Abstract

We study the pro-competitive effects of international trade, or lack thereof, in models with monopolistic competition, firm-level heterogeneity, and variable markups. Under standard restrictions on consumers’ demand and the distribution of firms’ productivity, we derive two theoretical results. First, although markups vary across firms, the distribution of markups and the share of aggregate profits in revenues are invariant to changes in openness to trade. Second, although the distribution of markups and the share of aggregate profits in revenues are unaffected by trade, gains from trade liberalization are weakly lower than those predicted by the models with constant markups considered in Arkolakis, Costinot, and Rodríguez-Clare (2012).
1 Introduction

How large are the gains from trade liberalization? In earlier work, Arkolakis, Costinot, and Rodríguez-Clare (2012), ACR hereafter, have shown that in an important class of trade models, the answer is pinned down by two statistics: (i) the share of expenditure on domestic goods, $\lambda$; and (ii) an elasticity of imports with respect to variable trade costs, $\varepsilon$, which we refer to as the trade elasticity. If a small change in trade costs raises trade openness in some country, $d \ln \lambda < 0$, then the associated welfare gain is given by

$$d \ln W = - \frac{d \ln \lambda}{\varepsilon},$$

where $d \ln W$ is the compensating variation associated with the shock expressed as a percentage of the income of the representative agent.

While the previous formula applies both to models with perfect and monopolistic competition, it relies on the assumption that all agents have Constant Elasticity of Substitution (CES) utility functions. This implies that the models with monopolistic competition considered by ACR necessarily feature constant markups, which de facto rules out any “pro-competitive” effects of trade. This is, potentially, a major limitation of our welfare formula. As Helpman and Krugman (1989) have noted, “the idea that international trade increases competition [...] goes back to Adam Smith, and it has long been one of the reasons that economists give for believing that the gains from trade and the costs from protection are larger than their own models seem to suggest.”

In this paper we drop the CES assumption and study how variable markups affect the gains from trade liberalization under monopolistic competition. Our main finding is that under standard assumptions on consumers’ demand and the distribution of firms’ productivity, which we describe in detail below, the welfare effect of a small trade shock is given by

$$d \ln W = - (1 - \eta) \frac{d \ln \lambda}{\varepsilon},$$

where $\eta$ is a structural parameter that depends, among other things, on the elasticity of markups with respect to firm productivity. Under parameter restrictions commonly imposed in the existing literature, $\eta$ is non-negative. Thus, perhaps surprisingly, gains from trade liberalization predicted by these new models are weakly lower than those predicted by models with constant markups.

Section 2 describes the basic environment in which this new formula applies. On the demand side, we start from a general demand system that encompasses the three main alternatives to CES utility functions considered in the trade literature: (i) separable, but
non-CES utility functions, as in the pioneering work of Krugman (1979) and the more recent work of Behrens and Murata (2009), Behrens, Mion, Murata, and Sudekum (2009), Saure (2009), Simonovska (2009), Dhingra and Morrow (2012) and Zhelobodko, Kokovin, Parenti, and Thisse (2011); (ii) a quadratic, but non-separable utility function, as in Ottaviano, Tabuchi, and Thisse (2002) and Melitz and Ottaviano (2008); and (iii) a translog expenditure function, as in Feenstra (2003), Bergin and Feenstra (2009), Feenstra and Weinstein (2010), Novy (2010), and Rodriguez-Lopez (2010). In addition, we assume that there exists a finite choke price for all varieties, implying that consumers are not willing to spend an infinite amount to consume an extra variety of a differentiated good. This allows us to abstract from the welfare effects associated with the entry and exit of firms selling at that price.

On the supply side, in order to isolate the contribution of variable markups to the gains from trade liberalization, we maintain the same assumptions as in the models of monopolistic competition with firm-level heterogeneity considered in ACR. In particular, we assume that the distribution of firm-level productivity is Pareto.

Section 3 characterizes the trade equilibrium. We first describe how markups vary across firms as a function of their productivity. We show that firm-level markups only depend on the log-difference between the choke price and firms’ marginal costs. When demand is log-concave, as assumed in Krugman (1979), markups are increasing with firm-level productivity, which implies incomplete pass-through of changes in marginal costs to prices. Log-concavity is one of the standard parameter restrictions alluded to before that leads to \( \eta \geq 0 \) and lower gains from trade liberalization.

Whether or not demand functions are log-concave, bilateral trade flows satisfy the same gravity equation as in ACR. Thus, conditional on an estimate of the trade elasticity, the macro-level predictions of models considered in this paper—namely, the predictions regarding the effects of changes in trade costs on wages and trade flows—are the same as in quantitative trade models with CES utility functions such as Krugman (1980), Eaton and Kortum (2002), Anderson and Van Wincoop (2003), and the versions of the Melitz (2003) model developed by Chaney (2008) and Eaton, Kortum, and Kramarz (2011).

Section 4 explores the pro-competitive effects of trade, or lack thereof, in the economic environment described in Sections 2 and 3. We establish two sets of results. First, although markups vary across firms, the distribution of firm-level markups and the share

1 Although the demand for differentiated goods in Ottaviano, Tabuchi, and Thisse (2002) and Melitz and Ottaviano (2008) is a special case of the demand system that we consider, it further requires the existence of an “outside good.” We discuss the robustness of our results to the introduction of such a good in Section 6.

2 The main drawback of this assumption is that the case of a separable utility function satisfying Inada conditions, including the CES case, is not formally covered by the present analysis.
of aggregate profits in any country are *invariant* to changes in trade costs. Second, although the distribution of markups and the share of aggregate profits are fixed, the gains from a small decline in trade costs are weakly lower than those predicted by ACR’s formula under parameter restrictions commonly imposed in the existing literature.

Our first set of results highlights the countervailing effects of a productivity change, in general, and a change in trade costs, in particular, on markups. Under standard assumptions, productivity gains tend to raise the markups of incumbent firms, but also lead to the entry of less efficient firms, which charge lower markups. Under Pareto, the second effect exactly offsets the first one.\(^3\)

Our welfare result is more subtle. It builds on the point made above that standard parameter restrictions imply incomplete pass-through of changes in marginal costs from firms to consumers. Firms that become more productive because of lower trade costs tend to raise their markups, which tends to lower the welfare gains from trade. Although the distribution of markups is not changing, what matters for welfare is not the unweighted average of markups, but instead the average of markups weighted by expenditure shares. In particular, the entry or exit of the least efficient firms is irrelevant for welfare since these firms have zero sales. For welfare purposes, only changes in the markups of incumbent firms matter. Our new welfare formula shows that, under standard parameter restrictions, once all general equilibrium effects have been accounted for, incumbents’ markups tend to go up.

