with endogenous mark-ups. This improves the model's predictions in terms of relative prices and allows generating PPP deviations linked to cross-country differences in the toughness of competition.

Consider a two-country economy with L^H and L^F consumers in each country. Each consumer supplies one unit of labor. Preferences are defined over a continuum of differentiated varieties indexed by $i \in \Omega$, and a homogenous good chosen as numeraire. All consumers share the same utility function given by:

$$U = q_0^c + \alpha \int_{i\in\Omega} q_i^c di - \frac{1}{2}\gamma \int_{i\in\Omega} (q_i^c)^2 di - \frac{1}{2}\eta \left(\int_{i\in\Omega} q_i^c di \right)^2$$

where q_0^c and q_i^c represent the individual consumption levels of the numeraire good and each variety i. The demand parameters α, η and γ are all positive. The parameters α and η index the substitution pattern between the differentiated varieties and the numeraire: increases in α and decreases in η both shift out the demand for the differentiated varieties relative to the numeraire. The parameter γ indexes the degree of product differentiation between the varieties. In the limit when $\gamma = 0$, consumers only care about their consumption level over all varieties, $Q_c = \int_{i\in\Omega} q_i^c di$. The varieties are then perfect substitutes. The degree of product differentiation increases with γ as consumers give increasing weight to the distribution of consumption levels across varieties. The marginal utilities for all goods are bounded, and a consumer may thus not have positive demand for any particular good. We assume that consumers have positive demands for the numeraire good ($q_0^c > 0$).
Labor is the only factor of production and is inelastically supplied in a competitive market. The numeraire good is produced under constant returns to scale at unit cost; its market is also competitive and perfectly integrated at the international level. Entry in the differentiated product sector is costly as each firm incurs product development and production startup costs. Subsequent production exhibits constant returns to scale at marginal cost c (equal to unit labor requirement). Research and development yield uncertain outcomes for c, and firms learn about this cost level only after making the irreversible investment f_E required for entry. This is modeled as a draw from a common (and known) distribution $G(c)$ with support on $[0; c_M]$. To simplify, it is assumed that $1/c$ has a Pareto distribution, or:

$$G(c) = \left(\frac{c}{c_M} \right)^k, \quad k \geq 1$$

Finally, technology, referenced by the entry cost and cost distribution, is assumed identical in both countries. Since the entry cost is sunk, firms that can cover their marginal cost survive and produce.

All other firms exit the industry. Surviving firms maximize their profits using the residual demand function. In so doing, given the continuum of competitors, a firm takes the average price level $\bar{p}$ and number of firms N as given. Finally, markets are segmented in the differentiated product sector: exporting firms incur a per-unit trade cost $\tau^l, \quad l = H/F$.

1. Solve the consumer's program to get individual demand functions.

Optimal demands are obtained by maximizing the following program:

$$\begin{cases} \max_{q_0^c, q_i^c} q_0^c + \alpha \int_{i\in\Omega} q_i^c di - \frac{1}{2}\gamma \int_{i\in\Omega} (q_i^c)^2 di - \frac{1}{2}\eta \left(\int_{i\in\Omega} q_i^c di\right)^2 \\ s.t. \quad q_0^c + \int_{i\in\Omega} p_i q_i^c \leq E^c \end{cases}$$

where E^c is the individual nominal income.
The inverse demand for each variety i is then given by:

$$p_i = \alpha - \gamma q_i^c - \eta Q^c$$

whenever $q_i^c > 0$.

2. Let $\Omega^* \subset \Omega$ be the subset of varieties that are consumed ($q_i^c > 0$). Derive the market demand system for a given variety as a function of the number of consumers L, the number of produced varieties N, the price of the variety p_i and the average price of differentiated varieties $\bar{p} = \frac{1}{N}\int_{i\in\Omega^*} p_i di$. Under which condition is a good in the subset Ω^*?
The previous inverse demand can be inverted and multiplied by the number of consumers to get the market demand for any variety i:

$$q_i \equiv Lq_i^c = \frac{\alpha L}{\gamma} - \frac{\eta L}{\gamma} Q^c - \frac{L}{\gamma} p_i$$

