Master EPP, International Macroeconomics
Homework
Agglomeration and Home Market Effect (Martin & Rogers, 1995)

This exercise has to be done for October, 30th. You are expected to provide all details concerning the derivation of analytical results and interpret them.

The model is a variant of Helpman and Krugman (1985) and Krugman (1991) that features "agglomeration effects", ie economic activity tends to generate forces that encourage further concentration of economic activity.

There are two regions, two sectors, and two productive factors. In the following, the regions are referred to as the north and the south (N and S); they are symmetric in terms of tastes, technology and openness to trade. The two sectors are referred to as manufacturing and agriculture (M and A). The manufacturing sector is marked by increasing returns, monopolistic competition and iceberg trade costs. On the other hand, the agricultural sector is assumed to produce a homogeneous good under Walrasian conditions (constant returns and perfect competition) and its output is traded costlessly.

The product factors are physical capital (K) and labor (L). Worldwide supplies of capital and labor are fixed, with the world's endowment denoted as L_W and K_W. $s_L = L_N/L_W$ and $s_K = K_N/K_W$ respectively denote the (exogenous) shares of worldwide endowments in labor and capital owned by the North. Both factors can freely move across sectors. Capital is perfectly mobile across regions, while labor is internationally immobile. Importantly, capital owners are immobile: physical capital can be employed in one region while its owner spends its reward in the other region. Thus, when pressures arise to concentrate production in one region, physical capital moves, but all of its reward is repatriated to its country of origin.

To simplify, it is assumed that technologies are the same in both regions. Technology in the agricultural sector only uses labor with the following production function:

$$Y_i^A = \frac{L_i^A}{a^A}$$

where Y_i^A is region i's production of agricultural good obtained from L_i^A labor units. The equilibrium price for labor is denoted $w_i,\ i = N/S$.

In the manufacturing sector, technology involves increasing returns. The cost function of a typical firm is non-homothetic: the factor intensity of the fixed cost differs from the factor intensity of the variable cost. To keep things simple, it is assumed that the fixed cost only involves capital while the variable cost involves labor. More specifically, each industrial firm requires one unit of capital, which equilibrium reward is denoted π_i, and a^M units of labor per unit of output.

Finally, while the agricultural good is freely traded across countries, exporting manufacturing goods involves an additional "iceberg" cost τ: to sell one unit abroad, the firm has to produce τ units because of real losses occuring during the transportation.

In each region, the representative consumer has preferences given by:

$$U_i = C_i, \quad C_i = {C_i^M}^{\mu}\, {C_i^A}^{1-\mu}, \quad C_i^M = \left(\int_0^{n_W} c_{is}^{\frac{\sigma-1}{\sigma}}\, ds\right)^{\frac{\sigma}{\sigma-1}}, \quad i = N/S,\ 0<\mu<1<\sigma$$

C_i^M and C_i^A respectively denote country i's consumption of manufacturing and agricultural goods. Each industry represents a constant share of aggregate consumption. Aggregate consumption of manufactured goods is a CES over all varieties s produced at the world level with σ the elasticity of substitution between any two varieties. The mass n_W of varieties available at the world level is endogenously determined, as well as its worldwide distribution. In the following, it is assumed that firms in the $[0, n]$ interval are located in the North and firms in the $[n, n_W]$ interval are located in the south. The share of firms located in the north, also equal to the region's share of worldwide capital, is denoted $s_n = n/n_W$.

1. Show that, in optimum, sectoral demand functions are:

$$C_i^M = \mu \frac{P_i C_i}{P_i^M}, \quad C_i^A = (1-\mu)\frac{P_i C_i}{P_i^A}, \quad i = N/S$$

Write the aggregate price index P_i as a function of sectoral price indices P_i^A and P_i^M.

2. Solve for the demand of individual varieties as a function of manufacturing consumption. Derive the manufacturing price index P_i^M.

3. Derive the equilibrium price of agricultural goods P_i^A. What does the free trade assumption in the agricultural sector implies concerning relative wages in both regions? (In the following, we will assume that some agricultural good is produced in both regions.)

4. Find equilibrium prices in the manufacturing sector.

5. Explain why the following "capital migration" equation holds:

$$\dot{s}_n = (\pi_N - \pi_S)(1 - s_n)s_n$$

where $\dot{s}_n$ is the change in the worldwide distribution of firms (and capital).

6. Use the free-entry condition in the manufacturing sector to find the equilibrium rewards for capital:

$$\begin{aligned} \pi_N &= \frac{\mu}{\sigma}\frac{E_W}{n_W}\left[\frac{s_E}{\Delta_N} + \tau^{1-\sigma}\frac{1-s_E}{\Delta_S}\right] \\ \pi_S &= \frac{\mu}{\sigma}\frac{E_W}{n_W}\left[\tau^{1-\sigma}\frac{s_E}{\Delta_N} + \frac{1-s_E}{\Delta_S}\right] \end{aligned}$$

where $E_W = P_N C_N + P_S C_S$ is the world expenditure, s_E is the (endogenous) share of north in world expenditures, $\Delta_N \equiv s_n + \tau^{1-\sigma}(1-s_n)$, $\Delta_S \equiv s_n\tau^{1-\sigma} + (1-s_n)$.

7. Show that the worldwide expenditure E_W can be written as:

$$E_W = \frac{wL_W}{1 - \mu/\sigma}$$

(Notice that expenditure and income are identical since there is no savings in this model: $E_W = P_N C_N + P_S C_S = ws_L L_W + \pi_N s_n K_W + w(1-s_L)L_W + \pi_S(1-s_n)K_W$.)

8. In the spirit of symmetry we make the straightforward assumption that half of the capital in each region belongs to northern capital owners regardless of s_n. That is, even if only a quarter of world capital is working in the south, we assume that half of that quarter comes from northern capital owners and half comes from southern capital owners. The ramification is that north's capital earns the world average reward.

Prove that the world average reward is equal to

$$\frac{\mu}{\sigma}\frac{E_W}{K_W}$$

Incorporating this in the expression for the north nominal expenditure ($P_N C_N$), show that the following relation linking s_E, s_L and s_K holds:

$$s_E = \left(1 - \frac{\mu}{\sigma}\right)s_L + \frac{\mu}{\sigma}s_K$$

Interpretation.

9. In the long run, capital migration stops. Prove that, in the long-run equilibria and when some manufacturing good is produced in each country (ie $s_n \in]0,1[$), the distribution of firms (and capital) is the following:

$$s_N = \frac{1}{2} + \frac{1+\tau^{1-\sigma}}{1-\tau^{1-\sigma}} \left(s_E - \frac{1}{2} \right)$$

Using this equilibrium relation and the constraint on feasible values for s_n, find the conditions under which the production of manufacturing goods is entirely concentrated in a single region. Interpretation. Explain why this model is said to be featured by a "Home Market Effect".

10. In the short-run, capital flows can occur. Prove that, outside the long-run equilibrium, the sign of the capital reward differential is:

$$sign(\pi_N - \pi_S) = sign \left[(1+\tau^{1-\sigma}) \left(s_E - \frac{1}{2} \right) - (1-\tau^{1-\sigma}) \left(s_n - \frac{1}{2} \right) \right]$$

Interpretation.