Master EPP, International Macroeconomics
Lecture 2
Real Exchange Rates, Balassa-Samuelson and Purchasing Power Parity

1 A Balassa-Samuelson model with non-traded inputs

Consider a variant of the standard Balassa-Samuelson model. There are two countries in the economy, Home and Foreign (in the following, foreign variables are indexed with a star *). Capital is perfectly mobile across sectors and countries. Labor is free to migrate between sectors of an economy but not between countries.
Each economy produces two composite goods, a traded good T and a non-traded good N. The production of T can be either consumed domestically or exported without any trade cost. N products are either consumed by the domestic household or used in the production of the T good. Outputs are given by the following constant-returns production functions:

$$\begin{aligned} Y_T &= A_T F(K_T, L_T, N_T) = A_T K_T^{\alpha} N_T^{\beta} L_T^{1-\alpha-\beta} \\ Y_N &= A_N G(K_N, L_N) = A_N K_N^{\gamma} L_N^{1-\gamma} \end{aligned}$$

where subscripts T denotes the traded good sector, subscript N the non-traded good sector and A a productivity shifter. N_T is the quantity of non-traded inputs incorporated in the production of traded goods. In the following, the traded good is taken as numeraire.
In each country, the total labor supply is fixed at L. The equalization of labor supply and demand ($L_N + L_T$) determines the equilibrium wage w (which is the same in both sectors when labor is mobile). Because capital is internationally mobile, there is a single world rate of return r, taken as exogenous. Capital accumulation takes place at the firm-level and there is no depreciation. Capital must be put in place a period before it is actually used.

1. Write and resolve the maximization problems of representative firms producing traded and non traded goods.

In each sector, firms' present-value profits measured in units of the numéraire are:

$$\begin{aligned} \Pi_{Tt} &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} [A_{Ts} F(K_{Ts}, L_{Ts}, N_{Ts}) - w_s L_{Ts} - \Delta K_{Ts+1} - P_{Ns} N_{Ts}] \\ \Pi_{Nt} &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} [P_{Ns} A_{Ns} G(K_{Ns}, L_{Ns}) - w_s L_{Ns} - \Delta K_{Ns+1}] \end{aligned}$$

The first-order conditions imply:

$$A_{Ts} F'_K(K_{Ts}, L_{Ts}, N_{Ts}) = r \tag{1}$$
$$A_{Ts} F'_L(K_{Ts}, L_{Ts}, N_{Ts}) = w_s \tag{2}$$
$$A_{Ts} F'_N(K_{Ts}, L_{Ts}, N_{Ts}) = P_{Ns} \tag{3}$$
$$P_{Ns} A_{Ns} G'_K(K_{Ns}, L_{Ns}) = r \tag{4}$$
$$P_{Ns} A_{Ns} G'_L(K_{Ns}, L_{Ns}) = w_s \tag{5}$$

or in intensive terms:

$$A_{Ts}f'_k(k_{Ts},n_{Ts}) = r \tag{6}$$
$$A_{Ts}[f(k_{Ts},n_{Ts}) - f'_k(k_{Ts},n_{Ts})k_{Ts} - f'_n(k_{Ts},n_{Ts})n_{Ts}] = w_s \tag{7}$$
$$A_{Ts}f'_n(k_{Ts},n_{Ts}) = P_{Ns} \tag{8}$$
$$P_{Ns}A_{Ns}g'(k_{Ns}) = r \tag{9}$$
$$P_{Ns}A_{Ns}[g(k_{Ns}) - g'(k_{Ns})k_{Ns}] = w_s \tag{10}$$

where:

$$f(k_{Ts},n_{Ts}) = \frac{F(K_{Ts},L_{Ts},N_{Ts})}{L_{Ts}} = F\left(\frac{K_{Ts}}{L_{Ts}},1,\frac{N_{Ts}}{L_{Ts}}\right)$$
$$g(k_{Ns}) = \frac{G(K_{Ns},L_{Ns})}{L_{Ns}} = G\left(\frac{K_{Ns}}{L_{Ns}},1\right)$$

2. Use the first-order conditions to solve for the equilibrium wage. Comparison with the standard Balassa-Samuelson model.

