Master EPP, International Macroeconomics
Lecture 1
Traditional Open Macro Models and Monetary Policy

1 The Cagan model (OR, 8.2)

Consider the Cagan model of Money and Prices. Let M denote a country's money supply and P its price level. The demand for real money balances M^d/P is explained by expected future price-level inflation.[1] In logarithms:

$$m_t^d - p_t = -\eta E_t\{p_{t+1} - p_t\}$$

with η the semielasticity of demand for real balances with respect to expected inflation.

1. Write the monetary equilibrium condition.

In equilibrium $m_t = m_t^d$ and thus

$$m_t - p_t = -\eta E_t\{p_{t+1} - p_t\}$$

This first-order stochastic difference equation explains price-level dynamics in terms of the money supply.

2. Write the general form of the equilibrium price level. Interpretation. (In the following, we will assume the following "no-speculative bubble" condition holds :

$$\lim_{T\to\infty}\left(\frac{\eta}{1+\eta}\right)^T E_t\{p_{t+T}\} = 0$$

$$\begin{aligned}
p_t &= \frac{1}{1+\eta}\left[m_t + \eta E_t\{p_{t+1}\}\right] \\
&= \frac{1}{1+\eta}\left[\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} E_t\{m_s\}\right] + \lim_{t\to\infty}\left(\frac{\eta}{1+\eta}\right)^T E_t\{p_{t+T}\} \\
p_t &= \frac{1}{1+\eta}\left[\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} E_t\{m_s\}\right]
\end{aligned}$$

In this model, money is fully neutral as $\frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} = 1$

3. Discuss the following particular cases : i) $m_t = \bar{m} \quad \forall t$, ii) $m_t = \bar{m} + \mu t$, iii) $m_t = \rho m_{t-1} + \varepsilon_t$ with $E_t\{E_{t+1}\} = 0$ and $0 < \rho < 1$.

i) When the money supply is constant, inflation is zero and $\bar{p} = \bar{m}$

[1] The Cagan's model seeks to study hyperinflation episodes and thus neglects the real determinants of money demand proposed by Keynes-Hicks.

ii) For a constant growth rate of the money supply, inflation is constant and the price level becomes:

$$\begin{aligned}
p_t &= \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}[m_t+\mu(s-t)] \\
&= m_t+\frac{\mu}{1+\eta}\left[\left(\frac{\eta}{1+\eta}\right)+\left(\frac{\eta}{1+\eta}\right)^2+\left(\frac{\eta}{1+\eta}\right)^3+...\right]\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} \\
&= m_t+\frac{\mu}{1+\eta}\eta(1+\eta) \\
p_t &= m_t+\mu\eta
\end{aligned}$$

iii) When m_t is an auto-regressive process, one gets:

$$p_t=\frac{m_t}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta\rho}{1+\eta}\right)^{s-t}=\frac{m_t}{1+\eta-\eta\rho}$$

In the limiting case $\rho=1$ in which money shocks are expected to be permanent, the solution reduces to $p_t=m_t$, in analogy with the nonstochastic case.

4. What happens to the dynamics of the price level when, at date 0, the government announces the money supply is going to rise from $\bar{m}$ to $\bar{m}'$ on a future date T?

$$p_t=\begin{cases}\bar{m}+\left(\frac{\eta}{1+\eta}\right)^{T-t}(\bar{m}'-\bar{m}) & ,t<T\\ \bar{m}' & ,t\geq T\end{cases}$$

The supply shock is integrated in the effective price level as long as it is announced by the government.

4. Consider a government that fixes the (constant) gross rate of money growth. What would be optimal to choose in order to maximize seignorage ?