Now, consider the total consumption of differentiated varieties:

$$\begin{aligned} Q^c \equiv \int_{i\in\Omega^*} q_i di &= \frac{N\alpha}{\gamma} - \frac{N\eta}{\gamma} Q^c - \frac{1}{\gamma}\int_{i\in\Omega^*} p_i di \\ \Rightarrow \quad Q^c &= \frac{N\alpha}{\gamma + N\eta} - \frac{N}{\gamma + N\eta}\bar{p} \end{aligned}$$

where N is the measure of consumed varieties in Ω^* and $\bar{p} = \frac{1}{N}\int_{i\in\Omega^*} p_i di$ is their average price.
Substituting this in q_i gives:

$$q_i = \frac{\alpha L}{\gamma + N\eta} - \frac{L}{\gamma} p_i + \frac{\eta N}{\gamma + N\eta}\frac{L}{\gamma}\bar{p}, \quad \forall i \in \Omega^*$$

The set Ω^* is the largest subset of Ω that satisfies $q_i \geq 0$ ie:

$$p_i \leq \frac{1}{\gamma + N\eta}(\gamma\alpha + \eta N \bar{p}) \equiv p_{max}$$

3. Discuss the implication of these preferences in terms of the price elasticity of demand.
The price elasticity of demand is defined as:

$$\varepsilon_i = \left|\frac{\partial q_i / q_i}{\partial p_i / p_i}\right| = \frac{1}{p_{max}/p_i - 1}$$

Contrary to the Dixit-Stglitz-Krugman case, the price elasticity of demand is not constant under these assumptions. Totally differentiating this expression gives:

$$\begin{aligned}
\frac{d\varepsilon_i}{\varepsilon_i} &= \frac{p_{max}}{p_{max}-p_i}\left(\frac{dp_i}{p_i}-\frac{p_{max}}{p_{max}}\right) \\
\frac{dp_{max}}{p_{max}} &= \frac{\alpha\gamma}{\alpha\gamma+\eta N\bar{p}}\frac{d\alpha}{\alpha}+\frac{\gamma^2\eta N(\alpha-\bar{p})}{(\alpha\gamma+\eta N\bar{p})(\eta N+\gamma)}\frac{d\gamma}{\gamma}+\frac{\eta N\bar{p}}{\alpha\gamma+\eta N\bar{p}}\frac{d\bar{p}}{\bar{p}}+ \\
&\quad \frac{\eta N\gamma(\bar{p}-\alpha)}{(\alpha\gamma+\eta N\bar{p})(\eta N+\gamma)}\left(\frac{dN}{N}-\frac{d\eta}{\eta}\right)
\end{aligned}$$

Note that the condition on the maximum price of varieties in the Ω^* subset implies that $\bar{p} \leq \alpha$. Thus, the price-elasticity of demand is:

- an increasing function of p_i: given the competitive environment (ie given N and $\bar{p}$), the price elasticity monotonically increases with the price p_i along the demand curve.
- an increasing function of η and a decreasing function of α ie the elasticity is smaller when consumers have a higher preference for the numeraire good.
- a decreasing function of γ, the differentiation degree (as in the CES case)
- a decreasing function of $\bar{p}$ and an increasing function of the number of competing varieties N. Both affect the price bound p_{max}

4. The model is first solved neglecting international trade. Solve the profit maximizing program of a firm that determines its production decisions. Show that, in optimum:

$$q_i(c) = \frac{L}{\gamma}[p_i(c) - c]$$

Once entered in the market, the firm maximizes its profit under the demand constraint:

$$\begin{cases} \max_{q_i(c)} [p_i(c)q_i(c) - wcq_i(c)] \\ s.t. \quad p_i(c) = \alpha - \eta Q - \frac{\gamma}{L}q_i(c) \end{cases}$$

The first order condition implies:

$$q_i(c) = \frac{L}{2\gamma}(\alpha - \eta Q^c - c)$$

or:

$$q_i(c) = \frac{L}{\gamma}[p_i(c) - c]$$

Remember that the profit maximizing price $p_i(c)$ may be above the price bound p_{max}, in which case the firm exits.

Let c_D reference the cost of the firm who is just indifferent about remaining in the industry. This firm earns zero profit as its price is driven down to its marginal cost, $p_i(c_D) = c_D = p_{max}$ and its demand level $q(c_D)$ is driven to zero. In the following, c_M is assumed high enough to be above c_D, so that some firms with cost draws between these two levels exit. All firms with cost $c < c_D$ earn positive profits (gross of the entry cost) and remain in the industry. The threshold cost c_D summarizes the effects of both the average price and number of firms on the performance measures of all firms.