This result may seem to go against Krugman (1979)’s finding that under monopolistic competition, if demand functions are log-concave—which is one of our standard parameter restrictions—then larger markets are associated with lower markups. After all, an increase in market size is formally equivalent to an infinite decrease in trade costs starting from autarky. The apparent contradiction between the two sets of results is resolved by noticing that Krugman (1979)’s analysis of markups implicitly focuses on domestic firms, whereas our welfare analysis is crucially affected by the behavior of foreign markups.

Our analysis is related to, and has implications for, a large number of theoretical and empirical papers in the international trade literature. On the empirical side, many authors have studied the relationship between international trade and firm-level markups; see e.g. Levinsohn (1993), Harrison (1994), Krishna and Mitra (1998), Konings, Van Cayseele, and Warzynski (2001), Chen, Imbs, and Scott (2009), Loecker and Warzynski (2012), and

\(^3\)A similar compositional effect is at play in Bernard, Eaton, Jensen, and Kortum (2003). In their model, Bertrand competition leads to variable markups at the firm-level, but distributional assumptions similar to ours make the distribution of markups invariant to changes in trade costs. de Blas and Russ (2010) and Holmes, Hsu, and Lee (2010) study how departures from this distributional assumptions affect markups.
Methodologies, data sources, and conclusions vary, but a common feature of the aforementioned papers is their exclusive focus on domestic producers. As just discussed, however, our results highlight that the overall pro-competitive effects of trade may be very different from the effects on domestic producers. Under standard assumptions, a decrease in trade costs reduces the markups of domestic producers. Yet, because it also increases the markups of foreign producers, gains from trade liberalization are actually lower than those predicted by standard models with CES utility functions.

A recent empirical paper by Feenstra and Weinstein (2010) is closely related to our analysis. The authors use a translog demand system—which is one of the demand systems covered by our analysis—to measure the contribution of new varieties and variable markups on the change in the U.S. consumer price index between 1992 and 2005. They find that the contribution of these two margins is of the same order of magnitude as the contribution of new varieties estimated by Broda and Weinstein (2006) under the assumption of CES preferences. Our theoretical results show that in the translog case, the overall gains from a hypothetical decline in trade costs are exactly the same as under CES, which resonates well with Feenstra and Weinstein (2010)’s empirical findings. It should be clear, however, that the two exercises are distinct. Feenstra and Weinstein (2010) is a measurement exercise that uses observed trade data to infer changes in particular components of the U.S. price index. The present paper is a counterfactual exercise that focuses on the welfare effect of a particular shock, namely a change in variable trade costs. This approach has both advantages and limitations: it relies on the full structure of the model, including the distribution of firm-level productivity being Pareto, but it allows us to take all general equilibrium effects into account when computing the exact welfare changes caused by trade liberalization.

On the theory side, the introduction of imperfect competition into trade models has been one of the major accomplishments in the international trade literature; see Helpman and Krugman (1985) and Helpman and Krugman (1989). One important message of this literature is that the consequences of changes in trade policy are very sensitive to the underlying market structure. Even in the case of an industry with a single monopolist, changes in trade costs have very different effects on consumer prices and welfare.

4 Among those, Goldberg, Loecker, Khandelwal, and Pavcnik (2012) is particularly relevant to the present analysis. Under general assumptions that strictly encompass ours, they estimate the elasticity of markups with respect to marginal costs for a large number of Indian firms. These estimates could provide one important piece of information to estimate η, which we plan to use in Section 5.

5 In the translog case, our new welfare formula readily extends to the case of large changes in trade costs, as we discuss in Section 6.
depending on whether the monopolist is a domestic or a foreign firm. As Helpman and Krugman (1989) note—and formalize in a model with one foreign monopolist—a “popular argument about tariffs is that they will be largely absorbed through a decline in foreign markups rather than passed onto consumers.” The flipside of that argument is that a decrease in trade costs may also be largely absorbed through an increase in foreign markups rather than passed onto consumers. In this regard, the main contribution of our paper is to show that under restrictive, but commonly used assumptions on market structure, demand, and firm-level productivity, this channel dominates the pro-competitive effects on domestic firms often emphasized in academic and policy discussions.

Since there are many ways to introduce imperfect competition and variable markups, we have made our modeling choices based on two main considerations. First, we have tried to stay as close as possible to existing trade papers in this area, such as Krugman (1979), Feenstra (2003), and Melitz and Ottaviano (2008). Second, we have tried to stay as close as possible to ACR’s assumptions in order to isolate how variable markups affect the mapping between trade openness, as measured by the share of domestic expenditure $\lambda$, and welfare. In recent work, Edmond, Midrigan, and Xu (2011) have used the model with two symmetric countries, Cournot competition, and CES utility functions developed by Atkeson and Burstein (2008) to study the magnitude of the gains from trade in economies with variable markups. When calibrating this model using data on manufacturing Taiwanese firms, they find gains from trade that are significantly larger than those predicted by ACR’s formula, though the numbers vary depending on assumptions made on the correlation of productivity between Taiwanese and non-Taiwanese firms. We come back to the comparison between their quantitative results and ours in Section 5.

2 Basic Environment

We consider a world economy comprising $i = 1, ..., n$ countries, one factor of production, labor, and a continuum of differentiated goods $\omega \in \Omega$. All individuals are endowed with one unit of labor, are perfectly mobile across the production of different goods, and are immobile across countries. $L_i$ denotes the total endowment of labor and $w_i$ denotes the wage in country $i$.

2.1 Consumers

The goal of our paper is to study the implications of trade models with monopolistic competition for the magnitude of the gains from trade in economies in which markups
are variable. This requires departing from the assumption of CES utility functions. The existing trade literature has proposed three alternatives: (i) separable, but non-CES utility functions, as in the pioneering work of Krugman (1979); (ii) a quadratic, but non-separable utility function, as in Ottaviano, Tabuchi, and Thisse (2002); and (iii) a translog expenditure function, as in Feenstra (2003). In this paper, we consider a general demand system for differentiated goods that encompasses all three.\(^6\)

All consumers have the same preferences. If a consumer with income \(w\) faces a schedule of prices \(p = \{p_\omega\}_{\omega \in \Omega}\), her Marshallian demand for any differentiated good \(\omega\) takes the form

\[
\ln q_\omega(p, w) = -\beta \ln p_\omega + \gamma \ln w + d \left( \ln p_\omega - \ln p^*(p, w) \right),
\]

where \(p^*(p, w)\) is symmetric in all prices. Three properties of our demand system are worth emphasizing. First, the price elasticity \(-\beta + d' \left( \ln p_\omega - \ln p^*(p, w) \right)\) is allowed to vary with prices, which will generate variable markups under monopolistic competition. Second, other prices only affect the demand for good \(\omega\) through their effect on the aggregator \(p^*(p, w)\).\(^7\) Third, the difference between the price elasticity and the cross-price elasticity, i.e. the elasticity with respect to \(p^*(p, w)\), is constant and equal to \(-\beta\). This parameter will play a crucial role in our welfare analysis.

