Lecture 1: Traditional Open Macro Models and Monetary Policy

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Introduction

- Many important questions in international macroeconomics involve monetary issues
- The main departure with respect to a closed economy is that several monetary authorities play independently, managing different currencies.
- Introducing money in a model allows addressing a number of issues: determinants of seignorage, mechanics of exchange-rate systems, long-run effects of money-supply changes on prices and exchange rates

Introduction (2)

- Role of money
  i) Medium of exchange
  ii) Store of value
  iii) Nominal unit of account
- Nature of money
  - Here, money is meant as currency (abstract from the banking system)
  - Money does not bear interest $\Rightarrow$ Simplifying assumption $\approx$ Liquidity premium

The Cagan Model

Hypotheses

- Simple empirical model of money and inflation used to study hyperinflations (ie inflation $>$ 50% per month, ex Zimbabwe: 100 000% in january 2008)
- Prices are fully flexible $\Rightarrow$ Adjust to clear product, factor and asset markets $\Rightarrow$ Long-run analysis
- Stochastic, discrete-time model
- Rational expectations

Hypotheses (2)

- Demand for real money balances depends on expected future price-level inflation:

$$m_t^d - p_t = -\eta E_t\{p_{t+1} - p_t\}$$

Higher expected inflation lowers the demand for real balances by raising the opportunity cost of holding money

⇒ Ignore real determinants to focus on hyperinflation period ⇒ Simplified form of Keynes LM curves:

$$m_t^d - p_t = \phi y_t - \mu \mathrm{i}_{t+1}, \quad \textit{with} \quad 1 + i_{t+1} = (1 + r_{t+1})\frac{P_{t+1}}{P_t}$$

- Money supply m_t exogenously determined

Monetary equilibrium

- In equilibrium:

$$\begin{aligned} & m_t = m_t^d \\ \Leftrightarrow \quad & m_t - p_t = -\eta E_t\{p_{t+1} - p_t\} \end{aligned}$$

$\Rightarrow$ First-order stochastic difference equation explaining price-level dynamics in terms of the money supply

Equilibrium price level

$$
\begin{aligned}
p_t &= \frac{1}{1+\eta}\left[m_t + \eta E_t\{p_{t+1}\}\right] \\
&= \frac{1}{1+\eta}\left[\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} E_t\{m_s\}\right] + \lim_{t\to\infty}\left(\frac{\eta}{1+\eta}\right)^{T} E_t\{p_{t+T}\} \\
p_t &= \frac{1}{1+\eta}\left[\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} E_t\{m_s\}\right]
\end{aligned}
$$

- "no-speculative bubble" condition:

$$
\lim_{T\to\infty}\left(\frac{\eta}{1+\eta}\right)^{T} E_t\{p_{t+T}\} = 0
$$

The limit is indeed zero unless the absolute value of the log price level grows exponentially at a rate of at least $(1+\eta)/\eta$

Equilibrium price level (2)

- The price level depends on a weighted average of future expected money supplies, with weights that decline geometrically as the future unfolds
- Note that:

$$\frac{1}{1+\eta}\left[\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}\right]=\frac{1}{1+\eta}\left(\frac{1}{1-\frac{\eta}{1+\eta}}\right)=1$$

$\Rightarrow$ Money is fully neutral in the absence of nominal rigidities or money illusion

Constant money supply

- $m_t = \bar{m}, \forall t$

$\Rightarrow$ Zero expected inflation : $E_t p_{t+1} - p_t = 0$,

$\Rightarrow$ Constant price level: $\bar{p} = \bar{m}$

Constant money supply growth

- $m_t = \bar{m} + \mu t$

$\Rightarrow$ Constant expected inflation : $E_t p_{t+1} - p_t = \mu$,

$\Rightarrow$ Constant price level growth:

$$\begin{aligned} p_t &= \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} [m_t + \mu(s-t)] \\ &= m_t + \frac{\mu}{1+\eta} \eta(1+\eta) \\ p_t &= m_t + \mu\eta \end{aligned}$$

Autoregressive money supply

- $m_t = \rho m_{t-1} + \varepsilon_t, \quad 0 \leq \rho \leq 1, \quad E_t\{\varepsilon_{t+1}\} = 0$
- Price level:

$$p_t = \frac{m_t}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta\rho}{1+\eta} \right)^{s-t} = \frac{m_t}{1+\eta-\eta\rho}$$

In the limiting case $\rho = 1$ in which money shocks are expected to be permanent, the solution reduces to $p_t = m_t$.