Equations (5)-(10) define a system of 5 equations in 5 unknowns: k_{Ts}, n_{Ts}, P_{Ns}, w_s and k_{Ns}. As a consequence, one can determine the equilibrium price of the non-traded good without any reference to the demand-side. First, use (9) and (10) to write P_{Ns} as a function of the equilibrium wage:

$$(9) \quad \Rightarrow \quad k_{Ns} = \frac{\gamma}{1-\gamma}\frac{w_s}{r}$$
$$(10) \quad \Rightarrow \quad P_{Ns} = \frac{r^{\gamma}w_s^{1-\gamma}}{A_{Ns}}\gamma^{-\gamma}(1-\gamma)^{\gamma-1}$$

Then, use (6)-(8) and the previous definition of the non-traded good price to solve for the equilibrium wage:

$$(8) \quad \Rightarrow \quad n_{Ts} = \left[A_{Ts}A_{Ns}r^{-\gamma}w^{\gamma-1}\gamma^{\gamma}(1-\gamma)^{1-\gamma}\beta k_{Ts}^{\alpha}\right]^{\frac{1}{1-\beta}}$$
$$(6) \quad \Rightarrow \quad r = A_{Ts}\alpha k_{Ts}^{\alpha-1}\left[\frac{A_{Ts}}{A_{Ns}}r^{-\gamma}w_s^{\gamma-1}\gamma^{\gamma}(1-\gamma)^{1-\gamma}\beta k_{Ts}^{\alpha}\right]^{\frac{\beta}{1-\beta}}$$
$$(7) \quad \Rightarrow \quad w_s = (1-\alpha-\beta)A_{Ts}k_{Ts}^{\alpha}\left[\frac{A_{Ts}}{A_{Ns}}r^{-\gamma}w_s^{\gamma-1}\gamma^{\gamma}(1-\gamma)^{1-\gamma}\beta k_{Ts}^{\alpha}\right]^{\frac{\beta}{1-\beta}}$$

The last two equations form a system of two equations in two unknowns w_s and k_{Ts}. Jointly, they thus allow to solve for the equilibrium wage:

$$k_{Ts} = \mathbf{A}w_s^{\frac{\beta(\gamma-1)}{1-\alpha-\beta}} \tag{11}$$
$$w_s = \mathbf{B}k_{Ts}^{\frac{\alpha}{1-\beta\gamma}} \tag{12}$$

with:

$$\mathbf{A} = \left[A_{Ts}A_{Ns}^{\beta}\alpha^{1-\beta}\gamma^{\gamma\beta}(1-\gamma)^{\beta(1-\gamma)}\beta^{\beta}r^{-1+\beta-\gamma\beta}\right]^{\frac{1}{1-\alpha-\beta}}$$
$$\mathbf{B} = \left[A_{Ts}A_{Ns}^{\beta}r^{-\gamma\beta}\gamma^{\gamma\beta}(1-\gamma)^{\beta(1-\gamma)}\beta^{\beta}(1-\alpha-\beta)^{1-\beta}\right]^{\frac{1}{1-\beta\gamma}}$$

In the (w_s, k_{Ts}) plan, equation (11) defines a decreasing relation while (12) is increasing. This implicitely gives the equilibrium wage. First-differenciating it

$$\frac{dw_s}{w_s} = \frac{1}{1-\beta\gamma-\alpha}\frac{dA_{Ts}}{A_{Ts}} + \frac{\beta}{1-\beta\gamma-\alpha}\frac{dA_{Ns}}{A_{Ns}} + \frac{-(\alpha+\gamma\beta)}{1-\beta\gamma-\alpha}\frac{dr}{r} \tag{13}$$

As in the standard BS model, w_s is increasing in A_{Ts} and decreasing in r. The rise in A_T and the attendant rise in k_T both push the marginal product of labor in tradeables. On the contrary, an increase in r is compensated by a drop in the capital intensity that tends to push w downward. The presence of non-traded inputs however increases the sensitivity of the equilibrium wage. Thus, a positive productivity shock in the traded good sector increases the demand for non-traded inputs and this exerces an additional upward pressure on the labor demand. Finally, the impact of a shock on the productivity of non-traded goods is positive as well, while it is null without non-traded inputs (when $\beta = 0$). The rise in A_N indeed increases the intensity in non-traded inputs. This tends to push the marginal product of labor upwards.