It is easy to check that the previous specification encompasses the case of separable utility functions considered in Krugman (1979). Using our notation, his model corresponds to a situation in which preferences are represented by a utility function, \(U = \int_{\omega \in \Omega} u(q_\omega)dw\) with \(u'(0) < \infty\). The first-order conditions associated with utility maximization imply \(u'(q_\omega) = \lambda(p, w) p_\omega\), where \(\lambda(p, w)\) is the Lagrangian multiplier of the budget constraint, i.e., \(\int_{\omega \in \Omega} q_\omega p_\omega dw = w\). Inverting the first-order conditions implies

\[
\ln q_\omega(p, w) = d \left( \ln p_\omega - \ln p^*(p, w) \right),
\]

where \(p^*(p, w) \equiv 1/\lambda(p, w)\) and \(d(x) \equiv u'^{-1}(e^x)\). Thus the case of separable utility functions correspond to \(\beta = \gamma = 0\). The mapping between our general demand system and the quadratic and translog cases can be established in a similar manner. We do so in the Appendix. For future reference, simply note that the translog case corresponds to \(\beta = 1\) and \(\gamma = 1\), whereas the quadratic case corresponds to \(\beta = -1\) and \(\gamma = 0\).\(^8\)

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\(^6\)As already mentioned in footnote 1, the quadratic utility function introduced by Ottaviano, Tabuchi, and Thisse (2002) assumes the existence of an “outside good,” which we will only introduce in Section 6.

\(^7\)In this regard, our specification is less general than the Almost Ideal Demand System of Deaton and Muellbauer (1980), though it does not impose any functional form restriction on \(p^*(p, w)\).

\(^8\)As mentioned earlier, the quadratic case can only be studied when an outside good is introduced, which we do formally in Section 6. At this point, we only want to clarify the relationship between the demand for differentiated goods in this paper and in the existing literature.
existing parameter values in the literature, we impose the following restriction.

**A1. [Existing Demand Systems]** $\beta \leq 1$ and $\gamma \leq 1$.

In addition, we impose the following restrictions on $d(\cdot)$.

**A2. [Choke Price]** For all $x \geq 0$, $d(x) = -\infty$.

Assumption A2 implies that the aggregator $p^*(p, w)$ introduced in equation (1) acts as a choke price. This additional assumption implies that, as in Melitz and Ottaviano (2008), there is selection of the most efficient firms into exports even in the absence of fixed exporting costs, which we will exploit below. As already mentioned, this also implies that the case of separable preferences with utility function $u$ satisfying Inada conditions is not covered by our analysis, most notably the CES case.

**A3. [Log-concavity]** For all $x \leq 0$, $d''(x) < 0$.

Assumption A3 is equivalent to the assumption that demand functions are log-concave for all differentiated goods. Though this is a fairly strong restriction, it is satisfied by the demand systems considered in Krugman (1979), Ottaviano, Tabuchi, and Thisse (2002), and Feenstra (2003). For future derivations, it is convenient to write the demand function in a way that makes explicit the symmetry assumption across goods as well as making explicit the way in which the choke–up price $p^*(p, w)$ affects the demand for all goods. Thus, we write $q_\omega(p, w) = q(p_\omega, p^*(p, w), w)$, with

$$\ln q(p_\omega, p^*, w) = -\beta \ln p_\omega + \gamma \ln w + d(\ln p_\omega - \ln p^*).$$  \hspace{1cm} (2)

### 2.2 Firms

Firms compete under monopolistic competition with free entry. There is a large number of ex ante identical firms in each country $i$ that have the option of hiring $F_i > 0$ units of labor to enter the industry. We denote by $N_i$ the measure of firms incurring this fixed entry cost in country $i$. After $w_iF_i$ has been paid, production of any differentiated good is subject to constant returns to scale. As in Melitz (2003), firm-level productivity $z$ is the realization of a random variable $Z_i$ drawn independently across firms from a distribution $G_i$. We assume that $G_i$ is Pareto with the same shape parameter around the world.

**A4. [Pareto]** For all $z \geq b_i$, $G_i(z) = \Pr(Z_i \leq z) = 1 - (b_i/z)^\theta$, with $\theta > \beta - 1$.

As we will demonstrate below, Assumption A4 implies that trade flows satisfy the same gravity equation as in models with CES utility functions. This is attractive for two

\footnote{In the case of separable preferences, this is equivalent to assuming that the elasticity of substitution between differentiated goods is decreasing with the level consumption.}
reasons. First, from an empirical standpoint, gravity models have long been one of the most successful empirical models in economics; see Anderson (2010). Second, from a theoretical standpoint, this implies that the key macro-level restriction affecting the magnitude of the gains from trade in ACR is satisfied, which will facilitate the comparison between the results in the two papers. The restriction $\theta > \beta - 1$ is a technical condition necessary for integrals to be finite in subsequent sections.

International trade is subject to iceberg trade costs $\tau_{ij} \geq 1$, where we normalize $\tau_{ii} = 1$. Thus, for a firm with productivity $z$ in country $i$, the constant cost of delivering one unit of the variety associated with that firm to country $j$ is given by $c_{ij}/z$, where $c_{ij} \equiv w_i\tau_{ij}$. Throughout our analysis, we assume that good markets are perfectly segmented across countries and that parallel trade is prohibited so that firms charge the optimal monopoly price in each market.

3 Trade Equilibrium

In this section we characterize the trade equilibrium for arbitrary values of trade costs. We proceed in three steps. We first study how the demand system introduced in Section 2 shapes firm-level markups. We then describe how firm-level decisions aggregate up to determine bilateral trade flows. We conclude by analyzing how free entry and labor market clearing determine wages and the measure of firms active in each market.

3.1 Firm-level Markups

Consider the optimization problem of a firm producing good $\omega$ in country $i$ and selling it in a certain destination $j$. To simplify notation, and without risk of confusion, we drop indexes and denote by $c \equiv c_{ij}/z$ the constant marginal cost of a firm with productivity $z$ of producing and delivering that good and by $p^*$ and $w$ the choke price and the wage in the destination country, respectively. Under monopolistic competition with segmented good markets and constant returns to scale, the firm chooses its market-specific price $p$ in order to maximize profits in the market

$$\pi(c, p^*, w) = \max_p \left\{ (p - c) q(p, p^*, w) \right\},$$

taking $p^*$ and $w$ as given. The associated first-order condition is

$$\frac{p - c}{p} = -\frac{1}{\frac{\partial \ln q(p, p^*, w)}{\partial \ln p}},$$
which states that monopoly markups are inversely related to the elasticity of demand.