Announced rise in money supply

- $m_t = \bar{m}, \forall t < T$
  $m_t = \bar{m}', \forall t \geq T$
- Price level:

$$p_t = \begin{cases} \bar{m} + \left(\frac{\eta}{1+\eta}\right)^{T-t} (\bar{m}' - \bar{m}), & t < T \\ \bar{m}', & t \geq T \end{cases}$$

- The supply shock is integrated in the effective price level as long as it is announced by the government.

Announced rise in money supply (2)

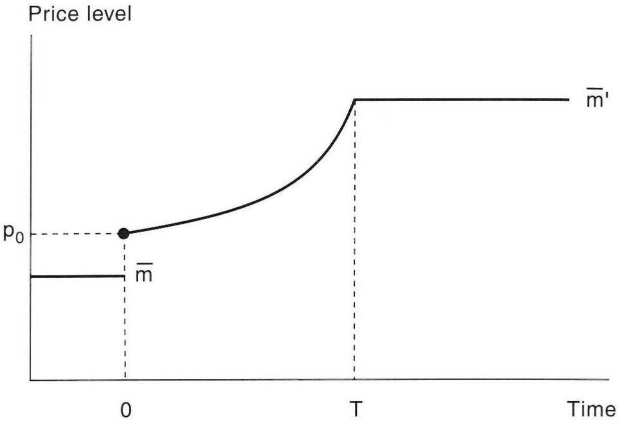

Figure 8.1
A perfectly anticipated rise in the money supply

Seignorage

- Real revenues a government acquires by using newly issued money to buy goods and nonmoney assets:

$$Seignorage = \frac{M_t - M_{t-1}}{P_t} \equiv \frac{M_t - M_{t-1}}{M_t} . \frac{M_t}{P_t}$$

⇒ If higher money growth raises expected inflation, the demand for real balances may fall, which exerts a negative influence on seignorage revenues ⇒ Marginal revenue from money growth can be negative ⇒ Limit to seignorage.

⇒ Optimal rate of inflation defined by:

$$\begin{cases} Max_\mu \frac{M_t - M_{t-1}}{M_t} . \frac{M_t}{P_t} \\ s.c. \quad \frac{M_t}{P_t} = \left(E_t \frac{P_{t+1}}{P_t} \right)^{-\eta} \\ with \quad \mu = \frac{M_t - M_{t-1}}{M_t} \end{cases}$$

Optimal Seignorage under Constant Money Growth

- $\frac{M_t}{M_{t-1}} = \frac{P_t}{P_{t-1}} = 1 + \mu$
- The optimal growth rate of money supply is then:

$$\mu^* = \frac{1}{\eta}$$

- Inverse function of the semielasticity of real balances with respect to inflation

How important is seignorage ?

Table: Average 1990-94 seignorage revenues in industrialized countries

Country	% Government spending	% GDP
Australia	0.95	0.31
Canada	0.84	0.09
France	-0.83	-0.23
Germany	2.89	0.56
Italy	3.11	0.32
New Zealand	0.04	0.01
Sweden	3.22	1.52
United States	2.19	0.44

Source: Obstfeld & Rogoff from IMF-IFS data

Limits

$\Rightarrow$ How can we explain periods of hyperinflation, in which governments obviously let money growth exceed the optimal rate? Backward-looking expectations ?

$\Rightarrow$ Credibility issues : On date 0, the government announces that it will stick to the revenue-maximizing rate of money growth $\rightarrow$ If agents believe it, they hold real balances $M/P = [(1+\eta)/\eta]^{-\eta}$ $\rightarrow$ On date 1, the government has an incentive to cheat and choose a higher money growth rate $\rightarrow$ If governments lack credit, agents will anticipate the government's temptation to cheat.

