Master EPP, International Macroeconomics
New Trade Models

1 Demand and prices under Dixit-Stiglitz preferences

Consider a utility function featured by Dixit-Stiglitz preferences:

$$U = C_M; \quad C_M = \left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}; \quad \sigma > 1$$

where N is the number of varieties available for consumption. Varieties i are differentiated and σ measures the elasticity of substitution between two varieties.

The consumer's aggregate nominal expenditure E is assumed exogenous. The indirect utility function can be written as:[1]

$$V = \frac{E}{P_M}$$

where P_M is the "ideal" price index associated with Dixit-Stiglitz preferences.

1. Write the consumer's utility maximization and derive the first-order conditions.

2. Use the first order conditions associated with two distinct varieties to derive relative consumption. Interpretation.

3. Combine the previously derived relative consumption with the budget constraint to get an expression of the "ideal" price index.

4. Finally, conclude on optimal demands under Dixit-Stiglitz.

5. Why is the CES utility function referred to as "love for variety" preferences?

6. Consider firms' price setting under Dixit-Stiglitz preferences. Each firm produces a symmetric variety subject to a homothetic cost function. The typical firm's objective function is revenue, $p_j c_j$ minus costs $a_m w c_j$ where w is the equilibrium wage and a_m is the firm's unit cost. What is the optimal price setting under Bertrand competition? under Cournot competition? What is the optimal price under monopolistic competition, when the number of firms is large enough so that each firm as a negligible impact on aggregate price and consumption?

2 Scale economies, Product differentiation and the Pattern of Trade (Krugman, 1980)

Motivation: Develop a model with economies of scale, product differentiation and imperfect competition explaining the pattern of international trade, notably the prevalence of intra-sectorial trade among the industrial countries.

[1] In economics, a consumer's indirect utility function $V(p,,w)$ gives the consumer's maximal utility when faced with a price level p and an amount of income w. It represents the consumer's preferences over market conditions. A consumer's indirect utility $V(p,w)$ can be computed from its utility function $U(c)$ by first computing the most preferred bundle $c(p,w)$ by solving the utility maximization problem; and second, computing the utility $U(c(p,w))$ the consumer derives from that bundle. The indirect utility function for consumers is analogous to the profit function for firms.

Consider a model à la Dixit-Stiglitz in which firms can costlessly differentiate their products and equilibrium takes the form of Chamberlian monopolistic competition: each firm has some monopoly power but entry drives monopoly profits to zero.
The model has two regions with two kinds of production: agriculture, which is constant-returns sector, and manufactures, an increasing-returns sector. The agriculture sector is featured by perfect competition and zero trade cost. Its role in the model is to simplify the analysis by equalizing wages across countries. It is taken as numéraire. In the increasing-returns sector, there are a large number of potential goods, all of which enter symmetrically into demand. All individuals have the same utility function:

$$U = C_M^{\mu} C_A^{1-\mu}, \quad C_M = \left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \quad 0 < \mu < 1$$

where C_A is consumption of the agricultural good, c_i is consumption of the ith variety of the manufacturing good and N the number of actually produced varieties, assumed to be large, although smaller than the potential range of products.
There are L individuals in the economy, each endowed with one unit of labor. In equilibrium, individual expenditure is thus equal to the wage rate. Labor is the only factor of production. The agricultural good is produced under constant returns, with one unit of labor producing one unit of good. In the manufacturing sector, all varieties are produced with the same cost function:

$$l_i = \alpha + \beta x_i$$

where l_i is labor used in producing the ith good and x_i is output of that good. α is the fixed producing cost and β the (constant) marginal cost.
Firms maximize profits, but there is free entry and exit of firms, so that in equilibrium profits will always be zero. Because firms can costlessly differentiate their products and all products enter symmetrically into demand, two firms will never want to produce the same product. At the same time, as the number of goods produced is large, the effect of the price of any one good on the demand for any other is negligible. As a result, each firm can ignore the effect of its actions on other firms' behavior. Under monopolistic competition, equilibrium is determinated.

1. Discuss the properties of the technological function in the manufacturing sector.

Equilibrium in a Closed Economy

2. Write the market-clearing conditions.

3. Derive optimal demand functions maximizing individual utility.

4. Find the equilibrium price of agricultural goods.

5. Find the optimal price set by the producer of a given manufacturing variety.

6. Use the free entry condition and the market-clearing conditions to derive the equilibrium number of produced goods.

Costly Trade Equilibrium

Assume two countries of size L and L^*, sharing the same tastes and technologies, open to trade. Trade is perfectly free in the agricultural sector but there is a transportation cost for manufacturing goods. Transportation costs are assumed to be of the "iceberg" type: to sell one unit of good abroad, the firm has to produce $\tau > 1$ units because $\tau - 1$ units are "lost" during transportation.
An individual in the home country has a choice over N products produced at home and N^* products produced abroad. She maximizes:

$$U = \left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}} + \sum_{i^*=1}^{N^*} c_{i^*}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\mu\sigma}{\sigma-1}} C_A^{1-\mu}, \quad \sigma > 1$$

where N^* is the number of varieties produced in the foreign country. Finally, trade costs induce market segmentation so that firms can charge different prices in their domestic and export markets.

7. Show that wages are equalized internationally under these assumptions.

8. Derive the price charged on domestic and foreign sales by a domestic producer. Discuss the impact of firms' location choices on aggregate manufacturing prices.

9. The equilibrium distribution of firms across countries ($s_n = \frac{N}{N+N^*}$) is determined by equalizing profits across countries. Determine s_n as a function of $s_L = \frac{L}{L+L^*}$. Interpretation.