Seignorage represents the real revenues a government acquires by using newly issued money to buy goods and nonmonetary assets:

$$Seignorage=\frac{M_t-M_{t-1}}{P_t}\equiv\frac{M_t-M_{t-1}}{M_t}.\frac{M_t}{P_t}$$

If higher money growth raises expected inflation, the demand for real balances may fall, which exerts a negative influence on seignorage revenues. Thus, for any money demand equation, there exists an optimal inflation rate μ^* maximizing seignorage revenues:

$$\begin{cases}Max_\mu\frac{M_t-M_{t-1}}{M_t}.\frac{M_t}{P_t}\\ s.c.\quad \frac{M_t}{P_t}=\left(\frac{P_{t+1}}{P_t}\right)^{-\eta}\end{cases}$$

Under constant gross rate of money growth, we know from question **3.** that : $\frac{M_t}{M_{t-1}}=\frac{P_t}{P_{t-1}}=1+\mu$. Maximizing seignorage revenues with respect to μ gives the first-order condition:

$$\begin{aligned}
&(1+\mu^*)^{-\eta-1}-(1+\eta)\mu^*(1+\mu^*)^{-\eta-2}=0\\
\Leftrightarrow\quad &\mu^*=\frac{1}{\eta}
\end{aligned}$$

The revenue-maximizing net rate of money growth thus depends inversely on the semielasticity of real balances with respect to inflation. This equation can be interpreted as a pricing formula for a monopolist with zero marginal cost of production.
Note that this view of the government's optimal strategy may suffer from credibility constraints. Suppose for instance that the government announces on date 0 that it will stick forever to the revenue-maximizing rate of money growth $1/\eta$. If this announcement is credible, the public will hold real balances $M/P = [(1+\eta)/\eta]^{-\eta}$. The government then has an incentive to cheat and set money growth greater than $1/\eta$ to increase revenues. The problem is that this cheating makes her lose its credibility. In the future, if the public is not gullible, it will anticipate the government's temptation to cheat and hold less real balances as a consequence.

2 A simple model of Exchange Rates (OR, 8.2)

Consider a small, open economy in which real output is exogenous and the demand for money is given by:

$$m_t - p_t = -\eta \mathrm{i}_{t+1} + \phi y_t$$

where $\mathrm{i}_{t+1} \equiv log(1+i_{t+1})$, p is the log of the price level, m is the log of the nominal money demand and y is the log of output.
PPP holds: $P_t = \mathcal{E} P_t^*$ with $\mathcal{E}$ the nominal exchange rate, defined as the price of foreign currency in terms of home currency, and P_t^* the world foreign-currency price of the consumption basket.
1. Write the uncovered interest parity (UIP) relation. (Assume away the existence of an exchange-rate risk premia)
In a world of perfect foresight, UIP holds via a simple arbitrage argument. Consider a domestic investor that owns 1 unit of domestic currency. She has the choice between investing it in her own country or in the foreign market. In case she chooses to buy domestic bonds, her investment in period t yields her $(1+i_{t+1})$ units of domestic currency in $t+1$. To invest in the foreign bond, she first has to convert her unit of domestic good in $1/\mathcal{E}_t$ units of foreign currency. In $t+1$, she recovers $(1+i^*_{t+1})/\mathcal{E}_t$ units of foreign currency, equivalent to $\mathcal{E}_{t+1}(1+i^*_{t+1})/\mathcal{E}_t$ units of domestic currency. In period t and in the absence of any exchange-rate risk, arbitrage thus implies :

$$1 + i_{t+1} = (1+i^*_{t+1}) E_t \left\{ \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} \tag{1}$$

$$\Leftrightarrow \mathrm{i}_{t+1} = \mathrm{i}^*_{t+1} + E_t e_{t+1} - e_t \tag{2}$$

Note that (2) is only an approximation since, by the Jensen's inequality, $\ln E_t\{\mathcal{E}_{t+1}\} > E_t\{\ln \mathcal{E}_{t+1}\}$.
2. What is the dynamics of the exchange rate in the model?
Incorporating the PPP and the IUP conditions into the money demand gives:

$$m_t - p_t^* - e_t = -\eta \mathrm{i}^*_{t+1} - \eta(E_t\{e_{t+1}\} - e_t) + \phi y_t \tag{3}$$

$$\Leftrightarrow \quad m_t - \phi y_t + \eta \mathrm{i}^*_{t+1} - p_t^* - e_t = -\eta(E_t\{e_{t+1}\} - e_t) \tag{4}$$

