

**Master EPP, International Macroeconomics**  
**Lecture 2**  
**Real Exchange Rates, Balassa-Samuelson and Purchasing Power Parity**

## 1 A Balassa-Samuelson model with non-traded inputs

Consider a variant of the standard Balassa-Samuelson model. There are two countries in the economy, Home and Foreign (in the following, foreign variables are indexed with a star \*). Capital is perfectly mobile across sectors and countries. Labor is free to migrate between sectors of an economy but not between countries.

Each economy produces two composite goods, a traded good  $T$  and a non-traded good  $N$ . The production of  $T$  can be either consumed domestically or exported without any trade cost.  $N$  products are either consumed by the domestic household or used in the production of the  $T$  good. Outputs are given by the following constant-returns production functions:

$$\begin{aligned} Y_T &= A_T F(K_T, L_T, N_T) = A_T K_T^\alpha N_T^\beta L_T^{1-\alpha-\beta} \\ Y_N &= A_N G(K_N, L_N) = A_N K_N^\gamma L_N^{1-\gamma} \end{aligned}$$

where subscripts  $T$  denotes the traded good sector, subscript  $N$  the non-traded good sector and  $A$  a productivity shifter.  $N_T$  is the quantity of non-traded inputs incorporated in the production of traded goods. In the following, the traded good is taken as numeraire.

In each country, the total labor supply is fixed at  $L$ . The equalization of labor supply and demand ( $L_N + L_T$ ) determines the equilibrium wage  $w$  (which is the same in both sectors when labor is mobile). Because capital is internationally mobile, there is a single world rate of return  $r$ , taken as exogenous. Capital accumulation takes place at the firm-level and there is no depreciation. Capital must be put in place a period before it is actually used.

1. Write and resolve the maximization problems of representative firms producing traded and non traded goods.
2. Use the first-order conditions to solve for the equilibrium wage. Comparison with the standard Balassa-Samuelson model.
3. Use the equilibrium wage to solve for the price of non-traded goods.
4. Discuss the impact of unanticipated productivity shocks in this setting.

## 2 Endogenous Tradability (Bergin & Glick, 2003)

Consider a very simple small open endowment economy. The country is endowed with a continuum of goods indexed by  $i$  on the unit interval, where  $y_i$  represents the level of endowment,  $C_i$  is the level of consumption, and  $p_i$  is the domestic price level of this good. All of these home goods have the potential of being exported, but some endogenously determined fraction of the goods,  $n$ , will be nontraded in equilibrium. For each traded home good there is a prevailing world price  $p_i^*$  that may differ from the home price because of trade costs. The small open economy may also import foreign goods for consumption purposes, with consumption level  $c_F$  and price level  $p_F$ . For simplicity, it is assumed that the endowments and world price levels of all home goods are uniform, implying,  $y_i = y$ ,  $p_i^* = p^*$  for all  $i$ .

The aggregate consumption index is modelled according to the following Cobb-Douglas specification:

$$c = \frac{c_H^\theta c_F^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}}$$

where  $c_H$  is an index of home goods consumption:

$$\begin{aligned} c_H^{\frac{\phi-1}{\phi}} &= \int_0^n c_i^{\frac{\phi-1}{\phi}} di + \int_n^1 c_i^{\frac{\phi-1}{\phi}} di \\ &= n \left( \frac{c_N}{n} \right)^{\frac{\phi-1}{\phi}} + (1-n) \left( \frac{c_T}{1-n} \right)^{\frac{\phi-1}{\phi}} \end{aligned}$$

$\phi > 1$  is the elasticity of substitution between varieties  $i$ . Price indexes are defined for each category of goods, in correspondence to the consumption indexes above:

$$\begin{aligned} p &= p_H^\theta p_F^{1-\theta} \\ p_H &= \int_0^n p_i^{1-\phi} di + \int_n^1 p_i^{1-\phi} di \\ &= n p_N^{1-\phi} + (1-n) p_T^{1-\phi} \end{aligned}$$

Note that if world prices are normalized to unity, i.e.  $p^* = 1$ ,  $p_F = 1$  and  $p$  can be interpreted as the reciprocal of the real exchange rate for this small open economy.

The home goods are distinguished from each other by the presence of good-specific iceberg costs,  $(\tau_i)$  where a certain fraction of the good disappears in transport. Assume that the home country pays for this cost so that the domestic price will be  $p^*/(1+\tau_i)$  if the country exports good  $i$ . These trade costs are specified to follow the distribution:

$$1 + \tau_i = \alpha i^{-\beta}, \quad \alpha \geq 1, \beta \geq 0$$

The decision of whether to export a good is determined solely on the basis of whether the export price (i.e. the world price) less iceberg costs, exceeds the domestic price. If the export price is higher, then the good is exported, if it is lower, then it is not traded.

1. Derive and interpret the distribution of export prices and the price index for traded goods.
2. Using the solution of the optimization problem faced by the domestic consumer, solve for the relative price of nontraded goods and the price index for nontraded goods.
3. Comment on the relative price of nontraded and traded goods.
4. Use the market clearing conditions to find the equilibrium value for  $n$  under balanced trade ( $p_H c_H = p c$ ).
5. Comment Table 1, that reports numerical simulations for a benchmark calibration of  $p^*/\alpha = 1$ ,  $\theta = 0.5$ .
6. In its two-period dynamic form, simulations of the model allow to study the volatility of the relative price of nontraded goods under endogenous tradability. Results are summarized in Table 2. Interpretation.

Table 1: Numerical simulations of the balanced trade economy ( $\phi = 10$ ,  $p^*/\alpha = 1$ ,  $\theta = 0.5$ )

$\phi = 10$		$\beta = 1.5$	
$\beta$	$\bar{n}$	$\phi$	$\bar{n}$
0.1	0.1966	5	0.5503
0.5	0.4507	10	0.5802
1.5	0.5802	20	0.5927
5	0.7184		
10	0.7963		

Source: Bergin & Glick, 2003

Table 2: Numerical simulations of the 2-period economy ( $\phi = 10$ ,  $p^*/\alpha = 1$ ,  $\theta = 0.5$ )

$\beta$	$\phi = 10$		$\beta = 1.5$	
	$\frac{sdev(p_N/p_T)}{sdev(1/p)}$		$\phi$	$\frac{sdev(p_N/p_T)}{sdev(1/p)}$
	Endogenous $n$	exogenous $n$		
0.1	3.5350	5.5679	5	
0.5	0.8988	2.5509	10	
1.5	0.3810	2.1689	20	
5	0.1623	2.0500		
10	0.1109	2.0251		

Source: Bergin & Glick, 2003