3. Use the equilibrium wage to solve for the price of non-traded goods.

Incorporating the equilibrium wage $w_s(A_{Ts}, A_{Ns}, r)$ on (9) and (10) allows to jointly solve for P_{Ns} and k_{Ns}:

$$\begin{aligned} P_{Ns} &= \frac{r}{\gamma A_{Ns}} k_{Ns}^{1-\gamma} \quad (MPK) \\ k_{Ns} &= \frac{\gamma}{1-\gamma}\frac{w_s}{r} \qquad (MPL) \end{aligned}$$

In the (k_{Ns}, P_{Ns}) plan, (MPK) defines an increasing function while (MPL) is decreasing. An increase in the price of N raises the marginal value product of capital and, given r the optimal capital intensity of production. Besides this, an increase in k_N raises the marginal physical product of labor. P_N must therefore fall to keep labor's marginal value product equal to the equilibrium wage in the traded good sector.

4. Discuss the impact of unanticipated productivity shocks in this setting.

To study the impact of unanticipated productivity shocks, take natural logarithms and differenciate the MPL and MPK relations:

$$\begin{aligned} \frac{dP_N}{P_N} &= -\frac{dA_N}{A_N} + (1-\gamma)\frac{dk_N}{k_N} \\ \frac{dk_N}{k_N} &= \frac{dw}{w} \end{aligned}$$

with dw/w given by equation (13). Solving for dP_N/P_N gives:

$$\frac{dP_N}{P_N} = \frac{1-\gamma}{1-\beta\gamma-\alpha}\frac{dA_T}{A_T} + \left[-1 + \frac{(1-\gamma)\beta}{1-\beta\gamma-\alpha}\right]\frac{dA_N}{A_N}$$

The impact of a productivity shock in the tradables sector is very similar as in the standard BS model. Namely, the increase in A_T pushes w upward. Accordingly, the MPL schedule shifts upward along the original MPK, giving higher values of P_N and k_N. With respect to the standard BS case, the wage reaction is amplified due to the presence of non-traded inputs in the production of T and the rise in P_N is larger.

The impact of an unanticipated rise in total factor productivity in nontraded goods is still negative but less than proportional than the shock, contrary to the standard BS case. Graphically, the MPK and the MPL schedules shift downward but the shift in MPL is of smaller magnitude than in the BS case. A sa consequence, the price decreases less and the capital intensity in the traded good sector increases.

Note: Another way to derive this result uses Equations (6)-(10). Combined together they imply:

$$A_T f(k_T, n_T) = rk_T + P_N n_T + w$$
$$P_N A_N g(k_N) = w + rk_N$$

Taking the natural logarithms and totally differenciating gives:

$$\frac{dA_T}{A_T} = \mu_{LT}\frac{dw}{w} + \mu_{NT}\frac{dP_N}{P_N}$$
$$\frac{dP_N}{P_N} + \frac{dA_N}{A_N} = \mu_{LN}\frac{dw}{w}$$

with μ_{Li} and μ_{Ni} respectively denoting the share of labor and non-traded inputs in the production of good i. Combining both eqautions finally gives:

$$\frac{dP_N}{P_N} = \frac{\mu_{LN}}{\mu_{LT} + \mu_{LN}\mu_{NT}}\frac{dA_T}{A_T} - \frac{\mu_{LT}}{\mu_{LT} + \mu_{LN}\mu_{NT}}\frac{dA_N}{A_N}$$

2 Endogenous Tradability (Bergin & Glick, 2003)

Consider a very simple small open endowment economy. The country is endowed with a continuum of goods indexed by i on the unit interval, where y_i represents the level of endowment, C_i is the level of consumption, and p_i is the domestic price level of this good. All of these home goods have the potential of being exported, but some endogenously determined fraction of the goods, n, will be nontraded in equilibrium. For each traded home good there is a prevailing world price p_i^* that may differ from the home price because of trade costs. The small open economy may also import foreign goods for consumption purposes, with consumption level c_F and price level p_F. For simplicity, it is assumed that the endowments and world price levels of all home goods are uniform, implying, $y_i = y$, $p_i^* = p^*$ for all i.