5. Write the revenue, profit, and (absolute) markup of a firm as functions of its marginal cost c and the threshold c_D.

Given the inverse demand function, one can rewrite the maximum price in the market as: $p_{max} = c_D = \alpha - \eta Q$. The price of a firm with marginal cost c can thus be rewritten as (subscripts i are now ignored as a firm/variety can be identified by its productivity c):

$$\begin{aligned} p(c) &= \alpha - \eta Q - \frac{\gamma}{L} q_i \\ &= \alpha - \eta Q - \frac{1}{2}(\alpha - \eta Q - c) \\ &= \frac{1}{2}(c_D + c) \end{aligned}$$

The firm's markup (difference between its price and marginal cost) is thus:

$$\mu(c) \equiv p(c) - c = \frac{1}{2}(c_D - c)$$

The quantity produced is:

$$q(c) = \frac{L}{\gamma}[p(c) - c] = \frac{L}{2\gamma}(c_D - c)$$

. The firm's revenue:

$$r(c) \equiv p(c)q(c) = \frac{L}{4\gamma}(c_D^2 - c^2)$$

and its profit:

$$\pi(c) = \frac{L}{4\gamma}(c_D - c)^2$$

As expected, lower cost firms set lower prices and earn higher revenues and profits than firms with higher costs. However, lower cost firms do not pass on all of the cost differential to consumers in the form of lower prices: they also set higher markups (in both absolute and relative terms) than firms with higher costs.

6. Assume entry is free in the market and find the value of the cost cutoff c_D under the Pareto assumption. Discuss the impact of market size on firms' performances.

Prior to entry, the expected firm profit is $\int_0^{c_D} \pi(c)dG(c) - f_E$. If this profit were negative, no firms would enter the industry. As long as some firms produce, the expected profit is driven to zero by the unrestricted entry of new firms. Using the expression for profit derived in question **5**, this yields the equilibrium free entry condition

$$\frac{L}{4\gamma} \int_0^{c_D} (c_D - c)^2 dG(c) = f_E$$

which determines the cost cutoff c_D.

This cutoff, in turn, determines the number of surviving firms, since $c_D = p(c_D)$ must also be equal to the zero demand price threshold:

$$c_D = \frac{1}{\eta N + \gamma}(\gamma\alpha + \eta N \bar{p})$$

This yields the zero cutoff profit condition. To derive it, first note that the average price can be expressed as a function of the cost cutoff and the average cost of surviving firms $\bar{c} = [\int_0^{c_D} c dG(c)]/G(c_D)$:

$$\begin{aligned} \bar{p} &= \frac{1}{G(c_D)} \int_0^{c_D} \frac{1}{2}(c_D + c)dG(c) \\ &= \frac{1}{2}\left(c_D + \frac{1}{G(c_D)} \int_0^{c_D} c dG(c)\right) \\ \bar{p} &= \frac{1}{2}(c_D + \bar{c}) \end{aligned}$$

Finally, one gets the number of surviving firms:

$$c_D = \frac{1}{\eta N + \gamma}(\gamma\alpha + \frac{\eta N}{2}(c_D + \bar{c}))$$
$$\Leftrightarrow \quad N = \frac{2\gamma}{\eta}\frac{\alpha - c_D}{c_D - \bar{c}}$$

The number of entrants is then given by $N_E = N/G(c_D)$.

Given a production technology referenced by $G(c)$, average productivity will be higher (lower $\bar{c}$) when sunk costs are lower, when varieties are closer substitutes (lower γ), and in bigger markets (more consumers L). In all these cases, firm exit rates are also higher (the pre-entry probability of survival $G(c_D)$ is lower). The demand parameters α and η that index the overall level of demand for the differentiated varieties (relative to the numeraire) do not affect the selection of firms and industry productivity - they only affect the equilibrium number of firms. Competition is "tougher" in larger markets as more firms compete and average prices $\bar{p} = (c_D + \bar{c})/2$ are lower. A firm with cost c responds to this tougher competition by setting a lower markup (relative to the markup it would set in a smaller market).