We use \( m \equiv \ln (p/c) \) as our measure of firm-level markups. Marginal cost pricing corresponds to \( m = 0 \). Combining the previous expression with equation (2), we can express \( m \) as the implicit solution of

\[
m - \ln \left( \frac{\beta - d'(m - v)}{\beta - 1 - d'(m - v)} \right) = 0,
\]

where \( v \equiv \ln (p^*/c) \) can be thought of as a market-specific measure of the efficiency of the firm relative to other firms participating in that market, as summarized by the choke price prevailing in that market. Equation (3) will play a crucial role in our analysis. It implies that the choke price \( p^* \) is a sufficient statistic for all general equilibrium effects that may lead a firm to change its price in a particular market.

We assume that for any \( v > 0 \), there exists a unique solution \( \mu(v) \) of equation (3) in \( m \), so that \( m = \mu(v) \). Assumption A3 is a sufficient, but not necessary condition for existence and uniqueness. The properties of the markup function \( \mu(v) \) derives from the properties of \( d(\cdot) \). Since \( \lim_{x \to 0} d(x) = -\infty \) by Assumption A2, we must also have \( \lim_{x \to 0} d'(x) = -\infty \), which implies \( \mu(0) = 0 \). The least-efficient firm in a market has zero markup and marginal cost equal to the choke price in that market. Whether markups are monotonically increasing in productivity depends on the monotonicity of \( d'(\cdot) \). As shown in the Appendix, if demand functions satisfy Assumption A3, then \( \mu'(v) > 0 \) so that more efficient firms charge higher markups.

### 3.2 Aggregate Trade Flows

In any given market, the log of the price charged by a firm with marginal cost \( c \) and relative efficiency \( v \) is equal to \( \ln p(c, v) = \ln c + \mu(v) \), where \( \mu(v) \) is the optimal markup given by equation (3). Given this pricing rule, we can use equation (2) to compute the total sales by a firm with marginal cost \( c \) and relative efficiency \( v \) in a market with wage \( w \) and population \( L \),

\[
x(c, v, w, L) \equiv Lw^\gamma c^{1-\beta} e^{\mu(v)} h(v),
\]

where \( \ln h(v) \equiv -\beta \mu(v) + d(\mu(v) - v) \). Let \( X_{ij} \) denote the total sales by firms from country \( i \) in country \( j \). In equilibrium, we know that only firms with marginal cost \( c \leq p^*_j \) sell in country \( j \). Thus there exists a productivity cut-off \( z^*_ij \equiv w_i \tau_{ij} / p^*_j \) such that a firm from country \( i \) sells in country \( j \) if and only if its productivity \( z \geq z^*_ij \). Accordingly, we can
express bilateral trade flows between country $i$ and $j$ as

$$X_{ij} = N_i \int_{z_{ij}^\frac{\tau_{ij}}{z}}^\infty x(w_i \tau_{ij}/z, \ln z - \ln z_{ij}^* w_j, L_j) dG_i(z).$$

Combining this expression with equation (4) and using our Pareto assumption A4, we get, after simplifications,

$$X_{ij} = N_i L_i w_i^{\gamma_j} \left( p_j^* \right)^{1-\beta+\theta} \theta b_i^{\theta_j} (w_i \tau_{ij})^{-\theta} \int_0^\infty e^{-(1-\beta+\theta)v+\mu(v)} h(v) dv. \quad (5)$$

Note that $\mu(v) \geq 0$ while $h(v)$ does not converge to zero as $v \to \infty$, hence a necessary condition for the above integral to be well defined is that $1 - \beta + \theta > 0$, as pointed out in Section 2 and assumed in A4. Note also that equation (5) implicitly assumes that the lower-bound of the Pareto distribution $b_i$ is small enough so that the firm with minimum productivity $b_i$ always prefer to stay out of the market, $b_i < z_{ij}^*$. This implies that the ‘extensive’ margin of trade is always active in our paper.

For many of our theoretical results, it will be convenient to work with bilateral trade shares rather than bilateral trade flows. We therefore define

$$\lambda_{ij} \equiv \frac{X_{ij}}{\sum_k X_{kj}}.$$ 

as the share of expenditure in country $j$ devoted to goods imported from country $i$. Equation (5) immediately implies

$$\lambda_{ij} = \frac{N_i b_i^{\theta_j} (w_i \tau_{ij})^{-\theta}}{\sum_k N_k b_k^{\theta_j} (w_k \tau_{kj})^{-\theta}}. \quad (6)$$

This expression is what ACR refers to as a strong CES import demand system. This macro-level restriction, which we will simply refer to as a gravity equation in this paper, is satisfied by many quantitative trade models using CES utility functions, such as Krugman (1980), Eaton and Kortum (2002), Anderson and Van Wincoop (2003), and Eaton, Kortum, and Kramarz (2011). In ACR, we have shown that this expression has strong implications for the welfare effects of changes in trade costs under CES utility functions, i.e., if markups are constant. In the next section, we will investigate how such results transpose to environments with variable markups.
3.3 Entry and Wages

To conclude the characterization of the equilibrium, we need to describe how free entry and labor market clearing determine the number of entrants and wages in each country. Given the firm’s pricing rules, one can check that the profits of a firm with marginal cost $c$ and relative efficiency $v$ selling in a market with wage $w$ and population $L$ are given by

$$ \pi(c, v, w, L) \equiv \left( \frac{e^{\mu(v)} - 1}{e^{\mu(v)}} \right) x(c, v, w, L). \quad (7) $$

The relationship between profits and sales is the same as in models of monopolistic competition with CES utility functions. The only difference is that markups are now allowed to vary across firms. Free entry requires that the sum of expected profits across all markets be equal to the entry costs, which can be expressed as

$$ \sum_j \int_{z_{ij}}^{\infty} \pi(w_{ij} \tau_{ij}/z, \ln z - \ln z_{ij}, w_j, L_j) dG(z) = w_i F_i. \quad (8) $$

Labor market clearing and free entry, in turn, require that total sales by firms is equal to the total wage bill,

$$ \sum_j \lambda_{ij} w_j L_j = w_i L_i. \quad (9) $$

The above completes the characterization of a trade equilibrium. Formally, a trade equilibrium corresponds to a measure of entrants, $N_j$, a wage, $w_j$, and a price schedule, $p_{ij}$ for $j = 1, ..., n$ such that (i) prices by firms from $i$ with productivity $z$ in country $j$ satisfy

$$ p_{ij}(z) = \left( w_i \tau_{ij}/z \right) e^{\mu(\ln p_{ij}^*(p_j, w_j) - \ln(w_i \tau_{ij}/z))}, \text{ if } z \geq w_i \tau_{ij}/p_{ij}^*(p_j, w_j), \quad (10) $$

and $p_{ij}(z) \geq w_i \tau_{ij}/z$ if $z < w_i \tau_{ij}/p_{ij}^*(p_j, w_j)$; (ii) the free entry condition (8) is satisfied, with firm-level profits being determined by equations (3), (4), and (7); and (iii) the labor market clearing condition (9) is satisfied, with trade shares given by equation (6). Note that budget constraint in all countries imply that one of these $n$ labor market conditions is redundant.