Open-economy extension
Obstfeld & Rogoff

Hypotheses of the model

- Small open economy
- Exogenous output
- Money demand defined by:

$$m_t - p_t = -\eta \mathrm{i}_{t+1} + \phi y_t$$

- Flexible prices and PPP:

$$p_t = e_t + p_t^*$$

with e_t the (log of) nominal exchange rate (home currency per unit of foreign currency) and p_t^* the world foreign-currency price

Hypotheses of the model (2)

- Uncovered interest parity:

$$\begin{aligned} 1 + i_{t+1} &= (1 + i^*_{t+1}) E_t \left\{ \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} \\ \Leftrightarrow \mathrm{i}_{t+1} &= \mathrm{i}^*_{t+1} + E_t e_{t+1} - e_t \end{aligned}$$

- Simple arbitrage argument under perfect foresight and no exchange-rate risk premium
- Note that the log UIP relation is only an approximation since, by the Jensen's inequality, $\ln E_t\{\mathcal{E}_{t+1}\} > E_t\{\ln \mathcal{E}_{t+1}\}$.

Exchange-rate dynamics

- Incorporating the PPP and the IUP conditions into the money demand gives:

$$\begin{aligned} & m_t - p_t^* - e_t = -\eta \mathrm{i}_{t+1}^* - \eta(E_t\{e_{t+1}\} - e_t) + \phi y_t \\ \Leftrightarrow \quad & m_t - \phi y_t + \eta \mathrm{i}_{t+1}^* - p_t^* - e_t = -\eta(E_t\{e_{t+1}\} - e_t) \end{aligned}$$

- Solving for e_t implies:

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta}\right)^{s-t} E_t\{m_s - \phi y_s + \eta \mathrm{i}_{s+1}^* - p_s^*\}$$

Exchange-rate dynamics (2)

$\Rightarrow$ Describes the behaviour of nominal exchange rates as a function of expectations of future variables ($\approx$ asset pricing equations).

- Nominal exchange-rate depreciation if:
  - the path of the home money supply raises, thus increasing the domestic price level and the exchange rate (through PPP)
  - the real domestic income goes down, thus contracting money demand which exerts a negative pressure on the domestic price level
  - the foreign interest rate increases
  - the foreign price level drops
- Note that this equation relies on a PPP assumption $\Rightarrow$ Long-run Model

Autoregressive money growth

- $m_t - m_{t-1} = \rho(m_{t-1} - m_{t-2}) + \varepsilon_t$ where ε iid, $E_{t-1}\{\varepsilon_t\} = 0$
- Expected rate of exchange rate depreciation:

$$E_t\{e_{t+1}\} - e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta}\right)^{s-t} E_t\{m_{s+1} - m_s\}$$

- Exchange rate level:

$$\begin{aligned} e_t &= m_t + \frac{\eta}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta}\right)^{s-t} E_t\{m_{s+1} - m_s\} \\ &= m_t + \frac{\eta\rho}{1+\eta-\eta\rho}(m_t - m_{t-1}) \end{aligned}$$

$\Rightarrow$ Impact of an unanticipated shock to m_t: direct exchange rate increase (raises the current nominal money supply) + when $\rho > 0$, increases expectations of future money growth, thereby pushing the exchange rate even higher.

Exchange rate fixing

- Fixed exchange rate: $e_t = \bar{e}$ and $\eta \mathrm{i}^* - \phi y - p^* = 0$

$\Rightarrow$ Fixed money supply: $m_t = \bar{m} = \bar{e}$

- Fixed exchange rate: $e_t = \bar{e}$ and $\eta \mathrm{i}^* - \phi y - p^* \neq 0$

$\Rightarrow$ Money supply endogenous, Adjustment to market-driven fluctuations in i^*

- Future fixing at some future date T: $e_t = \bar{e}, \forall t \geq T$
  In period $T-1$:

$$m_{T-1} - \phi y_{T-1} + \eta i^*_T - p^*_{T-1} - e_{T-1} = -\eta(E_{T-1}e_T - e_{T-1}) = 0$$
$$i_T = i^*_T + E_{T-1}e_T - e_{T-1} = i^*_T$$

$\Rightarrow$ i adjusts to satisfy the UIP relation. The monetary equilibrium implies that m also adjusts, whatever the exchange rate level the private sector expects $\Rightarrow$ The announcement is not a well-adapted solution for the exchange rate market to converge towards an "equilibrium" value.