Solving for e_t implies:

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} E_t\{m_s - \phi y_s + \eta \mathrm{i}^*_{s+1} - p_s^*\} \tag{5}$$

The model thus describes the behaviour of nominal exchange rates as a function of expectations of future variables, like in asset pricing equations. Namely, the nominal exchange rate depreciates (i.e. goes up) if:

- the path of the home money supply raises, thus increasing the domestic price level and the exchange rate (through PPP)
- the real domestic income goes down, thus contracting money demand which exerts a negative pressure on the domestic price level
- the foreign interest rate increases
- the foreign price level drops

An important component underlying this equation is the PPP assumption which only holds at the long-run level. Thus, this model must be thought as explaining the behaviour of equilibrium exchange rates.

3. Suppose $\eta \mathrm{i}^* - \phi y - p^* = 0$ and the money supply follows the process:

$$m_t - m_{t-1} = \rho(m_{t-1} - m_{t-2}) + \varepsilon_t$$

where ε is a serially uncorrelated mean-zero shock such that $E_{t-1}\{\varepsilon_t\} = 0$. Write the dynamics of the exchange rate.

Using (5), the expected rate of exchange rate depreciation writes:

$$E_t\{e_{t+1}\} - e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta}\right)^{s-t} E_t\{m_{s+1} - m_s\}$$

which can be incorporated in the money demand equation to get:

$$\begin{aligned} e_t &= m_t + \frac{\eta}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta}\right)^{s-t} E_t\{m_{s+1} - m_s\} \\ &= m_t + \frac{\eta\rho}{1+\eta-\eta\rho}(m_t - m_{t-1}) \end{aligned}$$

Under these assumptions, an unanticipated shock to m_t has two impacts. First, it raises the exchange rate directly by raising the current nominal money supply. Second, when $\rho > 0$, it also raises expectations of future money growth, thereby pushing the exchange rate even higher.

3. Assume now that the government wishes to fix the nominal exchange rate permanently at $\bar{e}$. What path of the money supply is consistent with having $e_t = \bar{e}$ permanently?

From (4), one gets:

$$\begin{aligned} & m_t - \bar{e} = -\eta(\bar{e} - \bar{e}) \\ \Rightarrow \quad & m_t = \bar{m} = \bar{e} \end{aligned}$$

Under our restrictive assumptions, a fixed exchange rate requires a level of the money supply that is permanently fixed. More generally, when $\eta \mathrm{i}^*_{t+1} - \phi y_t - p^*_t \neq 0$, the money supply becomes an endogenous variable. Fluctuations in m_t allows the home nominal interest rate to satisfy the UIP condition without any exchange rate adjustments.

4. During the beginning of the US Reagan administration in 1981, some US officials seriously discussed the possibility of making a transition to a fixed exchange rate for the dollar. However, they argued that it would be presumptuous for government officials to decide the best exchange rate and that they should instead let the market decide. The policy they proposed was to announce today that at some future date T they would permanently fix the exchange rate (using monetary policy) at whatever level prevailed in the market at time $T-1$. Is this a coherent policy?