Under the Pareto assumption, the cut-off can be found analytically:

$$\frac{L}{4\gamma}\int_0^{c_D}(c_D - c)^2 dG(c) = f_E$$
$$\Leftrightarrow \quad \frac{L}{4\gamma}\frac{k}{c_M^k}\int_0^{c_D}(c_D - c)^2 c^{k-1} dc = f_E$$
$$\Leftrightarrow \quad \left[\frac{c_D^{k+2}}{k} - \frac{2c_D^{k+2}}{k+1} + \frac{c_D^{k+2}}{k+2}\right] = \frac{4\gamma f_E c_M^k}{kL}$$
$$\Leftrightarrow \quad c_D = \left(\frac{2\gamma f_E c_M^k (k+1)(k+2)}{L}\right)^{\frac{1}{k+2}}$$

In the following, it is assumed that $c_M > \sqrt{(2(k+1)(k+2)\gamma f_E)/L}$ in order to ensure that $c_D < c_M$. This parametrization also yields simple derivations for the averages of all firm-level performance measures:

$$\bar{c} = \frac{1}{(c_D/c_M)^k}\int_0^{c_D} c\frac{k}{c_M}\left(\frac{c}{c_M}\right)^{k-1} dc = \frac{k}{k+1}c_D$$
$$N = \frac{2\gamma}{\eta}\frac{\alpha - c_D}{c_D - \frac{k}{k+1}c_D} = \frac{2\gamma(k+1)}{\eta}\frac{\alpha - c_D}{c_D}$$
$$\bar{p} = \frac{1}{2}(c_D + \bar{c}) = \frac{2k+1}{2k+2}c_D$$
$$\bar{\mu} = \frac{1}{2(c_D/c_M)^k}\int_0^{c_D}(c_D - c)\frac{k}{c_M}\left(\frac{c}{c_M}\right)^{k-1} dc = \frac{c_D}{2(k+1)}$$
$$\bar{q} = \frac{L}{2\gamma(c_D/c_M)^k}\int_0^{c_D}(c_D - c)\frac{k}{c_M}\left(\frac{c}{c_M}\right)^{k-1} dc = \frac{Lc_D}{2\gamma(k+1)}$$
$$\bar{r} = \frac{L}{4\gamma(c_D/c_M)^k}\int_0^{c_D}(c_D^2 - c^2)\frac{k}{c_M}\left(\frac{c}{c_M}\right)^{k-1} dc = \frac{Lc_D^2}{2\gamma(k+2)}$$
$$\bar{\pi} = \frac{L}{4\gamma(c_D/c_M)^k}\int_0^{c_D}(c_D - c)^2\frac{k}{c_M}\left(\frac{c}{c_M}\right)^{k-1} dc = \frac{Lc_D^2}{2\gamma(k+1)(k+2)}$$

We previously mentioned that bigger markets induced tougher selection (lower cutoff c_D), leading to higher average productivity (lower $\bar{c}$) and lower average prices. In addition, under the assumed parametrization of cost draws, average firm size (both in terms of output and sales) and profits are higher in larger markets: the direct market size effect outweighs its indirect effect through lower prices and markups. Similarly, average markups are lower as the direct effect of increased competition on firm-level markups ($\mu(c)$ shifts down) outweighs the selection effect on firms with lower cost (and relatively higher markups). Moreover, average profits and sales increase by the same proportion when market size increases. Thus, average industry profitability $\bar{\pi}/\bar{r}$ does not vary with market size.

7. We now turn on the open economy version of the model. Let $p_D^l(c)$ and $q_D^l(c)$ represent the domestic levels of the profit maximizing price and quantity sold for a firm producing in country l with cost c. Such a firm may also decide to produce some output $q_X^l(c)$ that it exports at a delivered price $p_X^l(c)$. Since the markets are segmented and firms produce under constant returns to scale, they independently maximize the profits earned from domestic and exports sales: $\pi_D^l(c) = [p_D^l(c) - c]q_D^l(c)$ and $\pi_X^l(c) = [p_X^l(c) - \tau^h c]q_X^l(c)$ (where $h \neq l$). Using the same reasoning as in the closed economy case, define the cost cut-offs for producing in the domestic market and for exporting.