The aggregate consumption index is modelled according to the following Cobb-Douglas specification:

$$c = \frac{c_H^{\theta} c_F^{1-\theta}}{\theta^{\theta}(1-\theta)^{1-\theta}}$$

where c_H is an index of home goods consumption:

$$\begin{aligned} c_H^{\frac{\phi-1}{\phi}} &= \int_0^n c_i^{\frac{\phi-1}{\phi}} di + \int_n^1 c_i^{\frac{\phi-1}{\phi}} di \\ &= n\left(\frac{c_N}{n}\right)^{\frac{\phi-1}{\phi}} + (1-n)\left(\frac{c_T}{1-n}\right)^{\frac{\phi-1}{\phi}} \end{aligned}$$

$\phi > 1$ is the elasticity of substitution between varieties i. Price indexes are defined for each category of goods, in correspondence to the consumption indexes above:

$$\begin{aligned} p &= p_H^{\theta} p_F^{1-\theta} \\ p_H &= \int_0^n p_i^{1-\phi} di + \int_n^1 p_i^{1-\phi} di \\ &= np_N^{1-\phi} + (1-n)p_T^{1-\phi} \end{aligned}$$

Note that if world prices are normalized to unity, i.e. $p^* = 1$, $p_F = 1$ and p can be interpreted as the reciprocal of the real exchange rate for this small open economy.
The home goods are distinguished from each other by the presence of good-specific iceberg costs, (τ_i) where a certain fraction of the good disappears in transport. Assume that the home country pays for this cost so that the domestic price will be $p^*/(1+\tau_i)$ if the country exports good i. These trade costs are specified to follow the distribution:

$$1 + \tau_i = \alpha i^{-\beta}, \quad \alpha \geq 1, \beta \geq 0$$

The decision of whether to export a good is determined solely on the basis of whether the export price (i.e. the world price) less iceberg costs, exceeds the domestic price. If the export price is higher, then the good is exported, if it is lower, then it is not traded.

1. Derive and interpret the distribution of export prices and the price index for traded goods.

Under the previous assumptions on the distribution of trade costs, the distribution of export prices can be written as:

$$p_i = \frac{p^* i^\beta}{\alpha}$$

The parameter β controls the curvature of the distribution, while α controls the level. The goods at the left end of the continuum (i near 0) tend to have lower prices when exported because the trade cost is large; these goods are less tradable. Goods toward the right end of the continuum (i near 1) have higher prices because the trade cost is low; they are more tradable. β characterizes how quickly the price of an individual good rises with the goods index – in fact, it can be viewed as an elasticity. For example, for a high β, the percent change in costs is high for a given percent change in the index. Given the distribution of export prices, the price index for traded goods becomes:

$$\begin{aligned} p_T &= \left(\frac{1}{1-n} \int_n^1 \left(\frac{p^* i^\beta}{\alpha} \right)^{1-\phi} di \right)^{\frac{1}{1-\phi}} \\ &= \frac{p^*}{\alpha} \left[\frac{1}{1-n} \frac{1}{\beta(\phi-1)-1} \left(\left(\frac{1}{n} \right)^{\beta(\phi-1)-1} - 1 \right) \right]^{\frac{1}{1-\phi}} \end{aligned} \tag{1}$$

The price of traded goods is expressed as a function of the (endogenous) share of traded goods n, the elasticity of substitution across domestic goods ϕ, and the trade cost parameters, β and α. It is straightforward to establish that $dp_T/dn > 0$; i.e. the price of traded goods increases with the share of nontraded goods. The reason is that, as the proportion of home goods that are nontraded rises, it is no longer profitable to export goods with marginally higher trade costs; as these goods are withdrawn from export markets, the average price of the remaining export goods rises.

2. Using the solution of the optimization problem faced by the domestic consumer, solve for the relative price of nontraded goods and the price index for nontraded goods.