4 The Elusive Pro-Competitive Effects of Trade

In this section we explore the pro-competitive effects of trade, or lack thereof, in the economic environment described in Sections 2 and 3. We first consider the impact of changes
in trade costs on the distribution of markups and the share of aggregate profits in the economy, which are two natural metrics of the degree of competition in a particular market. We then turn to the welfare implications of changes in trade costs and compare them to those predicted by models with constant markups.

4.1 Distribution of Markups

Let \(M_{ij}(m; \tau)\) denote the distribution of markups set by firms from country \(i\) in country \(j\) in a trade equilibrium if trade costs are equal to \(\tau \equiv \{\tau_{ij}\}\). Since firm-level markups only depend on the relative efficiency of firms, we can express

\[
M_{ij}(m; \tau) = \Pr \{ \mu(v) \leq m \mid v \geq 0 \},
\]

where the distribution of \(v\) depends, in principle, on the identity of both the exporting and the importing country. Recall that \(v \equiv \ln(p^*/c)\) and \(c = c_{ij}/z\). Thus for a firm with productivity \(z\) located in \(i\) and selling in \(j\), we have \(v = \ln p^*_j - \ln c_{ij} + \ln z = \ln z - \ln z^*_i\). Combining this observation with Bayes’ rule, we can rearrange the expression above as

\[
M_{ij}(\mu; \tau) = \frac{\Pr \left\{ \mu(\ln z^*_i - \ln z) \leq m, \ln z^*_i \leq \ln z \right\}}{\Pr \left\{ \ln z^*_i \leq \ln z \right\}}.
\]

Under Assumption A3, \(\mu(\cdot)\) is strictly monotone, and under Assumption A4, firm-level productivity \(z\) is drawn from a Pareto distribution. Thus the previous equation implies

\[
M_{ij}(m; \tau) = \int_{\ln z^*_i}^{\mu^{-1}(m)} \tilde{g}_i(u) du \left( \frac{b_i}{z^*_i} \right)^\theta,
\]

where \(\tilde{g}_i(u) \equiv \theta b_i^\theta e^{-\theta u} \) is the density of \(u \equiv \ln z\). This can be further simplified into

\[
M_{ij}(m; \tau) = 1 - e^{\theta \mu^{-1}(m)}.
\]

Since the function \(\mu(\cdot)\) is identical across countries and independent of \(\tau\), by equation (3), we have established the following result.

**Proposition 1** Suppose that Assumptions A2-A4 hold. Then for any exporter \(i\) and any importer \(j\), the distribution of markups \(M_{ij}(\cdot; \tau)\) is independent of the identity of the exporter \(i\), the identity of the importer \(j\), and the level of trade costs \(\tau\).
A corollary of Proposition 1 is that the overall distribution of markups in any country $j$ is also invariant to changes in trade costs. The fact the distribution of markups does not vary at all with the level of trade costs is clearly functional-form dependent. But the two countervailing forces behind this stark neutrality result are intuitive. Suppose that a change in trade cost leads to an increase in the productivity cut-off $z_{ij}^*$. On the one hand, incumbent firms from country $i$ become relatively less efficient, which leads them to lower their markups by Assumption A3. On the other hand, the least efficient firms from country $i$ exit. By Assumption A3, they were charging the lowest markup in the initial equilibrium. Thus this compositional effect tends to increase markups. Under Assumption A4, these two forces exactly cancel out. In general, which of the two forces dominate would depend on whether the distribution of log-productivity is log-concave or log-convex.\footnote{Assumption A4 corresponds to the case in which the distribution of log-productivity is log-linear. One can show that if the distribution of log-productivity is log-concave, then an increase in country size—which is equivalent to a large (i.e., infinite) decrease in trade costs—lowers the distribution of markups in terms of monotone likelihood ratio dominance. Conversely, if the distribution is log-convex, it decreases it. Details are available upon request.}

### 4.2 Share of Aggregate Profits

A concern with the previous measure of the pro-competitive effects of trade is that it is silent about the correlation between markups and other firm-level variables. In principle, Proposition 1 is consistent with a situation in which the overall distribution of markups is fixed, but the share of aggregate profits goes down with trade liberalization because, say, larger firms now charge lower markups. We now demonstrate that this is not the case.

Let $\Pi_{ij}(\tau)$ denote aggregate profits by firms from country $i$ in country $j$ (gross of fixed entry costs) as a function of trade costs $\tau \equiv \{\tau_{ij}\}$. The same logic used to derive equation (8) implies

$$
\Pi_{ij}(\tau) = N_i \int_{z \geq z_{ij}^*} \pi(w_i \tau_{ij}/z, \ln z_{ij}^* - \ln z, w_j, L_j) dG_i(z).
$$

Using Assumption A3 and equations (4) and (7), we can rearrange this expression as

$$
\Pi_{ij}(\tau) = N_i L_j w_j \left( p_j^* \right)^{1-\beta+\theta} \theta b_{ij}^{\theta} (w_i \tau_{ij})^{-\theta} \int_0^\infty \left( e^{\mu(v)} - 1 \right) e^{-(1-\beta+\theta)v} h(v) dv.
$$

If we denote by $X_{ij}(\tau)$ the total revenues by firms from country $i$ in country $j$ trade costs
\[ \frac{\Pi_{ij}(\tau)}{X_{ij}(\tau)} = \frac{\int_0^\infty \left(e^{\mu(v)} - 1\right) e^{-(1-\beta+\theta)v} h(v) dv}{\int_0^\infty e^{-(1-\beta+\theta)v+\mu(v)} h(v) dv}. \]

Since, as before, the function \( \mu(\cdot) \) is identical across countries and independent of \( \tau \), we have established the following result.

**Proposition 2** Suppose that Assumptions A2-A4 hold. Then for any exporter \( i \) and any importer \( j \), the share of aggregate profits in total revenues \( \Pi_{ij}(\tau)/X_{ij}(\tau) \) is independent of the identity of the exporter \( i \), the identity of the importer \( j \), and the level of trade costs \( \tau \).

Proposition 1 implies that \( \frac{\sum_j \Pi_{ij}(\tau)}{\sum_j X_{ij}(\tau)} \), i.e. the share of total profits in total revenues, is also invariant to changes in trade costs. For future reference, note that this further implies that the measure of entrants \( N_i \) is also independent of the level of trade costs; see Appendix for details.

To summarize, we have established that under standard restrictions on consumers’ demand and the distribution of firms’ productivity, trade models with monopolistic competition and firm-level heterogeneity, as in Melitz (2003), and variable markups, as in Krugman (1979), do not generate pro-competitive effects, either measured by changes in the distribution of markups or the share of aggregate profits in total revenues. While these are two natural measures of the pro-competitive effects of trade, one ultimately cares about welfare, not the distribution of markups or the share of aggregate profits per se. With this in mind, we turn to the welfare consequences of changes in trade costs.