Analogously to the closed economy case, the profit maximizing prices and output levels must satisfy: $q_D^l(c) = \frac{L^l}{\gamma}[p_D^l(c) - c]$ and $q_X^l(c) = \frac{L^h}{\gamma}[p_X^l(c) - \tau^h c]$ and prices are bounded by the following price threshold:

$$p_{max}^l = \frac{1}{\eta N^l + \gamma}(\gamma\alpha + \eta N^l \bar{p}^l)$$

where n^l is the total number of (domestic or foreign) firms selling in country l and $\bar{p}^l$ the average price. As was the case in the closed economy, only firms earning non-negative profits in a market (domestic or export) will choose to sell in that market. This leads to similar cost cutoff rules for firms selling in either market. Let c_D^l denote the upper bound cost for firms selling in their domestic market, and let c_X^l denote the upper bound cost for exporters from l to h. These cutoffs must satisfy:

$$\begin{aligned} c_D^l &= sup\{c : \pi_D^l(c) > 0\} = p_{max}^l \\ c_X^l &= sup\{c : \pi_X^l(c) > 0\} = \frac{p_{max}^h}{\tau^h} \end{aligned}$$

As was the case in the closed economy, the cutoffs summarize all the effects of market conditions relevant for firm performance. In particular, the optimal prices and output levels can be written as functions of the cutoffs:

$$\begin{aligned} p_D^l(c) &= \frac{1}{2}(c_D^l + c) \\ p_X^l(c) &= \frac{\tau^h}{2}(c_X^l + c) \\ q_D^l(c) &= \frac{L^l}{2\gamma}(c_D^l - c) \\ q_X^l(c) &= \frac{L^h}{2\gamma}\tau^h(c_X^l - c) \\ \pi_D^l(c) &= \frac{L^l}{4\gamma}(c_D^l - c)^2 \\ \pi_X^l(c) &= \frac{L^h}{4\gamma}(\tau^h)^2(c_X^h - c)^2 \end{aligned}$$

8. Define the free entry condition in the open economy and derive the cutoffs.

Entry is unrestricted in both countries. Firms choose a production location prior to entry and paying the sunk entry cost. Free entry of domestic firms in country l implies zero expected profits in equilibrium, hence:

$$\int_0^{c_D^l} \pi_D^l(c)dG(c) + \int_0^{c_X^l} \pi_X^l(c)dG(c) = f_E$$

Under the Pareto assumption, the free-entry condition can be re-written:

$$L^l\, {c_D^l}^{k+2} + L^h\, {\tau^h}^2\, {c_X^l}^{k+2} = 2\gamma c_M^k(k+1)(k+2)f_E$$

where $\phi = 2(k+1)(k+2)c_M^k f_E$ is a technology index that combines the effects of better distribution of cost draws (lower c_M) and lower entry costs f_E. This free entry condition will hold so long as there is a positive mass of domestic entrants $N_E^l > 0$ in country l.

Since $c_X^h = c_D^l/\tau^l$, the free entry condition can be re-written

$$L^l\, {c_D^l}^{k+2} + L^h \rho^h\, {c_D^h}^{k+2} = \gamma\phi$$

where $\rho^l \equiv {\tau^l}^{-k} \in [0,1]$ is an inverse measure of trade costs (the "freeness" of trade). This system (for $l = H; F$) can then be solved for the cutoffs in both countries:

$$c_D^l = \left[\frac{\phi\gamma(1-\rho^h)}{L^l(1-\rho^h\rho^l)}\right]^{\frac{1}{k+2}}$$

9. Find the number of firms selling in each market and the number of entrants.

The prices in country l reflect both the domestic prices of country-l firms, $p_D^l(c)$, and the prices of exporters from h, $p_X^h(c)$. These prices can be written:

$$\begin{aligned} p_D^l(c) &= \frac{1}{2}(p_{max}^l + c), \quad c \in [0, c_D^l] \\ p_X^h(c) &= \frac{1}{2}(p_{max}^l + \tau^l c), \quad c \in [0; c_D^l/\tau^l] \end{aligned}$$

where p_{max}^l is the price threshold defined by $p_{max}^l = \frac{\gamma\alpha+\eta N^l\bar{p}^l}{\eta N^l+\gamma}$. In addition, the cost of domestic firms $c \in [0; c_D^l]$ and the delivered cost of exporters $\tau^l c \in [0; c_D^l]$ have identical distributions over this support, given by $G^l(c) = (c/c_D^l)^k$. The price distribution in country l of domestic firms producing in l, $p_D^l(c)$, and exporters producing in h, $p_X^h(c)$, are therefore also identical. The average price in country l is thus given by $\bar{p}^l = \frac{2k+1}{2k+2}c_D^l$.