In period $T-1$, the following relations hold:

$$m_{T-1} - \phi y_{T-1} + \eta i_T^* - p_{T-1}^* - e_{T-1} = -\eta(E_{T-1}e_T - e_{T-1}) = 0$$
$$i_T = i_T^* + E_{T-1}e_T - e_{T-1} = i_T^*$$

3 The Mundell-Fleming-Dornbusch model (OR, 9.2)

Consider a small open economy facing an exogenous world (foreign-currency) interest rate i^*. With open capital markets and perfect foresight, uncovered interest parity holds. Only domestic residents holds the domestic money, according to the following relationship:

$$m_t - p_t = \phi y_t - \eta \mathrm{i}_{t+1} \tag{6}$$

The Purchasing Power Parity needs not hold so that the real exchange rate $q_t = e_t + p_t^* - p_t$ can vary. (In the following, the foreign price p^* is assumed constant.)
The Dornbusch model effectively aggregates all domestic output as a single composite commodity and assumes that aggregate demand for home-country output, y_d, is an increasing function of the home real exchange rate:

$$y_t^d = \bar{y} + \delta(q_t - \bar{q}), \quad \delta > 0 \tag{7}$$

where $\bar{y}$ is the "natural" rate of output and $\bar{q}$ the "equilibrium" real exchange rate consistent with full-employment. For simplicity, $\bar{y}$ and $\bar{q}$ are assumed to be constant.
Prices are rigid in the short-run and flexible in the long-run. Namely, p_t is assumed to be predetermined so that unanticipated shocks can lead to excess demand or supply. In the Keynesian tradition, it is assumed that output is demand determined. The price level adjusts according to the inflation-expectations-augmented Phillips curve:

$$p_{t+1} - p_t = \psi(y_t^d - \bar{y}) + (\tilde{p}_{t+1} - \tilde{p}_t) \tag{8}$$

where $\tilde{p}_t \equiv e_t + p_t^* - \bar{q}_t$ is the price level that would prevail if the output market cleared.
In the following, it is assumed that $\psi\delta < 1$ which assures a monotonic adjustment of the real exchange rate. To simplify, simplifying nomalizations are also assumed, namely: $p^* = \bar{y} = \mathrm{i}^* = 0$.

1. Interpret the assumption $\delta > 0$ in equation (7).

Assuming $\delta > 0$ means that an increase in the relative price of foreign goods shifts the world demand towards domestic goods. This prediction survives in many microfounded theoretical models.

2. Interpret equation (8).

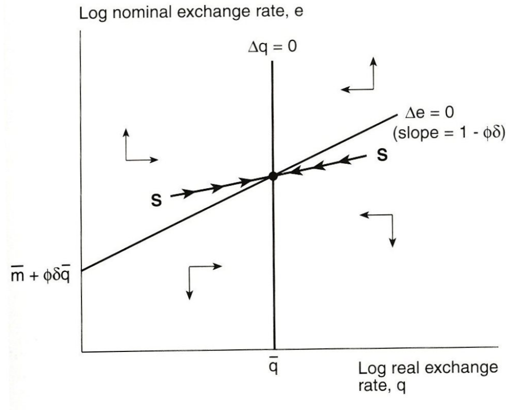
The first right-hand side term in equation (8) embodies the price inflation caused by date t excessive demand. The second-term provides for the price-level adjustment needed to keep up with expected inflation or productivity growth, ie to keep $y = \bar{y}$ if the output market were in equilibrium.

3. Assume that money supply is constant ($m_t = \bar{m}$). What is the the dynamics of the nominal and real exchange rates?

Using the aggregate demand (7), the Phillips curve equation (8) can be written as:

$$q_{t+1} - q_t = -\psi\delta(q_t - \bar{q}) \tag{9}$$

Figure 1: Phase diagram



Using the UIP relationship, the money demand (6) can be written in the following way:

$$e_{t+1} - e_t = \frac{e_t}{\eta} - \frac{1-\phi\delta}{\eta} q_t - \frac{\phi\delta\bar{q} + m_t}{\eta} \tag{10}$$