The consumer's optimization problem can be decomposed as follows:

$$\begin{cases} \max_{c_H, c_F} c \\ u.c. \quad p_H c_H + p_F c_F = R \end{cases}$$

$$\begin{cases} \max_{c_i} c_H \\ u.c. \quad \int_0^n p_i c_i di + \int_n^1 p_i c_i di = p_H c_H \end{cases}$$

It is easy to verify that the first step (the consumption arbitrage between domestic and foreign goods) implies:

$$c_F = (1-\theta)\frac{R}{p_F} = (1-\theta)\frac{pc}{p_F} \qquad (2)$$

$$c_H = \theta\frac{R}{p_H} = \theta\frac{pc}{p_H} \qquad (3)$$

As for the second step, the first-order condition for any i good is:

$$\left[\int_0^n c_i^{\frac{\phi-1}{\phi}}\,di + \int_n^1 c_i^{\frac{\phi-1}{\phi}}\,di\right]^{\frac{\phi}{\phi-1}-1} c_i^{-1/\phi} = p_i$$

In particular, for two nontraded goods i and j:

$$\left(\frac{c_i}{c_j}\right)^{-1/\phi} = \frac{p_i}{p_j}$$

Since consumption must equal the endowment of nontraded goods, and endowments are uniform for all goods here (i.e. $y_i = y$ for all i), we can conclude that for any pair of nontraded goods it will be true that $c_i/c_j = y_i/y_j = 1$ and so $p_i/p_j = 1$. In other words, the price of each nontraded good will be identical, because they each are by definition not affected by the trade costs which vary by good. This logic applies equally well to the home good that is just on the margin between being traded and nontraded ($i = n$). The marginal trader decides to export solely on the basis of whether the world price less iceberg costs exceeds the domestic price. But because this good is on the margin of being traded, the domestic price must be the same as that as if it were sold in the world market: $p_n = \frac{p^* n^\beta}{\alpha}$. As a result, the price index of nontraded goods is pinned down as the price of the marginal traded good:

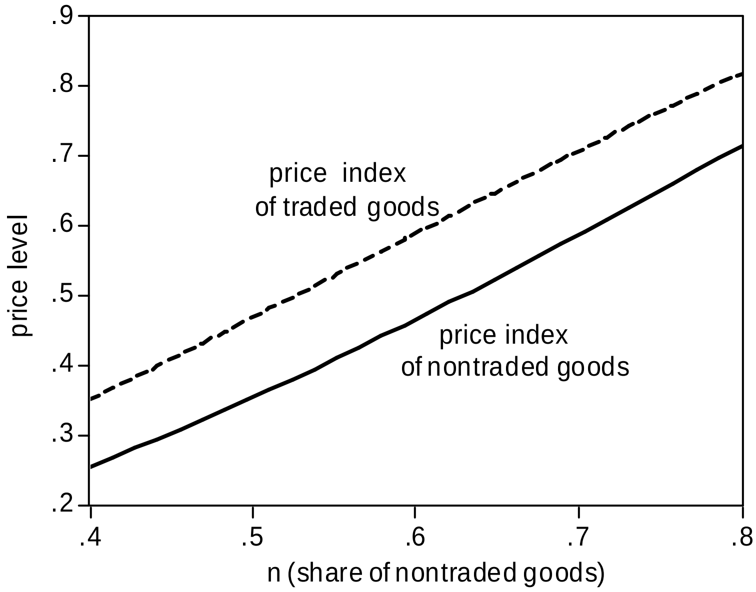
$$p_N = \left(\frac{1}{n}\int_0^n \left(\frac{p^*}{\alpha}\right)^{1-\phi} n^{\beta(1-\phi)}\,di\right)^{\frac{1}{1-\phi}}$$