Combining this with the threshold price determines the number of firms selling in country l:

$$N^l = \frac{2(k+1)\gamma}{\eta}\frac{\alpha - c_D^l}{c_D^l}$$

These results for product variety and average prices are identical to the closed economy case. This is driven by the matching price distributions of domestic firms and exporters in that market.

The number of entrants in each market is then obtained using the following identity (holding for each country):

$$G(c_D^l)N_E^l + G(c_X^h)N_E^h = N^l$$

Combining both equations together gives:

$$\begin{aligned} N_E^l &= \frac{c_M^k}{1-\rho^l\rho^h}\left[\frac{N^l}{{c_D^l}^k}-\rho^l\frac{N^h}{{c_D^h}^k}\right] \\ &= \frac{c_M^k}{1-\rho^l\rho^h}\frac{2(k+1)\gamma}{\eta}\left[\frac{\alpha-c_D^l}{{c_D^l}^{k+1}}-\rho^l\frac{\alpha-c_D^h}{{c_D^h}^{k+1}}\right] \end{aligned}$$

10. In a non-specialized equilibrium the condition $N_E^l > 0$ implies that $c_X^l < c_D^l$, so that only a subset of relatively more productive firms export. Comment the implication of this finding in terms of price discrimination.

Given the optimal price functions $p_D^l(c) = (c_D^l + c)/2$ and $p_X^l(c) = \tau^h(c_X^l + c)/2$, $c_X^l < c_D^l$ implies that $p_X^l(c) = /\tau^h < p_D^l(c), \forall c \geq c_X^l$. Therefore, all exporters set F.O.B. export prices (net of incurred trade costs) strictly below their prices in the domestic market. As described by Feenstra et al (2001), dumping behavior is closely linked to arbitrage conditions for the re-sale of goods across markets. This same link holds in our model, where dumping by exporters from country l ($p_X^l(c)/\tau^h < p_D^l(c)$) is equivalent to a no-arbitrage condition precluding the pro.table export resale by a third party of a good produced and sold in country l. The dumping condition also precludes pro.table resale of a good exported to country l, back in its origin country h ($p_D^h(c)\tau^h < p_X^h(c)$).

11. Compare the outcome of the closed and the open economy model to discuss the impact of trade.

In the open economy model, the distribution of the exporters' delivered cost $\tau^l c$ to country l matches the distribution of the domestic firms'cost c in country l. This leads to matching price distributions for both domestic firms in a country, and exporters to that country. This argument extends to the distribution of all the other firm-level variables (markups, output, revenue, and profit). Thus, the distribution of all these firm performance measures in the open economy equilibrium are identical to those in a closed economy case with a matching cost cutoff c_D. When analyzing the impact of trade, we can therefore focus on the determination of the cost cutoff. In the closed economy, the cut-off is given by:

$$c_D^l = \left[\frac{\gamma\phi}{L^l}\right]^{\frac{1}{k+2}}$$

while in the open economy:

$$c_D^l = \left[\frac{\gamma\phi}{L^l}\frac{1-\rho^h}{1-\rho^l\rho^h}\right]^{\frac{1}{k+2}}$$

The cost cutoff is lower in the open economy: trade increases aggregate productivity by forcing the least productive firms to exit. This effect is similar to that analyzed in Melitz (2003) but works through a different economic channel. In Melitz (2003), trade induces increased competition for scarce labor resources as real wages are bid up by the relatively more productive firms who expand production to serve the export markets. The increase in real wages forces the least productive firms to exit. In that model, import competition does not play a role in the reallocation process due to the C.E.S. specification for demand (residual demand price elasticities are exogenously fixed and unaffected by import competition). In the current model, the impact of these two channels - via increased factor market or product market competition - is reversed: increased product market competition is the only operative channel. Increased factor market competition plays no role in the current model, as the supply of labor to the differentiated goods sector is perfectly elastic. On the other hand, import competition increases competition in the domestic product market, shifting up residual demand price elasticities for all firms at any given demand level. This forces the least productive firms to exit. This effect is very similar to an increase in market size in the closed economy: the increased competition induces a downward shift in the distribution of markups across firms. Although only relatively more productive firms survive (with higher markups than the less productive firms who exit), the average markup is reduced. The distribution of prices shifts down due to the combined effect of selection and lower markups. Again, as in the case of larger market size in a closed economy, average firm size and profits increase - as does product variety. In this model, welfare gains from trade thus come from a combination of productivity gains (via selection), lower markups (pro-competitive effect), and increased product variety.