Equations (9) and (10) constitute a system of two first-order difference equations in q and e that can be solved analytically.
In the phase diagram (q, e) illustrated in Figure 1, the $\Delta q = 0$ schedule is vertical at $q_t = \bar{q}$ while the $\Delta e = 0$ schedule has vertical-axis intercept $\phi\delta\bar{q} + \bar{m}$ and (positive) slope $1 - \phi\delta$. The steady-state pair is defined by $\bar{e} = \bar{q} + \bar{m}$ which implies $\bar{p} = \bar{m}$. Last, the model has a saddle-path property. Outside this saddle path, the exchange rate has implosive/explosive trajectories. In itself, the model does not allow to argue rigorously that it won't be the case. But deriving the money demand from microfoundations shows that the no-bubbles path is the one that tightly links prices to fundamentals.
The analytical expression for the saddle-path is obtained by solving (9)-(10). First, rewrite equation (9) in deviation from the steady state $(\bar{q}, \bar{e} \equiv \bar{m} + \bar{q})$:

$$q_{t+1} - \bar{q} = (1 - \psi\delta)(q_t - \bar{q})$$

Given any date t deviation of the real exchange rate from its long-run value, the solution to equation (9) is:

$$q_s - \bar{q} = (1 - \psi\delta)^{s-t}(q_t - \bar{q}) \tag{11}$$

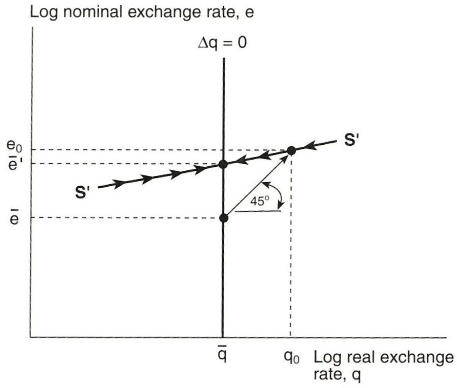
Incorporating this in (10) and solving for e_t gives:

$$\begin{aligned}
& e_t = \frac{\eta}{1+\eta}\left[e_{t+1} + \frac{1}{\eta}(m_t + \phi\delta\bar{q}) + \frac{1-\phi\delta}{\eta} q_t\right] \\
\Leftrightarrow \quad & e_t - \bar{q} = \frac{\eta}{1+\eta}(e_{t+1} - \bar{q}) + \frac{1-\phi\delta}{1+\eta}(q_t - \bar{q}) + \frac{m_t}{1+\eta} \\
\Leftrightarrow \quad & e_t - \bar{q} = \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s + \frac{1-\phi\delta}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}(q_s - \bar{q}) + \lim_{t\to\infty}\left(\frac{\eta}{1+\eta}\right)^{T} e_{t+T}
\end{aligned}$$

In the following, we impose a "Non-speculative bubbles" condition that assures the exchange rate will jump on the saddle-path after a shock:

$$\lim_{t\to\infty}\left(\frac{\eta}{1+\eta}\right)^{T} e_{t+T} = 0$$

Figure 2: Impact of an unanticipated monetary shock



When the money supply is constant ($m_t = \bar{m}$), the dynamics of the exchange rate is thus:

$$e_t - \bar{q} = \bar{m} + \frac{1-\phi\delta}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}(q_s - \bar{q})$$

$$\Leftrightarrow \quad e_t - \bar{q} = \bar{m} + \frac{1-\phi\delta}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}(1-\psi\delta)^{s-t}(q_t - \bar{q})$$

$$\Leftrightarrow e_t = \bar{q} + \bar{m} + \frac{1-\phi\delta}{1+\eta\psi\delta}(q_t - \bar{q}) \tag{12}$$

Equation (12) corresponds to the saddle-path.