$$= \frac{p^*}{\alpha}n^\beta \qquad (4)$$

It implies that the price of nontraded goods rises with the share of nontraded goods with elasticity β. It is easily verified that there can be no discontinuous jump in price either up or down between the last nontraded good and the first traded good. Note that the iceberg trading costs for adjacent goods are essentially identical and that there is no fixed cost to trade. Suppose that the price of the first traded good jumped discontinuously above the price of the last nontraded good; then it would be profitable for the last nontraded good to become traded instead. Similarly, suppose that the price of the first traded good jumped discontinuously below the price of the last nontraded good; then it would be profitable for the first traded good to become nontraded instead. Finally, one can show that the price indices of traded and nontraded goods are related to each other. Figure 1 shows their relationship as the share of nontraded goods varies. Observe that (i) p_T is everywhere higher than p_N, since traded goods are less costly to transport, and (ii) both p_N and p_T rise with n.
Finally, note that the second part of the consumer's problem can be rewritten as:

$$\begin{cases} \max_{c_T, c_N} c_H \\ u.c. \quad p_N c_N + p_T c_T = p_H c_H \\ \qquad c_H = \left[n\left(\frac{c_N}{n}\right)^{\frac{\phi-1}{\phi}} + (1-n)\left(\frac{c_T}{1-n}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \end{cases}$$

Figure 1: Price indexes of traded and nontraded goods as a function of n ($\beta = 1.5$, $p^*/\alpha = 1$, $\phi = 10$)



The first-order conditions are:

$$\left[n\left(\frac{c_N}{n}\right)^{\frac{\phi-1}{\phi}} + (1-n)\left(\frac{c_T}{1-n}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}-1}\left(\frac{c_N}{n}\right)^{\frac{-1}{\phi}} = p_N$$

$$\left[n\left(\frac{c_N}{n}\right)^{\frac{\phi-1}{\phi}} + (1-n)\left(\frac{c_T}{1-n}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}\left(\frac{c_T}{1-n}\right)^{\frac{-1}{\phi}} = p_T$$

$$p_N c_N + p_T c_T = p_H c_H$$

After some calculations, one obtains:

$$c_N = \left(\frac{p_N}{p_H}\right)^{-\phi} n c_H \qquad (5)$$

$$c_T = \left(\frac{p_T}{p_H}\right)^{-\phi} (1-n) c_H \qquad (6)$$

3. Comment on the relative price of nontraded and traded goods.

The definitions of traded and nontraded goods prices can be combined to obtain a characterization of how the relative price structure is pinned down by the share of nontraded goods n, the elasticity of transportation costs β, and the elasticity of substitution of home goods ϕ:

$$\frac{p_N}{p_T} = \left[\frac{n}{1-n}\frac{1}{\beta(\phi-1)-1}(1-n^{\beta(\phi-1)-1})\right]^{\frac{1}{\phi-1}} \qquad (7)$$

4. Use the market clearing conditions to find the equilibrium value for n under balanced trade ($p_H c_H = pc$).

The market clearing condition for non-traded goods implies:

$$p_N c_N = \int_0^n p_i y_i di \Leftrightarrow c_N = ny$$

Combining it with the optimal consumption of non-traded goods allows to solve for the nominal consumption of domestic goods:

$$\begin{aligned} c_H &= \frac{c_N}{n}\left(\frac{p_H}{p_N}\right)^{-\phi} \\ &= y\left(\frac{p^* n^\beta}{\alpha p_H}\right)^{\phi} \\ \Leftrightarrow \quad p_H c_H &= y\alpha^{-\phi} n^{\beta\phi} p^{*\phi} p_H^{1-\phi} \end{aligned}$$

Second, use the equilibrium price indices to get:

$$\begin{aligned} p_H^{1-\phi} &= n p_N^{1-\phi} + (1-n) p_T^{1-\phi} \\ &= n\left(\frac{p^* n^\beta}{\alpha}\right)^{1-\phi} + \left(\frac{p^*}{\alpha}\right)^{1-\phi} \frac{1}{\beta(\phi-1)-1}\left[\left(\frac{1}{n}\right)^{\beta(\phi-1)-1} - 1\right] \\ &= \left(\frac{p^*}{\alpha}\right)^{1-\phi} \frac{1}{\beta(\phi-1)-1}\left[\left(\frac{1}{n}\right)^{\beta(\phi-1)-1} \beta(\phi-1) - 1\right] \end{aligned}$$