### 4.3 Welfare

In this section, we focus on a small change in trade costs from \( \tau \equiv \{\tau_{ij}\} \) to \( \tau' \equiv \{\tau_{ij} + d\tau_{ij}\} \). In ACR, we have established that under monopolistic competition with Pareto distributions of firm-level productivity and CES utility functions, the compensating variation associated with such a change—i.e., the net revenue of a planner who must compensate a representative agent in country \( j \)—is given by

\[ d \ln W_j = -\frac{d \ln \lambda_{jj}}{\theta}. \]

where the planner’s revenue \( d \ln W_j \) is expressed as a percentage of the income of a representative agent in country \( j \); \( \theta \) is equal to the shape parameter of the Pareto distribution, like in the present paper; and \( d \ln \lambda_{jj} \) is the change in the share of domestic expenditure
on domestic goods caused by the change from \( \tau \) to \( \tau' \). By construction, the compensating variation \( d \ln W_j \) is positive if a change in trade costs leads to more trade, \( d \ln \lambda_{ij} < 0 \), because the planner would have to take money away from the consumer to bring her back to her original utility level. We now investigate how going from CES utility functions to the demand system described in equation (1) affects the above formula.

Let \( e_j \equiv e(p_j, u_j) \) denote the expenditure function of a representative consumer in country \( j \) and let \( u_j \) be the utility level of such a consumer at the initial equilibrium. By Shephard’s lemma, we know that \( de_j/dp_{\omega,j} = q(p_{\omega,j}, p_j^*, w_j) \) for all \( \omega \in \Omega \). Since all price changes associated with a change from \( \tau \) to \( \tau' \) are infinitesimal,\(^{11}\) we can therefore express the associated change in expenditure as

\[
de_j = \int_{\omega \in \Omega} [q(p_{\omega,j}, p_j^*, w_j)dp_{\omega,j}]d\omega,
\]

where \( dp_{\omega,j} \) is the change in the price of good \( \omega \) in country \( j \) caused by the change from \( \tau \) to \( \tau' \). The previous expression can be rearranged in logs as

\[
d \ln e_j = \int_{\omega \in \Omega} \left[ \lambda(p_{\omega,j}, p_j^*, w_j)d \ln p_{\omega,j} \right]d\omega,
\]

where \( \lambda(p_{\omega,j}, p_j^*, w_j) = p_{\omega,j}q(p_{\omega,j}, p_j^*, w_j)/e_j \) is the share of expenditure on good \( \omega \) in country \( j \) in the initial equilibrium. Using equation (4), equation (10), and the fact that firms from country \( i \) only sell in country \( j \) if \( z \geq z_{ij}^* \), we obtain

\[
d \ln e_j = \sum_i \int_{z_{ij}^*}^{\infty} \lambda_i(\ln z) \left[ -\mu'(\ln z - \ln z_{ij}^*)d \ln z_{ij}^* + d \ln(w_1 \tau_{ij}) \right]dG_i(z), \tag{11}
\]

where

\[
\lambda_i(\ln z) \equiv \frac{N_i e^{\mu(\ln z - \ln z_{ij}^*) - (1-\beta)(\ln z - \ln z_j^*)}h(\ln z - \ln z_{ij}^*)}{\sum_k N_k \int_{z_{kj}^*}^{\infty} e^{\mu(\ln z' - \ln z_{kj}^*) - (1-\beta)(\ln z' - \ln z_j^*)}h(\ln z' - \ln z_{kj}^*)dG_i(z')}.
\tag{12}
\]

According to equation (11), the percentage change in expenditure is equal to a weighted sum of the percentage change in prices, with the percentage in prices themselves being the sum of the percentage change in markups, \(-\mu'(\ln z - \ln z_{ij}^*)d \ln z_{ij}^* \), and marginal costs,

\(^{11}\)In principle, price changes may not be infinitesimal because of the creation of “new” goods or the destruction of “old” ones. This may happen for two reasons: (i) a change in the number of entrants \( N \) or (ii) a change in the productivity cut-off \( z^* \). Since the number of entrants is independent of trade costs, as argued above, (i) is never an issue. Since the price of goods at the productivity cut-off is equal to the choke price, (ii) is never an issue either. This would not be true under CES utility functions. In this case, changes in productivity cut-offs are associated with non-infinitesimal changes in prices since goods at the margin go from a finite (selling) price to an (infinite) reservation price, or vice versa.
\( d \ln (w_i \tau_{ij}) \). Combining equations (11) and (12) with Assumption A3 and equation (6), we get, after simplifications,

\[
d \ln e_j = \sum_i \lambda_{ij} d \ln (w_i \tau_{ij}) - \rho \sum_i \lambda_{ij} d \ln z_{ij}^*
\]

where \( \rho \) is a weighted average of the markup elasticities \( \mu'(v) \) across all firms,

\[
\rho = \int_0^\infty \frac{\mu'(v) - (1-\beta)v h(v)}{\int_0^\infty \mu'(v' - (1-\beta)v') h(v')} e^{\theta v} dv',
\]

with weights given by the share of total expenditure on firms with relative efficiency \( v \).

Finally, using the definition of the productivity cut-off \( z_{ij}^* \equiv w_i \tau_{ij} / p_j^* \), we can rearrange the expression above as

\[
d \ln e_j = \underbrace{\sum_i \lambda_{ij} d \ln (w_i \tau_{ij})}_{\text{Change in marginal costs}} + \underbrace{(-\rho) \sum_i \lambda_{ij} d \ln (w_i \tau_{ij})}_{\text{Direct markup effect}} + \underbrace{\rho d \ln p_j^*}_{\text{GE markup effect}}. \tag{13}
\]

To fix ideas, consider a “good” trade shock, \( \sum_i \lambda_{ij} d \ln (w_i \tau_{ij}) < 0 \). If markups were constant, the only effect of such a shock would be given by the first term on the RHS of (13). Here, the fact that firms adjust their markups in response to a trade shock leads to two additional terms. The second term on the RHS of (13) is a direct effect. Ceteris paribus, a decrease in trade costs makes exporting firms relatively more productive, which leads to changes in markups, by equation (3). Interestingly, under Assumption A3, this direct effect tends to lower gains from trade liberalization. The reason is simple. There is incomplete pass-through of changes in marginal costs from firms to consumers: firms that become more productive because of lower trade costs tend to raise their markups \( (\mu'(v) > 0) \), leading to lower welfare gains \( (\rho > 0) \). The third term on the RHS of (13) is a general equilibrium effect. It captures the change in markups caused by changes in the choke price \( p_j^* \). This is the channel emphasized, for instance, by Melitz and Ottaviano (2008). If trade liberalization leads to a decline in the choke price, reflecting a more intense level of competition, then \( \rho > 0 \) implies a decline in markups and higher gains from trade liberalization.

