

**Master EPP, International Macroeconomics**  
**Lecture 4**  
**Heterogenous firms and international trade**

## 1 Endogenous mark-ups and firms' heterogeneity (Melitz & Ottaviano, 2006)

*Motivation: In Melitz' seminar model of firms' heterogeneity, Dixit-Stiglitz preferences imply that mark-ups are homogenous across firms and destination markets. This does not allow reproducing empirical evidence linking the extent of trade barriers to the distribution of prices and markups across firms and the pro-competitive effects of trade liberalization.*

*Melitz and Ottaviano (2006) instead build a model of heterogenous firms with endogenous mark-ups. This improves the model's predictions in terms of relative prices and allows generating PPP deviations linked to cross-country differences in the toughness of competition.*

Consider a two-country economy with  $L^H$  and  $L^F$  consumers in each country. Each consumer supplies one unit of labor. Preferences are defined over a continuum of differentiated varieties indexed by  $i \in \Omega$ , and a homogenous good chosen as numeraire. All consumers share the same utility function given by:

$$U = q_0^c + \alpha \int_{i \in \Omega} q_i^c di - \frac{1}{2} \gamma \int_{i \in \Omega} (q_i^c)^2 di - \frac{1}{2} \eta \left( \int_{i \in \Omega} q_i^c di \right)^2$$

where  $q_0^c$  and  $q_i^c$  represent the individual consumption levels of the numeraire good and each variety  $i$ . The demand parameters  $\alpha$ ,  $\eta$  and  $\gamma$  are all positive. The parameters  $\alpha$  and  $\eta$  index the substitution pattern between the differentiated varieties and the numeraire: increases in  $\alpha$  and decreases in  $\eta$  both shift out the demand for the differentiated varieties relative to the numeraire. The parameter  $\gamma$  indexes the degree of product differentiation between the varieties. In the limit when  $\gamma = 0$ , consumers only care about their consumption level over all varieties,  $Q_c = \int_{i \in \Omega} q_i^c di$ . The varieties are then perfect substitutes. The degree of product differentiation increases with  $\gamma$  as consumers give increasing weight to the distribution of consumption levels across varieties. The marginal utilities for all goods are bounded, and a consumer may thus not have positive demand for any particular good. We assume that consumers have positive demands for the numeraire good ( $q_0^c > 0$ ).

Labor is the only factor of production and is inelastically supplied in a competitive market. The numeraire good is produced under constant returns to scale at unit cost; its market is also competitive and perfectly integrated at the international level. Entry in the differentiated product sector is costly as each firm incurs product development and production startup costs. Subsequent production exhibits constant returns to scale at marginal cost  $c$  (equal to unit labor requirement). Research and development yield uncertain outcomes for  $c$ , and firms learn about this cost level only after making the irreversible investment  $f_E$  required for entry. This is modeled as a draw from a common (and known) distribution  $G(c)$  with support on  $[0; c_M]$ . To simplify, it is assumed that  $1/c$  has a Pareto distribution, or:

$$G(c) = \left( \frac{c}{c_M} \right)^k, \quad k \geq 1$$

Finally, technology, referenced by the entry cost and cost distribution, is assumed identical in both countries. Since the entry cost is sunk, firms that can cover their marginal cost survive and produce.

All other firms exit the industry. Surviving firms maximize their profits using the residual demand function. In so doing, given the continuum of competitors, a firm takes the average price level  $\bar{p}$  and number of firms  $N$  as given. Finally, markets are segmented in the differentiated product sector: exporting firms incur a per-unit trade cost  $\tau^l$ ,  $l = H/F$ .

1. Solve the consumer's program to get individual demand functions.
2. Let  $\Omega^* \subset \Omega$  be the subset of varieties that are consumed ( $q_i^c > 0$ ). Derive the market demand system for a given variety as a function of the number of consumers  $L$ , the number of produced varieties  $N$ , the price of the variety  $p_i$  and the average price of differentiated varieties  $\bar{p} = \frac{1}{N} \int_{i \in \Omega^*} p_i di$ . Under which condition is a good in the subset  $\Omega^*$ ?
3. Discuss the implication of these preferences in terms of the price elasticity of demand.
4. The model is first solved neglecting international trade. Solve the profit maximizing program of a firm that determines its production decisions. Show that, in optimum:

$$q_i(c) = \frac{L}{\gamma} [p_i(c) - c]$$

Let  $c_D$  reference the cost of the firm who is just indifferent about remaining in the industry. This firm earns zero profit as its price is driven down to its marginal cost,  $p_i(c_D) = c_D = p_{max}$  and its demand level  $q(c_D)$  is driven to zero. In the following,  $c_M$  is assumed high enough to be above  $c_D$ , so that some firms with cost draws between these two levels exit. All firms with cost  $c < c_D$  earn positive profits (gross of the entry cost) and remain in the industry. The threshold cost  $c_D$  summarizes the effects of both the average price and number of firms on the performance measures of all firms.

5. Write the revenue, profit, and (absolute) markup of a firm as functions of its marginal cost  $c$  and the threshold  $c_D$ .
6. Assume entry is free in the market and find the value of the cost cutoff  $c_D$  under the Pareto assumption. Discuss the impact of market size on firms' performances.
7. We now turn on the open economy version of the model. Let  $p_D^l(c)$  and  $q_D^l(c)$  represent the domestic levels of the profit maximizing price and quantity sold for a firm producing in country  $l$  with cost  $c$ . Such a firm may also decide to produce some output  $q_X^l(c)$  that it exports at a delivered price  $p_X^l(c)$ . Since the markets are segmented and firms produce under constant returns to scale, they independently maximize the profits earned from domestic and exports sales:  $\pi_D^l(c) = [p_D^l(c) - c]q_D^l(c)$  and  $\pi_X^l(c) = [p_X^l(c) - \tau^h c]q_X^l(c)$  (where  $h \neq l$ ). Using the same reasoning as in the closed economy case, define the cost cutoffs for producing in the domestic market and for exporting.
8. Define the free entry condition in the open economy and find the cutoff values.
9. Find the number of firms selling in each market and the number of entrants.
10. In a non-specialized equilibrium the condition  $N_E^l > 0$  implies that  $c_X^l < c_D^l$ , so that only a subset of relatively more productive firms export. Comment the implication of this finding in terms of price discrimination.
11. Compare the outcome of the closed and the open economy model to discuss the impact of trade.
12. Discuss the impact of market size differences on firm performance measures.
13. What is the impact of trade liberalization i) if trade costs are symmetric ( $\tau^H = \tau^F = \tau$ ) and the liberalization is bilateral ? ii) if the liberalization is asymmetric ?
14. Show that this model allows deriving a gravity equation.