Incorporating this value of the price index of domestic consumption in the nominal consumption gives:

$$p_H c_H = y\frac{p^*}{\alpha}\frac{n^{\beta\phi}}{\beta(\phi-1)-1}\left[\beta(\phi-1)n^{1-\beta(\phi-1)} - 1\right]$$

Now, note that the domestic value of aggregate home production can be derived as:

$$\begin{aligned} p_H y_H &= \int_o^n p_i y_i di + \int_n^1 p_i y_i di \\ &= \frac{p^*}{\alpha} y n^{\beta+1} + \frac{p^*}{\alpha} y \frac{1-n^{1+\beta}}{1+\beta} \\ &= \frac{p^*}{\alpha} y \frac{1+\beta n^{1+\beta}}{1+\beta} \end{aligned}$$

Finally, using the condition for balanced trade: $p_H y_H - pc = p_H y_H - \frac{p_H c_H}{\theta} = 0$, one gets an implicit solution for the endogenous share of nontraded goods n:

$$\begin{aligned} & \theta p_H y_H - p_H c_H = 0 \\ \Leftrightarrow \quad & \theta\frac{1+\beta n^{1+\beta}}{1+\beta} - \frac{1}{\beta(\phi-1)-1}\left[n^{1+\beta}\beta(\phi-1) - n^{\beta\phi}\right] = 0 \end{aligned} \qquad (8)$$

One can easily verify that condition (8) defines a unique equilibrium value for n in the range of possible values. Defining

$$Z(n) = \frac{1}{\beta(\phi-1)-1}\left[n^{1+\beta}\beta(\phi-1) - n^{\beta\phi}\right] - \theta\frac{1+\beta n^{1+\beta}}{1+\beta}$$

Table 1: Numerical simulations of the balanced trade economy ($\phi = 10$, $p^*/\alpha = 1$, $\theta = 0.5$)

$\phi = 10$		$\beta = 1.5$	
β	$\bar{n}$	ϕ	$\bar{n}$
0.1	0.1966	5	0.5503
0.5	0.4507	10	0.5802
1.5	0.5802	20	0.5927
5	0.7184		
10	0.7963		

Source: Bergin & Glick, 2003

one verifies that:

$$\begin{aligned} Z(0) &= -\theta \\ Z(1) &= 1-\theta \\ \frac{\partial Z(n)}{\partial n} &= \beta n^{\beta}(1-\theta) + \frac{\beta\phi n^{\beta}[1-n^{\beta(\phi-1)-1}]}{\beta(\phi-1)-1} > 0 \end{aligned}$$

The trade balance thus falls as n increases. Intuitively, increasing n implies trade in fewer varieties of goods and lowers the trade surplus.
Condition (8) provides a number of insights concerning the determinants of the equilibrium share of nontraded goods. One observation is that the curvature parameter in the distribution of trade costs (β) plays an important role in determining n.

5. Comment Table 1, that reports numerical simulations for a benchmark calibration of$p^*/\alpha = 1$, $\theta = 0.5$.
Table 1 shows that a rise in β progressively raises the share of home goods that are nontraded. This result is fairly intuitive: if trade costs rise very quickly as one exports more classes of goods, it is optimal to export a smaller number of goods. A country should then concentrate its exports in those commodities for which international trade is so much less costly.
Another important determinant of tradedness is the elasticity of substitution between home goods (ϕ). As this elasticity rises, n rises gradually. The intuition is that if home goods are highly substitutable in consumption, one can conserve on trade costs by concentrating one's exports in the goods that are easiest to trade. This means there will be a smaller quantity of these particular classes of goods to consume, but under a high elasticity, it is easy to compensate for this by consuming a greater quantity of other types of goods. On the other hand, if home goods were less substitutable with each other, one would want to consume a more even distribution of home goods, thereby requiring the country to export a smaller portion of a larger number of goods to pay the bill for imports.
Lastly, observe that the scale parameter in the distribution of trade costs, α, does not appear in equation (8) above. When one considers the effects of trade costs here, it is their relative levels between goods (summarized in β), not their overall level (summarized in α) which determines the varieties of goods that are nontraded. In part, this last implication results from the assumption of Cobb-Douglas preferences over home and foreign goods, which is a common assumption in this literature, known to have certain implications that help simplify analytical solutions. Some intuition can be found in the fact that a unitary elasticity of substitution between home and foreign goods implies that a constant share of consumption expenditure goes toward foreign goods, regardless of the

relative price between goods, and hence regardless of the size of transport costs. A sufficient quantity of home goods then must be traded and exported to pay for these imports under balanced trade.