In order to compare the direct and general equilibrium markup effects, we now need to compare the change in marginal costs, \( \sum_i \lambda_{ij} d \ln (w_i \tau_{ij}) \), to the change in the choke price, \( d \ln p_j^* \). We can do so by using the labor market clearing condition (9). Totally differentiating \( \sum_i X_{ij} = w_j L_j \), using equation (5), and setting labor in country \( j \) as the
numeraire, \( w_j = 1 \), we obtain

\[
d \ln p_j^* = \frac{\theta}{1 - \beta + \theta} \sum_i \lambda_{ij} d \ln (w_i \tau_{ij}). \tag{14}
\]

Plugging equation (14) into equation (13), we finally get

\[
d \ln e_j = \left[ 1 - \rho \left( \frac{1 - \beta}{1 - \beta + \theta} \right) \right] \left[ \sum_i \lambda_{ij} d \ln (w_i \tau_{ij}) \right]. \tag{15}
\]

At this point, we can follow the exact same strategy as in ACR. First, we can use the gravity equation (6) to show that \( \sum_i \lambda_{ij} d \ln (w_i \tau_{ij}) \) is equal to \( d \ln \lambda_{jj}/\theta \). Second, we can use the fact that total income is fixed in country \( j \), by free entry and our choice of numeraire, to show that \( d \ln W_j \) is equal to \(-d \ln e_j \). We do so formally in the Appendix. Combining these two observations with equation (15), we obtain our main result.

**Proposition 3** Suppose that Assumptions A1-A4 hold. Then the compensating variation associated with a small trade shock in country \( j \) is given by

\[
d \ln W_j = -(1 - \eta) \frac{d \ln \lambda_{jj}}{\theta}, \text{ with } \eta \equiv \rho \left( \frac{1 - \beta}{1 - \beta + \theta} \right) \geq 0.
\]

According to Proposition 3, the negative direct markup effect dominates the positive general equilibrium markup effect. By Assumptions A1, A3 and A4, \( \eta \) is non-negative. Thus an increase in openness to trade, \( d \ln \lambda_{jj} < 0 \), is associated with welfare gains that are lower than \(-d \ln \lambda_{jj}/\theta \). Since bilateral trade flows satisfy the gravity equation (6) and the measure of entrants is independent of trade costs, one can further check that the value of \( d \ln \lambda_{jj}/\theta \) associated with a trade shock is the same as in ACR; see Appendix. We conclude that under standard restrictions on consumers’ demand and the distribution of firms’ productivity, gains from trade liberalization are weakly lower than those predicted by the models with constant markups considered in ACR.

The fact that the markup elasticity, \( \rho \), matters for welfare in an environment with variable markups is intuitive enough. To understand why the parameter \( \beta \) also plays a crucial role in our welfare analysis, consider the following thought experiment. Suppose that the prices of all goods were to go down by the same percentage, \( d \ln p \). Equation (2) and
budget balance in country \( j \) require that\(^\text{12}\)

\[
\frac{d \ln p_j^*}{d \ln p} = 1 + \frac{(1 - \beta)}{\int_{\omega \in \Omega} d\omega \left( \ln p_\omega - \ln p_j^* \right) \lambda(p_{\omega,j}, p_j^*, w_j) d\omega}.
\]  

(16)

Since \( d' < 0 \), equation (16) establishes that the choke price reacts less than proportionally to a decrease in prices if and only if \( \beta \leq 1 \). Since markups depend positively on the log-difference between \( p_j^* \) and a firms’ marginal costs under Assumption A3, the previous thought experiment suggests that if \( \beta \leq 1 \), “good” productivity shocks, in general, and “good” trade shocks, in particular, should tend to increase markups, and in turn, to lower the welfare gains associated with these shocks. This is what Proposition 3 formally establishes.

The logic of our welfare results—gains from trade liberalization are lower because lower trade costs tend to push markups up—may seem at odds with Proposition 1. After all, we have established that the distribution of markups is invariant to changes in trade costs. The key to reconcile both results is to note that what matters for welfare is not the unweighted average of markups, but instead the average of markups weighted by expenditure shares; see equation (11). In particular, the entry or exit of the least efficient firms—which is crucial for establishing the invariance of the distribution of markups—is irrelevant for welfare since these firms have zero sales. For welfare purposes, only changes in the markups of incumbent firms matter. Under standard restrictions on consumers’ demand and the distribution of firms’ productivity, Proposition 3 demonstrates that these markups tend to go up.

5 Empirical Results

Whether parameter restrictions commonly imposed in the existing literature hold is, ultimately, an empirical question. In ongoing empirical work we use disaggregated U.S. trade data to estimate \( \eta \), and in turn, to quantify the importance of the pro-competitive

\(^{12}\)To see this, note that equation (2) and budget constraint imply

\[
\int_{\omega \in \Omega} e^{d \left( \ln p_\omega - \ln p_j^* \right) w_j \gamma p_\omega^{1-\beta}} d\omega = w_j.
\]

If the price of all goods go down by \( d \ln p \), we therefore get

\[
\int_{\omega \in \Omega} \left[ \left( d' \left( \ln p_\omega - \ln p_j^* \right) + (1 - \beta) \right) d \ln p - d' \left( \ln p_\omega - \ln p_j^* \right) d \ln p_j^* \right] \lambda(p_{\omega,j}, p_j^*, w_j) d\omega = 0,
\]

which implies equation (16) after simple rearrangements.
effects of trade in practice.

6 Robustness

In ongoing theoretical work we explore the robustness of our results to: (i) large changes in trade costs; (ii) alternative market structures; and (iii) outside sectors.

7 Concluding Remarks

We have studied the pro-competitive effects of international trade, or lack thereof, in models with monopolistic competition, firm-level heterogeneity, and variable markups. Under standard restrictions on consumers’ demand and the distribution of firms’ productivity, we have derived two theoretical results. First, although markups vary across firms, the distribution of markups and the share of aggregate profits in revenues are invariant to changes in openness to trade. Second, although the distribution of markups and the share of aggregate profits in revenues are unaffected by trade, gains from trade liberalization are weakly lower than those predicted by the models with constant markups considered in Arkolakis, Costinot, and Rodríguez-Clare (2012).
A Proofs

A.1 Existing Demand Systems

**Linear demand.** A quadratic utility function as in Ottaviano, Tabuchi, and Thisse (2002) implies the following linear demand for differentiated goods:

\[
q(p_\omega, p^*, w) = \frac{\kappa_1}{\kappa_2 + \kappa_3 M(p^*)} - \frac{p_\omega}{\kappa_2} + \frac{\kappa_3}{\kappa_2} \int_{p_\omega \leq p^*} p_\omega d\omega,
\]

where \( M(p^*) \equiv \int_{p_\omega \leq p^*} d\omega \) and \( p^* \) is implicitly defined as the solution of

\[
p^* = \frac{\kappa_1 \kappa_2}{\kappa_2 + \kappa_3 M(p^*)} + \frac{\kappa_3}{\kappa_2 + \kappa_3 M(p^*)} \int_{p_\omega \leq p^*} p_\omega d\omega.
\]

Demand in (17) can be rewritten as

\[
\ln q(p_\omega, p^*, w) = \ln p_\omega - \ln \kappa_2 + \ln (\exp (-[\ln p_\omega - \ln p^*]) - 1).
\]

In terms of the demand system introduced in Section 2, this implies \( \beta = -1, \gamma = 0 \) and \( d(x) = -\ln \kappa_2 + \ln (e^{-x} - 1) \).