4. What is the impact of an unanticipated permanent rise in the money demand at date 0?

In the long-run, the nominal exchange rate and the home price level increase in proportion to the change in the money supply. In the short-run however, p is predetermined and thus $p_0 = \bar{m}$ and $q_0 = e_0 + \bar{m}$. The nominal exchange rate adjusts immediately to the monetary shock and allows the economy to jump on the new saddle-path point (q_0, e_0). As can be shown graphically in Figure 2, $e_0 > \bar{e}'$, ie the exchange rate initially changes more than proportionately to the money shock. This is Dornbusch's "overshooting" result. Its popularity is that it allows explaining the surprising volatility of floating exchange rates in a rational expectations framework.
To show analytically the overshooting effect, use the initial conditions: $p_0 = \bar{m}$ (price predetermined) and $q_0 = e_0 - \bar{m}$ with the saddle-path equation at date 0:

$$e_0 = \bar{m}' + \bar{q} + \frac{1-\phi\delta}{1+\psi\delta\eta}(q_0 - \bar{q})$$

From this, one can compute the initial effect of the monetary shock on the real exchange rate:

$$q_0 = \bar{q} + \frac{1+\psi\delta\eta}{\phi\delta+\psi\delta\eta}(\bar{m}' - \bar{m}) > \bar{q}$$

Incorporating in the definition of the nominal exchange rate gives:

$$e_0 = \bar{m}' + \bar{q} + \frac{1-\phi\delta}{\phi\delta+\psi\delta\eta}(\bar{m}' - \bar{m}) > \bar{e}' \; if \; \phi\delta < 1$$

This shows that the nominal exchange rate initially overshoots as long as $\phi\delta < 1$. Finally, the dynamics of the exchange rate on the transition path writes:

$$\begin{aligned} e_t &= \bar{m}' + \bar{q} + \frac{1-\phi\delta}{1+\psi\delta\eta}(q_t - \bar{q}) \\ &= \bar{m}' + \bar{q} + \frac{1-\phi\delta}{\phi\delta+\psi\delta\eta}(1-\psi\delta)^t(\bar{m}' - \bar{m}) \end{aligned}$$

The underlying mechanism is as follows. On impact, the monetary shock causes an increase in real money balances since p is initially fixed:

$$\bar{m}' - p = \Delta\bar{m}$$

If the exchange rate jumped immediately to its new steady state, output would rise by $\delta\Delta\bar{m}$ (equation (7). Money demand would rise by $\phi\delta\Delta\bar{m}$. As long as $\phi\delta < 1$, money demand rises less than the rise in money supply and thus, the home nominal interest i must fall below i^* to restore the money-market equilibrium (6). This implies an expected fall in e by the IUP condition, which contradicts our initial supposition that e jumps to its new steady state. When $\phi\delta < 1$, the only possibility is for the currency initially to overshoot its long-run level. As a consequence, the aggregate demand increases and the nominal interest rate decreases. $i < i^*$ is a solution since future appreciation is a rational expectation if the initial exchange rate overshoots. The nominal depreciation of domestic currency implies a real depreciation under sticky prices. This real depreciation raises aggregate demand, so output rises temporarily above its steady-state value. Finally, note that the correlation between nominal interest rates and exchange rates is negative when monetary shocks are permanent.

5. Consider now the same model with $\bar{y} \neq 0$. Discuss and show graphically (and partially analytically) the short-run and the long-run effects of a rise in natural output.

The drop in the natural rate of output shifts the stationary schedule for the nominal exchange rate downwards. The stationary real exchange rate is unchanged but the long-run nominal exchange rate is reduced as a consequence. As in the case of the monetary shock, the long-run price level adjusts: $\Delta\bar{p} = \Delta\bar{e} = -\phi\Delta\bar{y}$ while the nominal interest rate remains $i*$. In the short-run however, the price level is sticky which again explains that the economy does not immediately jumps on its new steady state. Using the saddle-path equation, one finds:

$$q_0 = \bar{q} - \frac{\phi(1+\eta\psi\delta)}{\phi\delta+\eta\psi\delta}(\bar{y}' - \bar{y}) < \bar{q}$$

$$e_0 = \bar{m} - \phi\bar{y}' + \bar{q} - \frac{\phi(1+\eta\psi\delta)}{\phi\delta+\eta\psi\delta}(\bar{y}' - \bar{y}) < \bar{e}$$

Thus, the exchange rate initially underadjusts in this case.