6. In its two-period dynamic form, simulations of the model allow to study the volatility of the relative price of nontraded goods under endogenous tradability. This relative volatility falls dramatically as the curvature of trade costs rises. For a value of $\beta = 1.5$, the model is able to approximately replicate the value of 0.37 found in the empirical study by Betts and Kehoe (2001a). Empirical work by Engel (1999) finds that the volatility of nontraded prices may yet be lower than this, but the model is capable of replicating even very low values of volatility as the curvature parameter β is assumed to be progressively larger. This result stands in sharp contrast to the standard result of open economy models in the literature, where the share of nontraded goods is taken to be exogenous. For example the classic Balassa-Samuelson model explains real exchange rate levels exclusively in terms of shifts in the relative price of nontraded goods. The same is true for the well-known two-period model of Dornbusch (1983), which is very similar to the model considered here, except for the assumption that the share of nontraded goods is fixed. Under such an assumption, a rise in consumption demand will tend to push up the price of consumption goods, but this will be expressed only for nontradeds, because the price of traded goods is pinned down to the world price level by arbitrage. A rise in the relative price of nontraded goods is necessary for equilibrium, to convince households to take their extra consumption in the form of additional imports of tradable goods, given that the consumption of nontraded goods is limited by definition to the domestic supply of such goods.

It can be shown that the relative price of nontraded goods moves much less under the assumption of endogenous tradedness than for the standard assumption of exogenous tradedness. Since the aggregate price level p is a weighted average of nontraded prices (p_N), traded home goods prices (p_T), and import prices (p_F), where the latter two are fixed by world levels, the movement in the first component must always be larger than the movement in the overall average that it induces. This explains why a small open economy model with exogenously determined nontraded goods has such difficulty explaining a low volatility in the price of nontraded goods relative to the overall real exchange rate. Making n endogenous has a very dramatic effect on the ability of the model to explain this empirical regularity. Now, as a rise in demand starts to push up the relative price of nontraded goods, some traded goods sellers on the margin will find it profitable to sell more in the home market, to the point of abandoning attempts to market their good abroad where they need to deal with costs of trade. This endogenous rise in the share of nontraded goods allows the supply of nontraded goods to rise, despite the fact that the endowment of each individual good is fixed. This rise in supply reduces the pressure for the relative price of nontradeds to rise in the face of the higher demand. The main insight here is that, when one begins to view nontraded goods as being endogenously determined, one can see there is a potentially strong force limiting the movement in the relative price of these nontraded goods. The marginal trading condition from the model (eqn. (4)) is useful in seeing how this result arises. Recall that this equation states that the price index of nontraded goods will equal the price of the marginal traded good. This linkage between nontraded and traded prices prevents one price index from straying too far from the other, and thus helps dampen the volatility in their ratio.

Why does this mechanism work best for high values of β? Looking at the marginal condition (equation 4), it becomes clear that β is the elasticity of the nontraded price index with respect to changes in n. It is at high values of β where the demand shock induces a small change in n and a large change in the price of nontraded goods. But this also requires a larger change in the price index of traded goods, so the overall price index changes more. One interesting implication of this logic, is that the mechanism outlined here to explain the stylized fact does not require an implausible degree of movement in the share of nontraded goods. The curvature parameter is not the only parameter to play an important role in this mechanism. Begrin & Glick show that a higher elasticity of substitution between home goods (ϕ) also plays an important role. As ϕ rises, the volatility in relative nontraded prices as a ratio to that of the real exchange rate falls. Intuitively, if the last nontraded good and the marginal traded good are highly substitutable, this makes the link between their two prices stronger. This in turn strengthens the linkages between the price indexes of traded and nontraded goods.