**Translog demand.** A translog expenditure function as in Feenstra (2003) implies the following demand for differentiated goods:

\[
q(p_\omega, p^*, w) = \frac{w}{p_\omega} \zeta \ln (p^*/p_\omega),
\]

where \( p^* \) is implicitly defined as the solution of

\[
\ln p^* = \frac{1}{\zeta} + \int_{p_\omega \leq p^*} \ln p_\omega d\omega
\]

\[
M(p^*) \equiv \int_{p_\omega \leq p^*} d\omega.
\]

In terms of the demand system introduced in Section 2, this implies \( \beta = \gamma = 1, \) and \( d(x) = \ln \zeta + \ln(-x) \).
A.2 Monotonicity of Markups

To see this, let \( f(m, v) \equiv m - \ln \left( \frac{\beta - d'(m - v)}{\beta - 1 - d'(m - v)} \right) \). Differentiating with respect to \( \mu \) and \( v \), we obtain

\[
\begin{align*}
    f_m(m, v) &= 1 + \left[ \frac{1}{\beta - d'(m - v)} - \frac{1}{\beta - 1 - d'(m - v)} \right] d''(m - v), \\
    f_v(m, v) &= -\left[ \frac{1}{\beta - d'(m - v)} - \frac{1}{\beta - 1 - d'(m - v)} \right] d''(m - v).
\end{align*}
\]

Note that \( \beta - d'(\mu - v) > \beta - 1 - d'(\mu - v) > 0 \), where the last equality follows from the condition that at firms’ chosen prices the elasticity of demand must be higher than one. But then \( d''(\mu - v) < 0 \) implies \( f\mu(\mu, v) > 0 \) and \( f_v(\mu, v) < 0 \). By the implicit function theorem, equation (3) therefore implies \( \mu'(v) = -f_v(\mu, v)/f\mu(\mu, v) > 0 \).

A.3 Measure of Entrants

Proposition 2 establishes that \( \Pi_{ij}(\tau)/X_{ij}(\tau) \) is a constant that is not dependent on \( i, j, \tau \). We now show that this implies that entry, \( N_i \), is invariant to \( \tau \). Using the definition of \( \Pi_{ij}(\tau) \), the free entry condition (8) can be rewritten as \( \sum_j \Pi_{ij}(\tau)/N_i = w_iF_i \). Letting \( \pi \equiv \frac{\Pi_{ij}(\tau)}{X_{ij}(\tau)} \) then this entails \( \pi \sum_j X_{ij}(\tau) = w_iF_iN_i \). But the labor market clearing condition (9) implies \( \sum_j X_{ij}(\tau) = w_iL_i \), hence we have \( \pi w_iL_i = w_iF_iN_i \) and so \( N_i = \pi L_i/F_i \).

A.4 Proposition 3

In the main text we have established that

\[
d \ln e_j = \left[ 1 - \epsilon \left( \frac{1 - \beta}{1 - \beta + \theta} \right) \right] \left[ \sum_i \lambda_{ij} d \ln (w_i \tau_{ij}) \right].
\]

Equation (6) and \( \sum_i \lambda_{ij} = 1 \) imply

\[
\sum_i \lambda_{ij} d \ln (w_i \tau_{ij}) = \frac{d \ln \lambda_{ij}}{\theta}.
\]

Combining the two previous expressions, we obtain

\[
d \ln e_j = \left[ 1 - \epsilon \left( \frac{1 - \beta}{1 - \beta + \theta} \right) \right] \frac{d \ln \lambda_{ij}}{\theta}.
\]

(18)

By free entry, we know that total income in country \( j \) is equal to \( w_jL_j \). Given our choice of numeraire, \( w_j = 1 \), this implies that the percentage change in income, \( d \ln W_j \), required to compensate a representative consumer in country \( j \) from a small change in trade costs is...
given by
\[ d \ln W_j = -d \ln e_j. \]

Proposition 3 directly derives from this observation and equation (18).

### A.5 Equivalence of Counterfactual Changes in Trade Flows

By equations (6) and (9), we know that
\[
\lambda_{ij} = \frac{N_i b_i^{-\theta} (w_i \tau_{ij})^{-\theta}}{\sum_k N_k b_k^{-\theta} (w_k \tau_{kj})^{-\theta}},
\]
\[
\sum_j \lambda_{ij} w_j L_j = w_i L_i.
\]

Consider a counterfactual change in variable trade costs from \( \tau \equiv \{ \tau_{ij} \} \) to \( \tau' \equiv \{ \tau'_{ij} \} \). Let \( \hat{\sigma} \equiv v'/v \) denote the change in any variable \( v \) between the initial and the counterfactual equilibrium. Since \( N_i \) fixed for all \( i \), using the exact same argument as in the proof of Proposition 2 in ACR, one can show that
\[
\hat{\lambda}_{jj} = \frac{1}{\sum_{i=1}^n \lambda_{ij} (\hat{w}_i \hat{\tau}_{ij})^{-\theta}},
\]
where \( \hat{w}_j = 1 \) by choice of numeraire, and \( \{ \hat{w}_i \}_{i \neq j} \) are implicitly given by the solution of
\[
\hat{w}_i = \sum_{j'=1}^n \frac{\lambda_{ij'} \hat{w}_{j'} Y_i \left( \hat{w}_i \hat{\tau}_{ij'} \right)^{-\theta}}{Y_i \sum_{j'=1}^n \lambda_{ij'} \left( \hat{w}_{j'} \hat{\tau}_{ij'} \right)^{-\theta}}.
\]

By Proposition 2 in ACR, this implies that conditional on trade flows and expenditures in the initial equilibrium and an estimate of the trade elasticity, counterfactual changes in trade flows are the same as in ACR.
References


Goldberg, P., J. D. Loecker, A. Khandelwal, and N. Pavcnik (2012): “Prices, Markups and Trade Reform,” *mimeo*.