12. Discuss the impact of market size differences on firm performance measures.

Once again, cross-country differences in firm performance measures will be determined by the differences in the cost cutoffs c_D^l. This immediately highlights how costly trade does not completely integrate markets as respective country size plays an important role in determining all firm performance measures: When trade costs are symmetric ($\rho^l = \rho^h$), the larger country will have a lower cutoff, and thus higher average productivity and product variety, along with lower markups and prices (relative to the smaller country). Moreover, the larger country will attract relatively more entrants and local producers. In short, all of the size-induced differences across countries in autarky persist (although not to the same extent). It is in this sense that costly trade does not completely integrate markets. Surprisingly, the expression for the cost cut-off also indicates that the size of a country's trading partner does not affect the cost cutoff (and hence all firm performance measures). This highlights some important offsetting effects of trading partner size - although the exact outcome of these trade-offs are naturally influenced by the functional form assumptions. On the export side, a larger trading partner represents increased export market opportunities. However, this increased export market size is offset by its increased competitiveness (a greater number of more productive firms are competing in that market, driving down markups). On the import side, a larger trading partner represents an increased level of import competition. In the long run, this is offset by a smaller proportion of entrants, and hence less competition in the smaller market.

13. What is the impact of trade liberalization i) if trade costs are symmetric ($\tau^H = \tau^F = \tau$) and the liberalization is bilateral ? ii) if the liberalization is asymmetric ?

When trade costs are symmetric, the cost cut-off is:

$$c_D^l = \left[\frac{\gamma\phi}{L^l} \frac{1}{1+\rho} \right]^{\frac{1}{k+2}}$$

Bilateral liberalization ($\downarrow \tau$, $\uparrow \rho$) thus increases competition in both markets, leading to proportional changes in the cutoffs (and hence proportional increases in aggregate productivity) in both countries. The effects of such liberalization are thus qualitatively identical to those described for the transition from autarky to the open economy: Product variety increases as a result of the increased competition, which also induces a decrease in markups and prices. Again, welfare rises from a combination of higher productivity, lower markups, and increased product variety.

In the case of a unilateral liberalization by country l (an increase in ρ^l, holding ρ^h constant), the cost cutoff c_D^l increases (less competition in the liberalizing country) whereas the cutoff c_D^h in the country's trading partner decreases, indicating an increase in competition there. These results are driven by the change in firm location induced by entry in the long run. The number of entrants N_E^l in the liberalizing country decreases, while the number of entrants in the other country, N_E^h, increases.

14. Show that this model allows deriving a gravity equation.

Export sales of an individual firm from country h selling good in country l are defined as:

$$r_X^{hl}(c) = p_X^{hl}(c) q_X^{hl}(c) = \frac{L^l}{4\gamma}({c_D^{lh}}^2 - {\tau^{lh}}^2 c^2)$$

Aggregating over all exporters yiels the aggregate bilateral exports from h to l:

$$Exp^{hl} = N_E^h \int_0^{c_X^{hl}} r_X^{hl}(c) dG(c) = \frac{N_e^h L^l \, {c_D^{hl}}^{k+2} \, {\tau^{hl}}^{-k}}{2\gamma c_M^k (k+2)}$$

This gravity equation determines bilateral exports as a log-linear function of bilateral trade barriers and country characteristics. It reflects the joint effects of country size, technology (comparative advantage), and geography on both the extensive (number of traded goods) and intensive (amount traded per good) margins of trade flows. Similarly, it highlights how -holding the importing country size fixed - tougher competition in that country (lower average prices, reflected by a lower c_D^l) dampens exports by making it harder for potential exporters to break into that market.